Calculations of $1/\nu$ transport in an $l=3$ stellarator magnetic field in presence of magnetic islands caused by the magnetic system errors

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Introduction

Relatively small errors in making a stellarator magnetic system can decrease a quality of the magnetic configuration. In particular, magnetic islands can arise for the rational values of a rotational transform ι. For example, such a phenomenon had been found for the $l=3$ Uragan-3M (U-3M) torsatron [1]. By numerical calculations and in experiments for certain values of the vertical magnetic field there were found rather big magnetic islands corresponding to $\iota = 1/4$ which can be explained by a small eccentricity of the vertical field coils. In the present paper calculations of the $1/\nu$ neoclassical transport (effective ripple) are carried out under the conditions corresponding to the formation of the above-mentioned magnetic islands. For the comparison the calculations are also made for the case of the negligible eccentricity.

Method of numerical investigation

It is well known that for the $1/\nu$ transport regime the characteristic features of the specific magnetic field geometry manifest themselves in particle and heat fluxes through the factor $\varepsilon_{eff}^{3/2}$, where $\varepsilon_{eff}^{3/2}$ is the so-called effective ripple. For the conventional stellarator field ε_{eff} coincides with the helical ripple ε_h. For an arbitrary stellarator magnetic field, the quantity ε_{eff} in accordance with [2] can be calculated with the help of the following expression:

$$\varepsilon_{eff}^{3/2} = \frac{\pi R_0^2}{8\sqrt{2}} \lim_{L_s \to \infty} \left(\int_0^{L_s} \frac{ds}{B} \right) \left(\int_0^{L_s} \frac{ds}{B} |\nabla\psi| \right)^{-2} \int_{B_{\min}^{abs}/B_0}^{B_{\max}^{abs}/B_0} db' \sum_{j=1}^{j_{\max}} \frac{\hat{H}_j^2}{\hat{I}_j}, \qquad (1)$$

$$\hat{H}_j = \frac{1}{b'} \int_{s_j^{\min}}^{s_j^{\max}} \frac{ds}{B} \sqrt{b' - \frac{B}{B_0}} \left(4\frac{B}{B_0} - \frac{1}{b'} \right) |\nabla\psi| k_G, \qquad \hat{I}_j = \int_{s_j^{\min}}^{s_j^{\max}} \frac{ds}{B} \sqrt{1 - \frac{B}{B_0 b'}}. \qquad (2)$$

Here R_0 is the major radius of the torus, B_0 is a reference magnetic field, ψ is the magnetic surfaces label, $k_G = \left(\vec{h} \times \left(\vec{h} \cdot \nabla\right)\vec{h}\right) \cdot \nabla\psi / |\nabla\psi|$ is the geodesic curvature of a magnetic field line with the unit vector $\vec{h} = \vec{B}/B$. The quantity ε_{eff} is calculated by integration over the magnetic field line length, s, over the sufficiently large interval $0 \div L_s$, and by integration over the perpendicular adiabatic invariant of trapped particles, $J_\perp$, by means of the variable b'. Here, $B_{\min}^{abs}$ and $B_{\max}^{abs}$ are the minimum and maximum values of B within the interval $0 \div L_s$. The quantities $s_j^{\min}$ and $s_j^{\max}$ within the sum over j in (1)-(2) correspond to the turning points of trapped particles.

Note that formulas (1)-(2) must be supplemented with the magnetic field line equations as well as with the equations for the vector $\vec{P} \equiv \nabla\psi$ (see Ref. [3])

$$\frac{dP_i}{ds} = -\frac{\partial B^j}{\partial \xi^i} P_j, \qquad (3)$$

where B^j are the contra-variant components of $\vec{B}$ in real-space coordinates ξ^i, and $P_j = \partial\psi / \partial\xi^j$ are the covariant components of $\vec{P}$.

