SU(2) HIGGS BOSON AND VECTOR BOSON MASSES ON THE LATTICE ☆

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Received 4 February 1986

Results are presented for the masses m_H and m_W of isoscalar and isovector states in the lattice SU(2) Higgs model with scalar field in the fundamental representation. The Monte Carlo study is done on a lattice of size $8^3 \times 16$ in the vicinity of three points of the Higgs-phase-transition sheet. The masses show only weak dependence on the quartic self-coupling λ and on the gauge coupling β, but an interesting dependence on the hopping parameter κ. At the phase transition m_H has a sharp dip consistent with critical behaviour, whereas m_W stays above $1/2a$. The mass ratio m_H/m_W becomes larger than one in the Higgs region shortly above the phase transition.

1. Introduction. Numerical simulations on a lattice allow the study of nonperturbative effects in quantum field theory. Recently there have been various investigations of gauge-Higgs systems for non-abelian gauge groups with dynamical radial mode of the Higgs fields [1–5] aiming at the nonperturbative aspects of the Higgs phenomenon. One of the most remarkable nonperturbative properties of Higgs models is the simultaneous occurrence of the Higgs phenomenon and of confinement of fundamental charges in a single phase [6].

Here we discuss the SU(2) lattice Higgs model with scalar field in the fundamental representation of the gauge group. In this model both the basic mechanisms of confinement and of the Higgs phenomenon occurring in gauge theories may be readily studied. This study should shed light on the nonperturbative realization of the electroweak lagrangian in the standard model. Although there the gauge group is SU(2) × U(1), the basic properties of the Higgs phenomenon should be the same, except that we cannot expect a massless photon and that the massive vector boson triplet is degenerate in mass. As usual we assume that fermions play no essential role in the Higgs mechanism and disregard them.

The central question in the lattice formalism is the approach to the continuum theory. This limit has to be taken at a critical point where a correlation length measured in multiples of the lattice constant diverges. This was the motivation in recent work to start with a careful search for such points [3]. The next step is to study physical quantities in the neighbourhood of the critical manifold.

The most interesting quantities are masses of gauge singlet states: isoscalars and isovectors, which are analogues of Higgs and vector bosons. Their masses will be noted by m_H and m_W, respectively. The final aim is to determine the number of relevant couplings at the continuum-limit point and the behaviour of the mass values when this limit is approached.

The SU(2) Higgs model may be formulated on the lattice with a minimal set of three couplings:

$$S = \tfrac{1}{4}\beta \sum_{p\in\Lambda} \mathrm{Tr}(U_p + U_p^+) - \kappa \sum_{x,\mu} (\Phi_x^+ U_{x\mu} \Phi_{x+\mu} + \mathrm{h.c.}) + \lambda \sum_x (\Phi_x^+\Phi_x - 1)^2 + \sum_x \Phi_x^+\Phi_x \,, \qquad (1)$$

with the gauge coupling $\beta = 4/g^2$, the hopping param-

☆ Supported by Deutsche Forschungsgemeinschaft, by Deutsches Bundesministerium für Wissenschaft und Technologie and by Fonds zur Förderung der Wissenschaftlichen Forschung in Österreich, project P5125.

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0370-2693/86/$ 03.50 © Elsevier Science Publishers B.V.
(North-Holland Physics Publishing Division)

eter κ (related to the bare Φ-mass) and the quartic coupling λ. As discussed earlier [1,3] the Higgs fields may be suitably parametrized by a pair of variables (σ, ρ) where σ denotes a 2×2 SU(2) matrix and ρ the real length of the field. The integration measure for a site variable is then $\rho^3\, d\rho\, d\sigma$, where $d\sigma$ is the Haar measure over the group SU(2). In this form the Higgs part of the action may be written

$$S_H = -\kappa \sum_{x,\mu} \rho_x \rho_{x+\mu} \operatorname{Tr}(\sigma_x^+ U_{x\mu} \sigma_{x+\mu}) + \lambda \sum_x (\rho_x^2 - 1)^2 + \sum_x \rho_x^2 . \qquad (2)$$

Other formulations may be transformed into the above form by suitable reparametrizations. One advantage of this form is its symmetry $\kappa \to -\kappa$.

