*32nd EPS Conference on Plasma Phys. Tarragona, 27 June - 1 July 2005 ECA Vol.**29C**, P-1.111 (2005)*

Calculation of neoclassical transport in stellarators with finite collisionality using integration along magnetic field lines *

W. Kernbichler[1], S.V. Kasilov[2], G.O. Leitold[1], V.V. Nemov[2]

[1] *Institut für Theoretische Physik - Computational Physics, Technische Universität Graz, Petersgasse 16, A–8010 Graz, Austria*

[2] *Institute of Plasma Physics, National Science Center "Kharkov Institute of Physics and Technology", Akademicheskaya Str. 1, 61108 Kharkov, Ukraine*

Methods of calculation of transport coefficients and equilibrium parallel currents using integration along magnetic field lines [1, 2] have certain advantages, such as high speed, good convergence in low collisionality regimes as well as the possibility of computations for magnetic fields given in magnetic and real space coordinates, in particular, for magnetic fields resulting directly from the Biot-Savart law or from new equilibrium codes such as PIES and HINT. So far these methods were developed for asymptotical collisionality regimes, namely, Pfirsch-Schlüter, plateau and $1/\nu$ regime. Here, the generalization of these methods for arbitrary collisionality regimes is described.

For the Lorentz collision model, the matrix of transport coefficients contains four coefficients - diffusion coefficient, bootstrap coefficient, Ware pinch coefficient and conductivity coefficient. The first pair is obtained from the linearized drift kinetic equation where only the radial gradient of the unperturbed distribution function is retained whereas the parallel electric field is put to zero (gradient drive). For the second pair this is vice versa (electric field drive). In regimes with small poloidal drift, the kinetic equation for both cases can be presented in the following dimensionless form,

$$\sigma \frac{\partial \hat{f}_I^\sigma}{\partial s} - \kappa \frac{\partial}{\partial \eta} \left(\frac{|\lambda| \eta}{\hat{B}} \frac{\partial \hat{f}_I^\sigma}{\partial \eta} \right) = q_I^\sigma, \tag{1}$$

where σ is the sign of parallel velocity, s is the distance counted along the m.f.l., $\eta = (1-\lambda^2)/\hat{B}$ is a normalized perpendicular invariant (magnetic moment), $\hat{B} = B/B_0$ is the magnetic field module normalized to some reference magnetic field B_0, $\lambda = v_\parallel/v$ is pitch, $\kappa = 4/l_c$ with $l_c = v/\nu_\chi$ and ν_χ being mean free path and pitch-angle scattering frequency, respectively. Subscript $I = G, E$ denotes drives by gradient and by parallel electric field, respectively. The normalized distribution function $\hat{f}_I^\sigma$ and the source term q_I^σ in these two cases are

*This work has been carried out within the Association EURATOM-ÖAW and with funding from the Austrian Science Fund (FWF) under contract P16797-N08. The content of the publication is the sole responsibility of its authors and it does not necessarily represent the views of the Commission or its services.

$$\hat{f}_G^\sigma = \left(\frac{v}{\omega_{c0}}\frac{\partial f_M}{\partial \psi}\right)^{-1}\tilde{f}^\sigma, \qquad q_G^\sigma = \frac{\partial}{\partial \eta}\left(\frac{|\lambda|}{\hat{B}}\hat{V}_G\right),$$
$$\hat{f}_E^\sigma = \left(\frac{eE_\parallel}{T\hat{B}}f_M\right)^{-1}\tilde{f}^\sigma, \qquad q_E^\sigma = \sigma\hat{B}, \tag{2}$$

where ψ is some flux surface label, ω_{c0} is the cyclotron frequency for the reference magnetic field B_0, e, T and $E_\parallel \propto \hat{B}$ are charge, temperature and parallel electric field, respectively, and

$$\hat{V}_G = \frac{1}{3}\left(\frac{4}{\hat{B}} - \eta\right)|\nabla\psi| k_G, \tag{3}$$

with k_G being the geodesic curvature [1]. The following integrals along the magnetic field line and in velocity space define transport coefficients,

$$\gamma_{II'}(s_0,s_1) = -\sum_{\sigma=\pm 1}\int_{s_0}^{s_1} \mathrm{d}s \int_0^{1/\hat{B}} \mathrm{d}\eta\, \hat{f}_I^\sigma q_{I'}^{-\sigma}. \tag{4}$$