Magnetic field configuration

Previous numerical investigations [1] of the U-3M magnetic configuration allow us to draw the following conclusions. If the confining magnetic field is unperturbed, and if we assume that there exists a magnetic well and the plasma-confinement region is sufficiently large, then one of the promising modes of operation for U-3M is the mode characterized by $B_\perp / B_0 \approx 1.2\%$, where B_0 is the toroidal magnetic field, $B_\perp$ is the total vertical field generated by the helical and compensating windings, and the direction of $B_\perp$ is such that the magnetic configuration is shifted toward the outer circumference of the torus. However, in this case, the rotational transform ι has a minimum (equal approximately to $\iota \approx 1/4$) at a certain distance from the magnetic axis and increases in two directions: toward the axis of the magnetic configuration and toward its boundary. On the other hand, experimental studies of the magnetic surfaces indicate that, because of the presence of magnetic islands, the plasma-confinement region is not as large as predicated by the above calculations. A comparison of the experimentally observed family of magnetic surfaces with that calculated numerically for the operating mode characterized by $B_\perp / B_0 \approx 1.2\%$ and by small eccentricity $d = 2mm$ of the external compensating coils suggests that the real picture of magnetic surfaces can be attributed, in particular, to this type of asymmetry. Pronounced magnetic islands corresponding to the $\iota = 1/4$ surfaces were observed in experiments carried out under above conditions [1].

These conditions will be used in subsequent calculations, which carried out in a cylindrical coordinate system (ρ, φ, z) whose z-axis coincides with the major axis of the torus for the parameters $d = 2mm$ and $B_\perp / B_0 \approx 1.2\%$. The vacuum magnetic field generated by torsatron helical windings was modeled by finite series of toroidal harmonics expressed in terms of the Legendre associated functions in the form prescribed in Ref.[4]. To calculate the amplitudes of these harmonics, we model each section of the helical windings by 20 filamentary coils divided into five layers each formed by four coils [5]. The magnetic field generated by compensating coils was modeled in the same fashion as in [1].

Figure 1a shows the Poincare plot computed by means of the above model representation for the magnetic field in the $\varphi = 0$ plane, and after a quarter and a half of the helical field period. We can see two main groups of magnetic islands with $\iota = 1/4$, which are located inside and outside of a narrow region of regular magnetic surfaces with $\iota < 1/4$. Tracing the magnetic field lines showed that the outer group of $\iota = 1/4$ magnetic islands is enclosed by a narrow layer consisting of the chains of numerous very small magnetic islands. For comparison, Figure 1b displays the Poincare plot of the magnetic surfaces for unperturbed magnetic configuration ($d = 0$).

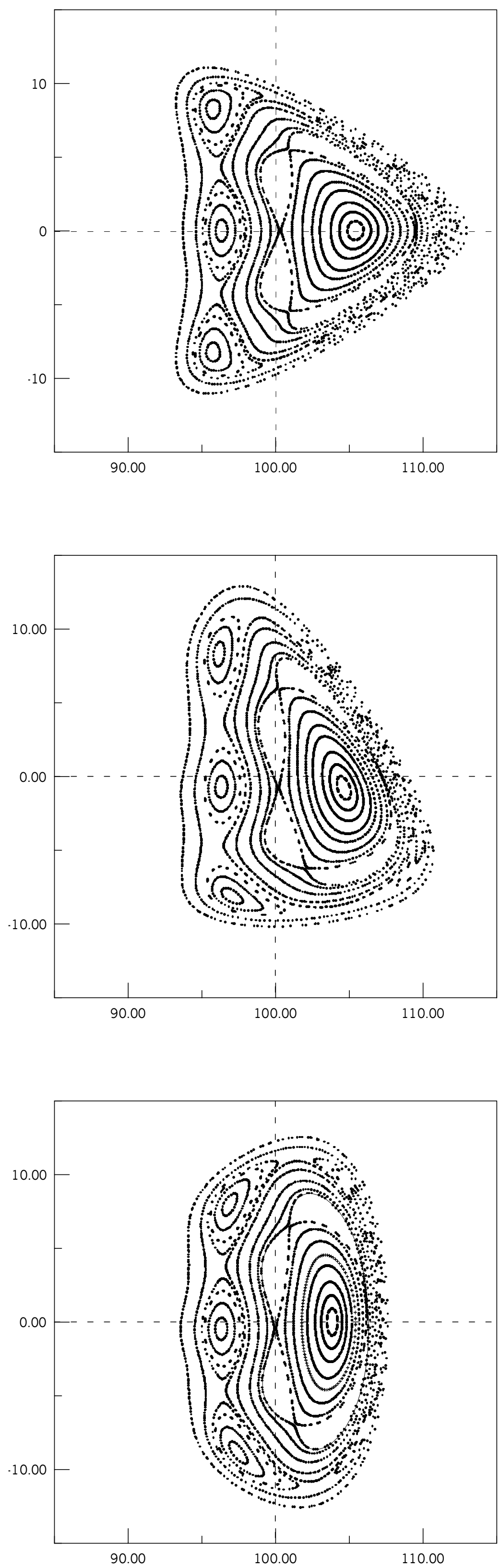

Fig.1a Magnetic surface cross-section for the configuration with small eccentricity ($d = 2mm$) of the compensating coils.