The phase structure of the model (1) can be summarized as follows [1–3]: For positive β in the space of three couplings there is a Higgs phase-transition (PT) sheet which includes the global SU(2) symmetry breaking PT of pure SU(2) Φ^4 theory at $\beta = \infty$. It was found that the Higgs region above this sheet is analytically connected to the confinement region below it for large λ. But for small enough λ and not too large β the Higgs phase transition extends to negative β. For small λ the PT is clearly of first order but it weakens towards larger λ (cf. ref. [3] for further details).

In this work we determine the behaviour of correlation lengths for isoscalars and isovectors across the Higgs PT for three different combinations of (λ, β): (3.0, 2.25), (0.5, 2.25) and (0.5, 2.40). These three points on the two-dimensional Higgs PT sheet were chosen for the following reasons:

(1) The calculation at these three points allows to estimate the dependence of the data both on λ and on β.

(2) Previous investigations showed that the first-order Higgs PT weakens with increasing λ. For $\lambda \simeq 0.5$ the Higgs PT shows signs of critical behaviour. Therefore $\lambda = 0.5$ was chosen for two points.

(3) The value $\lambda = 3.0$ is a compromise between the need to increase λ substantially with respect to $\lambda = 0.5$ and the wish to keep the radial mode active.

(4) The width of the step in β from $\beta = 2.25$ to $\beta = 2.4$ was expected to be sufficiently large to cause a substantial change of lattice spacing. The basis for the estimate was the increase of correlation length by a factor of $\sqrt{2}$ in pure SU(2). (This estimate turned out to be misleading.)

2. *Technical details.* The results presented here have been obtained for lattice size $8^3 \times 16$. The elongated shape was chosen to get a better decay signal for the correlation functions. The gauge group SU(2) has been approximated by its icosahedral subgroup Y (cf. the discussion in ref. [3]).

In addition to the local gauge symmetry

$$\Phi_x \to g_x \Phi_x , \quad U_{x\mu} \to g_x U_{x\mu} g_x^+ , \quad g_x \in \mathrm{SU}(2) , \qquad (3)$$

there is also a global SU(2) isospin symmetry

$$\Phi_x \to \Phi_x \Lambda , \quad \Lambda \in \mathrm{SU}(2) . \qquad (4)$$

The gauge-invariant physical states are multiplets under this symmetry operation.

The isoscalar sector contains a variety of operators suitable for studying states with quantum number $(I)\, J^{PC} = (0)\, 0^{++}$. (Other contributions would come from states with spin higher than three.) They can be used for the calculation of the Higgs boson mass ($\mu = 1, ..., 4$ denote unit vectors):

H_1: $\Phi_x^+ \Phi_x = \rho_x^2$,

H_2: $\sum_{\mu=1}^{3} \Phi_x^+ U_{x\mu} \Phi_{x+\mu} + \text{h.c.}$

$= \sum_{\mu=1}^{3} \rho_x \rho_{x+\mu} \operatorname{Tr}(\sigma_x^+ U_{x\mu} \sigma_{x+\mu})$,

H_3: $\sum_{\mu=1}^{3} \operatorname{Tr}(\sigma_x^+ U_{x\mu} \sigma_{x+\mu})$ (notably for $\lambda = \infty$) ,

H_4: $P_{x,12} + P_{x,23} + P_{x,31}$,
where $P_{x,12}$ is Tr U_p for the plaquette at site x in direction 12,

H_5: $\Phi_x^+ U_{x4} \Phi_{x+4} + \text{h.c.} = \rho_x \rho_{x+4} \operatorname{Tr}(\sigma_x^+ U_{x4} \sigma_{x+4})$
(link in time direction),

H_6: $\operatorname{Tr}(\sigma_x^+ U_{x4} \sigma_{x+4})$,

H_7: $P_{x,14} + P_{x,24} + P_{x,34}$.

In the isovector sector the simplest operators with quantum numbers including (1) 1^{--} are

$$W_1: \quad \rho_x \rho_{x+\mu} \operatorname{Tr}(\sigma_x^+ U_{x\mu} \sigma_{x+\mu} \boldsymbol{\tau}) , \quad \mu = 1, 2, 3 ,$$

$$W_2: \quad \operatorname{Tr}(\sigma_x^+ U_{x\mu} \sigma_{x+\mu} \boldsymbol{\tau}) , \quad \mu = 1, 2, 3 ,$$

$\boldsymbol{\tau}$: Pauli matrices.