Here, two normalized quantities are of interest, the ratio of mono-energetic diffusion coefficient D_{mono} to the plateau diffusion coefficient $D_{plateau}$

$$\frac{D_{mono}}{D_{plateau}} = -\frac{2\sqrt{2}}{\pi}\iota R_0 \lim_{L\to\infty}\int_0^L \frac{\mathrm{d}s}{B}\left(\int_0^L \frac{\mathrm{d}s}{B}|\nabla\psi|\right)^{-2}\gamma_{GG}(0,L), \tag{5}$$

where ι is rotational transform angle, R_0 - reference value of big radius, and $D_{plateau} = \pi v^3/(8\sqrt{2}\iota R_0 \omega_{c0}^2)$, and the bootstrap factor [2]

$$\lambda_b \equiv \left(\frac{c}{B_0}\frac{\mathrm{d}p}{\mathrm{d}r}\right)^{-1}\frac{\langle j_\parallel \hat{B}\rangle}{\langle \hat{B}^2\rangle} = -\frac{3}{4}\lim_{L\to\infty}\int_0^L \frac{\mathrm{d}s}{\hat{B}}\left(\int_0^L \mathrm{d}s\hat{B}\right)^{-1}\left(\int_0^L \frac{\mathrm{d}s}{\hat{B}}|\nabla\psi|\right)^{-1}\gamma_{GE}(0,L), \tag{6}$$

where p is plasma pressure and r is plasma radius defined in [1]. In the numerical computations, L must be large enough in order that the m.f.l. covers the magnetic surface rather densely (hundreds of toroidal turns).

For the solution of (1), the method of Green's functions together with an adaptive third order conservative finite-difference discretization scheme over η is used. It should be noted that in the long mean free path regime, which is of main interest, dependence of $\hat{f}_I^\sigma$ on η is highly non-uniform: it is rather steep in boundary layers between different classes of trapped particles (positions of these layers are determined by the magnetic field values at nearest local magnetic field maxima) and it is rather smooth elsewhere. Since local magnetic field maxima are distributed between its global maximum and global minimum, its is not possible to introduce an adaptive grid which resolves all boundary layers in the field line interval required for averaging. To overcome this problem, the field line is split into sub-intervals called “ripples” where a local adaptive grid is introduced in order to resolve only the local boundary layers. Denoting with s^l and s^r left and right boundaries of this ripple, the distribution function of particles leaving the ripple is expressed through the distribution function of particles entering the ripple as follows,

$$\begin{aligned}
\hat{f}_I^+(s^r,\eta) &= \int\limits_0^{1/\hat{B}(s^l)} \mathrm{d}\eta' A^{++}(\eta,\eta')\hat{f}_I^+(s^l,\eta') + \int\limits_0^{1/\hat{B}(s^r)} \mathrm{d}\eta' A^{-+}(\eta,\eta')\hat{f}_I^-(s^r,\eta') + Q_I^+(\eta), \\
\hat{f}_I^-(s^l,\eta) &= \int\limits_0^{1/\hat{B}(s^l)} \mathrm{d}\eta' A^{+-}(\eta,\eta')\hat{f}_I^+(s^l,\eta') + \int\limits_0^{1/\hat{B}(s^r)} \mathrm{d}\eta' A^{--}(\eta,\eta')\hat{f}_I^-(s^r,\eta') + Q_I^-(\eta),
\end{aligned}$$

where $Q_I^\sigma(\eta)$ corresponds to a particular source q_I^σ in (2). Respectively, the averages (4) are given by sums of ripple averages

$$\gamma_{II'}(0,L) = \sum_{ripples} \gamma_{II'}(s^l,s^r), \tag{7}$$

where ripple averages are expressed through the distribution function of incoming particles and contributions to averages of local particle sources, $\gamma_{II'}^{(loc)}$, as follows

$$\gamma_{II'}(s^l,s^r) = \int\limits_0^{1/\hat{B}(s^l)} \mathrm{d}\eta\, g_{I'}^+(\eta)\hat{f}_I^+(s^l,\eta) + \int\limits_0^{1/\hat{B}(s^r)} \mathrm{d}\eta\, g_{I'}^-(\eta)\hat{f}_I^-(s^r,\eta) + \gamma_{II'}^{(loc)}. \tag{8}$$

The set of functions $A^{\sigma\sigma'}$, Q_I^σ, g_I^σ and $\gamma_{II'}^{(loc)}$ which is called a “propagator”, is obtained numerically on the local η-grid which is specific for each ripple. Neighboring propagators are linked with each other by boundary conditions. Namely, at ripple boundaries, the distribution function of outgoing particles from one ripple is the same as the distribution function of incoming particles to the next ripple. Since this distribution function is discretized on different grids in these ripples, a high order conservative rediscretization scheme is used for matching ripples. The resulting discrete matching conditions define group relations of the propagators. With help of these relations, propagators can be combined into joint propagators