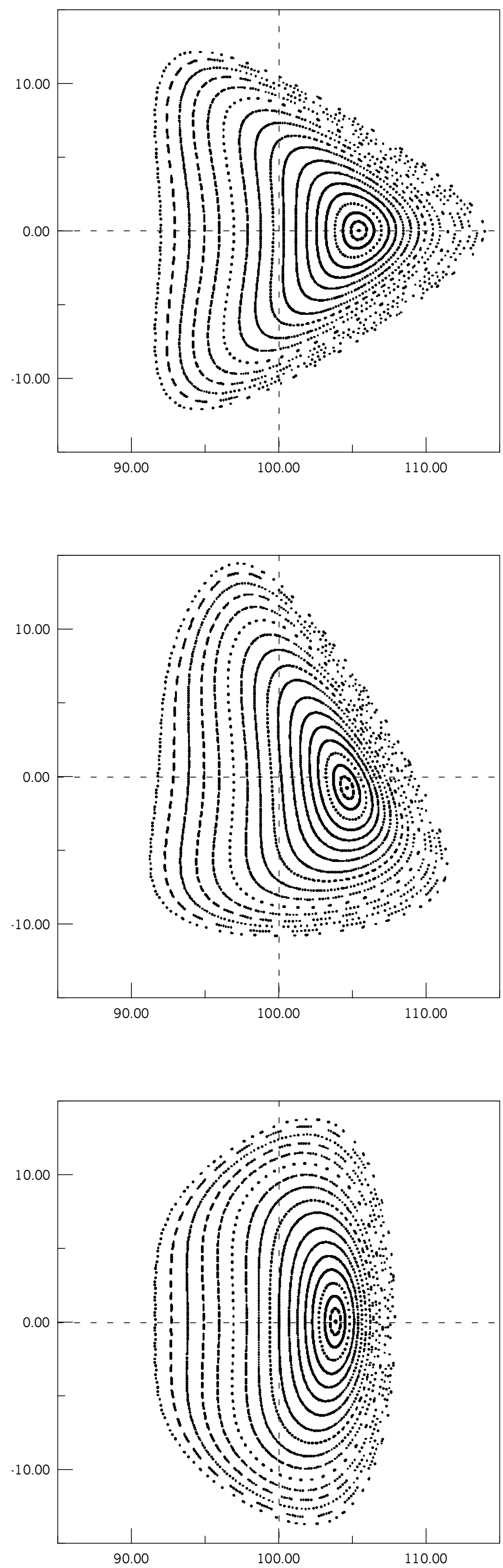

Fig.1b Magnetic surface cross-section for unperturbed magnetic configuration ($d = 0$).

Computational results

Results of $\varepsilon_{eff}^{3/2}$ computations (except of islands) are shown in Figure 2. These results are presented as functions of the ratio r/a where r is the mean radius of a given magnetic surface and a is the mean radius of the outermost magnetic surface for the unperturbed magnetic configuration ($a \approx 9.59 cm$, see Fig.1b). The thick curve in Figure 2 corresponds to the case of the magnetic configuration in presence of magnetic islands, and the thin curve corresponds to the unperturbed magnetic configuration case. From Figure 2 follows that both curves for $\varepsilon_{eff}^{3/2}$ factor practically coincide in those regions of the magnetic configurations where the magnetic island structure is absent. So, for the significant parts of the magnetic configurations corresponding to the $0 < r/a < 0.4$ region effective ripple $\varepsilon_{eff}^{3/2}$ for both configurations is approximately within the limits 0.006÷0.013, and it is within the limits 0.019÷0.025 for the $0.54 < r/a < 0.66$ region. On the other hand, it follows from the obtained results that the configuration with magnetic islands is characterized by increased values of effective ripple in the magnetic island region of the configuration. The value of $\varepsilon_{eff}^{3/2}$ in this case (not shown in Fig.2) increases to 0.03÷0.06 and reaches values commensurable with the $\varepsilon_{eff}^{3/2}$ value near $r/a = 1$. As follows from the computational results for the outer region in Figure 2 for rather large r/a values $\varepsilon_{eff}^{3/2}$ increases to its maximum magnitude $\varepsilon_{eff}^{3/2} \approx 0.06$. So, one finds larger $\varepsilon_{eff}^{3/2}$ values in the magnetic island region of the configuration with small eccentricity of the vertical field coils compared to $\varepsilon_{eff}^{3/2}$ values for the unperturbed magnetic configuration. This fact is resulting from differences in the magnetic field geometry (see Figure 1). Note that the $\varepsilon_{eff}^{3/2}$ value of 0.06 at the edge of the unperturbed configuration is commensurable with that for the edge of the standard CHS configuration [6].