In our investigation we considered all these operators except for H_3, H_6 and W_2. (Montvay [4] considered both W_1 and W_2 but concludes that W_1 gives a better signal.) The results for different operators with the same isospin are compatible but the best signal was obtained with W_1 and H_2, which were then used for the final analysis.

The correlation functions were actually determined for each quantity (like e.g. the single link operator in a fixed direction) separately and then combined accordingly. Like in ref. [4] we found no difference in the decay properties for a single link versus the correct sum in operator H_2. This indicates that the states with higher spin have a higher mass and cannot be observed with our resolution. In the 1^{--} channel we summed correlations over space directions of W_1.

Operators suitable for studying (0) 2^{++} states include

$$D_1: \quad 2\Phi_x^+ U_{x1} \Phi_{x+1} - \Phi_x^+ U_{x2} \Phi_{x+2} - \Phi_x^+ U_{x3} \Phi_{x+3}$$

and permutations of 1,2,3 as well as corresponding combinations of plaquettes. We determined correlations for D_1 at some points in the Higgs phase and found them to decay rapidly. This further supports the tacit assumption that states with higher spins which might possibly contribute in any channel are very massive. We also calculated correlations for the isovector scalar operator

$$\rho_x \rho_{x+4} \operatorname{Tr}(\sigma_x^+ U_{x4} \sigma_{x+4} \boldsymbol{\tau})$$

and found no signal for distances greater than zero.

All operators were summed over all space points for given time slice t to project onto the zero-momentum state, i.e.

$$O_i(t) = \sum_x O_i(x, t) . \tag{5}$$

The field configurations were obtained by Monte Carlo techniques in the usual way (cf. ref. [3] for more details). Once the system was in equilibrium we determined the correlation functions

$$C_{ij}(\Delta t) = \sum_t \langle (O_i(t) - \langle O_i(t) \rangle) \times (O_j(t + \Delta t) - \langle O_j(t + \Delta t) \rangle) \rangle \tag{6}$$

by averaging over an ensemble of configurations. The sum over the time coordinates takes into account time-translation invariance and enhances the statistics. The error estimates were obtained by constructing block correlations

$$C_{ij}(\Delta t, b) = \sum_t [(O_i(t) - \langle O_i(t) \rangle) \times (O_j(t + \Delta t) - \langle O_j(t + \Delta t) \rangle)] , \tag{7}$$

where the square brackets denote the average over block b of consecutive configurations. The variance of $C_{ij}(b)$ with regard to C_{ij} provided the estimate of the errors.

For given quantum numbers we expect exponential decay of the connected correlation function, dominated asumptotically by the state with lowest mass. In practice we can resolve only one mass and have to take into account the effects of periodicity of the lattice. For time period N_t we thus parametrize

$$C_{ij}(\Delta t) = \text{const.} \times \{\exp(-am\Delta t) + \exp[-am(N_t - \Delta t)]\} , \tag{8}$$

if both operators i and j support a state with mass m.

3. Monte Carlo results. For all three (λ, β) combinations we have found long time correlations in the Higgs PT region, with a length of up to about 8000 sweeps in some cases. Specific heat peaks are very narrow and large [3]. Their positions and widths correspond to the dips we have found in the Higgs boson mass (table 1).

Table 1
Phase transition regions determined from dips in m_H.

λ	β	κ_{PT}	$\Delta\kappa$
3.0	2.25	0.3610–0.3630	0.0020
0.5	2.25	0.2703–0.2707	0.0004
0.5	2.40	0.2580–0.2595	0.0015

At the phase transition for $(\lambda, \beta) = (0.5, 2.25)$ we have seen an indication of metastable states. The specific heat peak is extremely narrow (see table 1). In addition we recently observed, on a 16^4 lattice, a two-state signal between $\kappa = 0.2706$ and 0.27062 with lifetime of metastable states of the order of 20000 sweeps [5]. This is in agreement with recent results by Langguth and Montvay who observed double peak distributions on a 12^4 lattice for $(\lambda, \beta) = (1.0, 2.3)$ [4]. Thus the phase transition appears to be of first order for these values of λ and β. Nevertheless, for (0.5, 2.25) the correlation length in the Higgs boson channel achieves the size of $5a$, comparable with the size of the lattice we use in this paper. At higher λ and at higher β the phase transition becomes weaker [3] and broader (table 1).