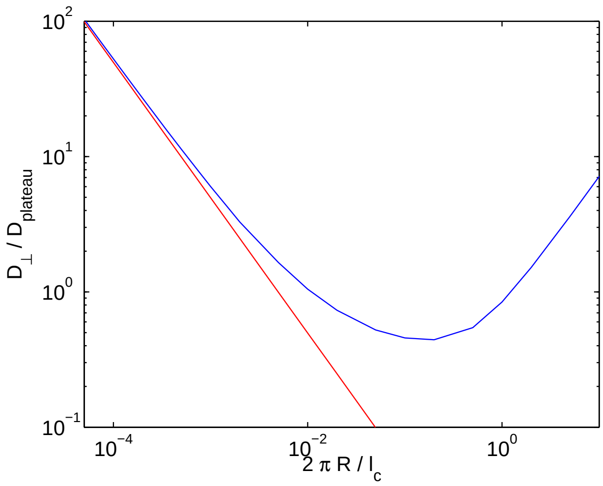

Figure 1: Normalized diffusion coefficient vs collisionality parameter (blue) compared to asymptotical value (red).

(for two and more ripples).

During the numerical solution, each new propagator for a single ripple is joined to a combined propagator for all previous ripples. Averages for this combined propagator become independent on the distribution function of incoming particles when the combined ripple length becomes much larger than the mean free path. In this case, averages $\gamma_{II'}^{(loc)}$ for the combined propagator approximate the desired averages (4). However, in the long mean free path regime the required integration length can be very large. Therefore, within a more efficient procedure, after large enough number of toroidal turns such that the field line re-enters a given small vicinity of the starting point, integration is terminated and the field line is claimed to be closed, as it would be on a rational magnetic surface of high order. With this, left and right boundary of the combined propagator ($s = 0$ and $s = L$, respectively) are joined together which provides final values of all averages.

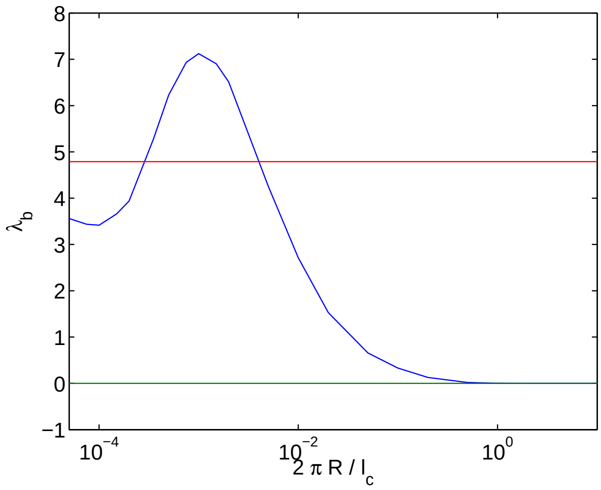

Figure 2: Normalized bootstrap coefficient vs collisionality parameter (blue) compared to asymptotical value (red).

For testing, the real space representation of the magnetic field of Wendelstein-7AS [3] has been used. The mono-energetic diffusion coefficient normalized to the plateau diffusion coefficient, Eq. (5), and bootstrap factor, Eq. (6), are plotted as functions of collisionality parameter, $2\pi R_0/l_c$, in Figures 1 and 2, respectively. For comparison, also plotted are the asymptotical values of these parameters for $1/\nu$-regime. These values have been computed by methods of Refs. [1] and [2], respectively. For the diffusion coefficient the agreement with the asymptotical value is good, as for λ_b, it can be seen that the asymptotical value is not reached within the considered collisionality range. Therefore, collisions are important for bootstrap current even in rather low collisionality regimes [4].

References

[1] V.V. Nemov, S.V. Kasilov, W. Kernbichler and M.F. Heyn, Phys. Plasmas **6**, 4622 (1999)

[2] S.V. Kasilov, V.V. Nemov, W. Kernbichler and M.F. Heyn, *27th EPS Conference on Contr. Fusion and Plasma Phys., Budapest, 12–16 June 2000*, ECA **24B** 668 (2000)

[3] W. Dommaschk, W. Lotz, and J. Nührenberg, Nuclear Fusion **24**, 794 (1984)

[4] H. Maaßberg, private communication