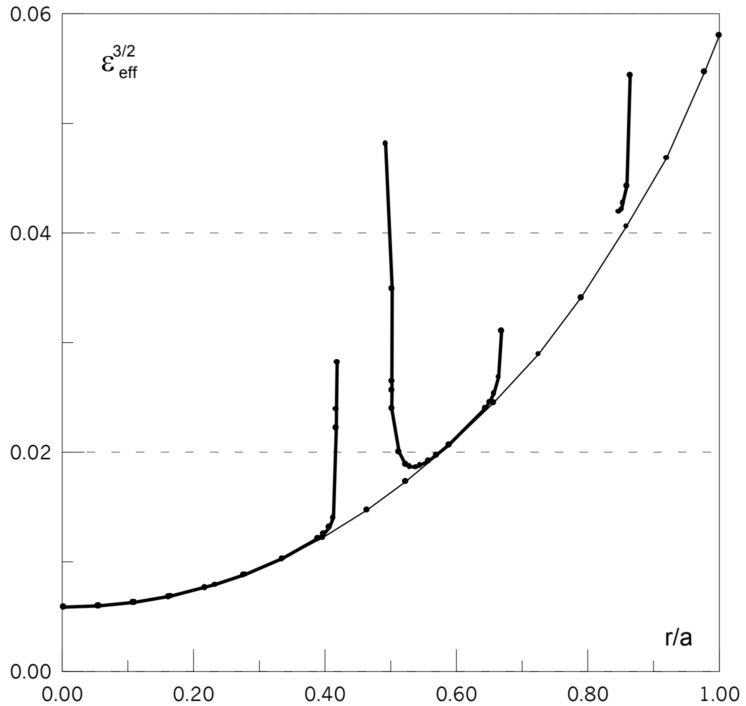

Fig. 2. Results of $\varepsilon_{eff}^{3/2}$ calculation for Uragan-3M torsatron.

Summary

A so-called $1/\nu$ transport regime, in which the transport coefficients are increased with reduction of particle collision frequency ν is considered for $l = 3$ stellarator magnetic field in presence of magnetic islands caused by the magnetic system errors. For calculating of transport coefficients a technique based on integration along magnetic field lines in given stellarator magnetic field is used. The obtained transport coefficients are presented in a standard form containing parameter ε_{eff} (effective ripple) depending on the magnetic field geometry. For the comparison the calculations of the effective ripple are also made for the case of the the unperturbed magnetic configuration. The obtained $\varepsilon_{eff}^{3/2}$ values in the magnetic island region of the configuration with the magnetic system errors are substantially greater than those when the confining magnetic field is unperturbed. Also, due to these errors the confinement region markedly decreases as compared to the unperturbed configuration.

References

[1] G.G.Lesnyakov, E.D.Volkov, A.V.Georgievskij, et al., *Nucl. Fusion* **32**, 2157 (1992).
[2] V.V.Nemov, S.V.Kasilov, W.Kernbichler and M.F.Hein, *Phys. Plasmas* **6**, 4622 (1999).
[3] V.V.Nemov, *Nucl. Fusion* **28**, 1727 (1988).
[4] V.N.Kalyuzhnyj, V.V.Nemov, *Vopr. At. Nauki Tekh., Term. Sintez* **2**, 35 (1985).
[5] V.N.Kalyuzhnyj, V.V.Nemov, V.E.Bykov, et al., *Vopr. At. Nauki Tekh., Term. Sintez* **2**, 33 (1986).
[6] M.F.Hein, M.Isobe, S.V.Kasilov, et al., *Plasma Phys. Control. Fusion* **43**, 1311 (2001).