The value of C_{ii} depends sensitively on $\langle O_i(t) \rangle$ which may take several thousand sweeps to adjust itself. Unthermalized $\langle O_i(t) \rangle$ cause correlations to be too strong and their statistical errors to be too small, thus leading to artificially low-mass values. With increasing statistics $C_{ii}(\Delta t)$ drops (improving from smaller to larger distance Δt), at first with an *increasing* statistical error.

Therefore the number of thermalization sweeps at each κ has to be large and the number of sweeps used for measurements should exceed the length of fluctuations in Monte Carlo time by a fairly large factor. Within the limited computer time available (about 380 CPU hours on a two-pipe Cyber 205) we could only partially satisfy these requirements at most κ points. We controlled the results by performing runs with high statistics at some κ values for each (λ, β) pair.

At $(\lambda, \beta) = (0.5, 2.4)$ we used about (13 ... 33) + (21 ... 31) thousand sweeps at most κ values (numbers are for thermalization + measurement). High statistics runs were done at $\kappa = 0.257$, 0.2595, 0.269 and 0.29 with about 50000 and at $\kappa = 0.259$ with 100000 measurement sweeps. For both $(\lambda, \beta) = (0.5, 2.25)$ and (3.0, 2.25) most points represent 2 + (13 ... 32) thousand sweeps, starting from thermalized configurations at nearby values of κ. Long runs with about 5000 + 50000 sweeps were done at $\kappa = 0.2699$, 0.2725, 0.29, 0.30 and at $\kappa = 0.365$, 0.38, respectively. Convergence of masses at $\beta = 2.4$ was found to be much slower than at $\beta = 2.25$. The signal to noise ratio was considerably better in the Higgs than in the confinement region for all correlations.

Data were combined into blocks of 1000 sweeps each for an error estimate as described above. The appearance of long fluctuations inside the PT regions (table 1) means that there the blocks are too short and the errors are probably underestimated. A proper estimate of errors would require a much higher statistics at each point. We preferred to determine an overall picture of the PT by scanning the corresponding κ region with more points over getting more precise results at only a few points.

Self correlations of site operators H_1 and of link operators H_5 give almost identical masses as H_2, well within the (somewhat larger) errors. Plaquette–plaquette correlations, on the other hand, show higher noise levels. Outside the critical region they allow only less reliable fits at small Δt. We also measured some non-diagonal correlations in the (0) 0^{++} channel. They again give the same masses, but with larger noise.

Fig. 1 shows correlations of H_2 and of W_1 versus κ in both channels for $(\lambda, \beta) = (0.5, 2.4)$. For κ well above κ_{PT} (Higgs region) the asymptotic behaviour seems to set in already at $\Delta t = 1$. Fig. 2 displays our results for the masses. To minimize systematical errors, masses are obtained by fits to correlations starting with $\Delta t = 2$ for κ above the values 0.26, 0.269 and 0.36 in figs. 2a, 2b and 2c, respectively. At much lower κ in the confinement region the correlations drop very fast with Δt and stable fits are possible only when $\Delta t = 1$ points are included. To allow a smooth transition to the fits starting with $\Delta t = 2$ we adopt a more elaborate procedure below the mentioned κ values. Here we average mass values from both fits, adding to the statistical errors also systematic errors obtained from the difference of the masses in both fits. The maximal Δt included in any fit is determined by requiring correlations at all lower Δt to lie at least 1.5 standard deviations above zero. In all cases we fit at least up to $\Delta t = 4$. Within the critical region and in part of the Higgs region correlations have small errors up to the maximal $\Delta t = 8$.

Fig. 2 shows the resulting Higgs boson and vector boson masses as functions of κ. The behaviour is qualitatively similar in all three cases when the PT regions are superposed. In the confinement region m_W exceeds m_H, whereas it is smaller than m_H in the Higgs region. The Higgs mass m_H shows a dip in the critical region for all three (λ, β), down to at least $0.3/a$. At (0.5, 2.25) $1/m_H$ reaches $5a$ and thus becomes com-

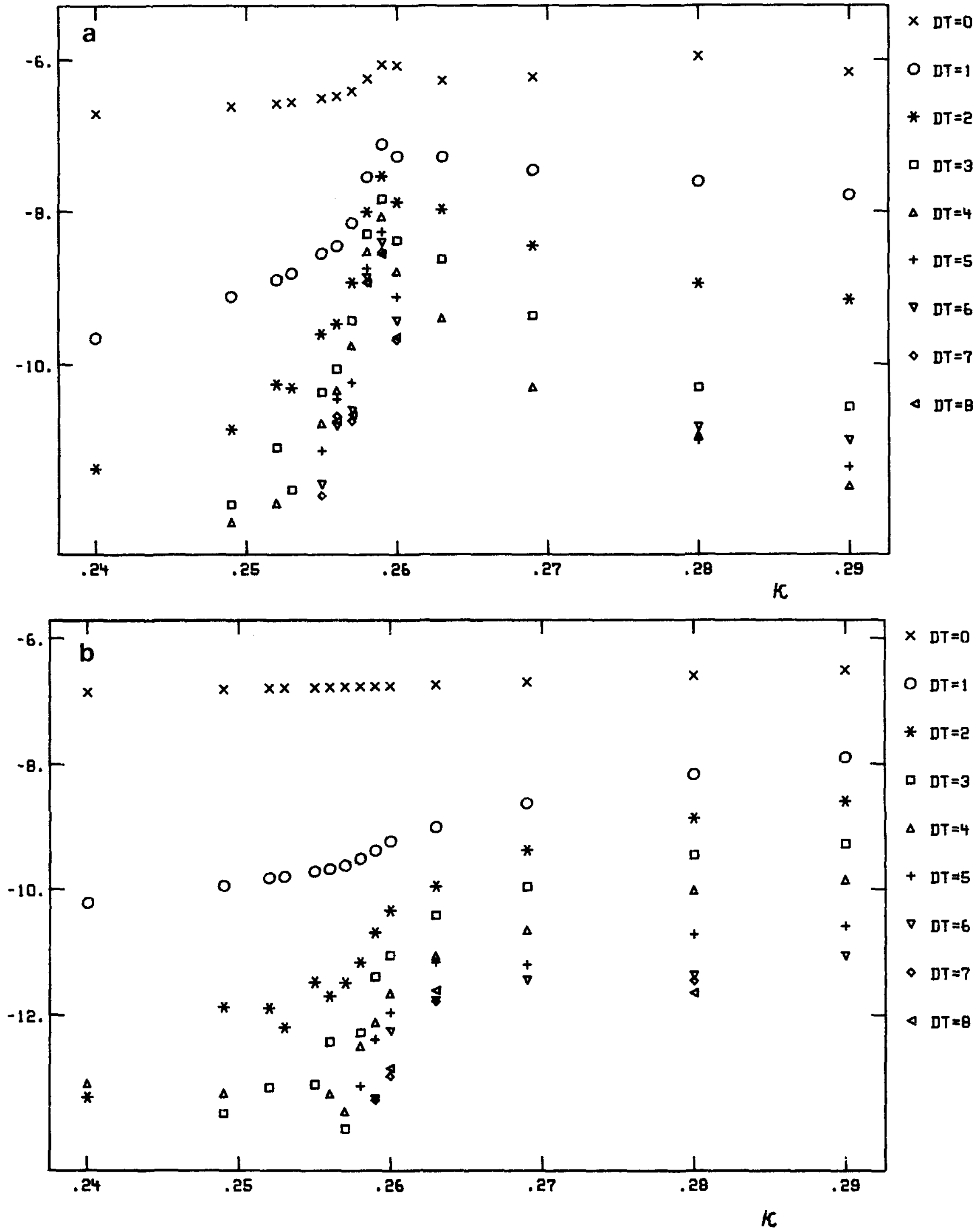
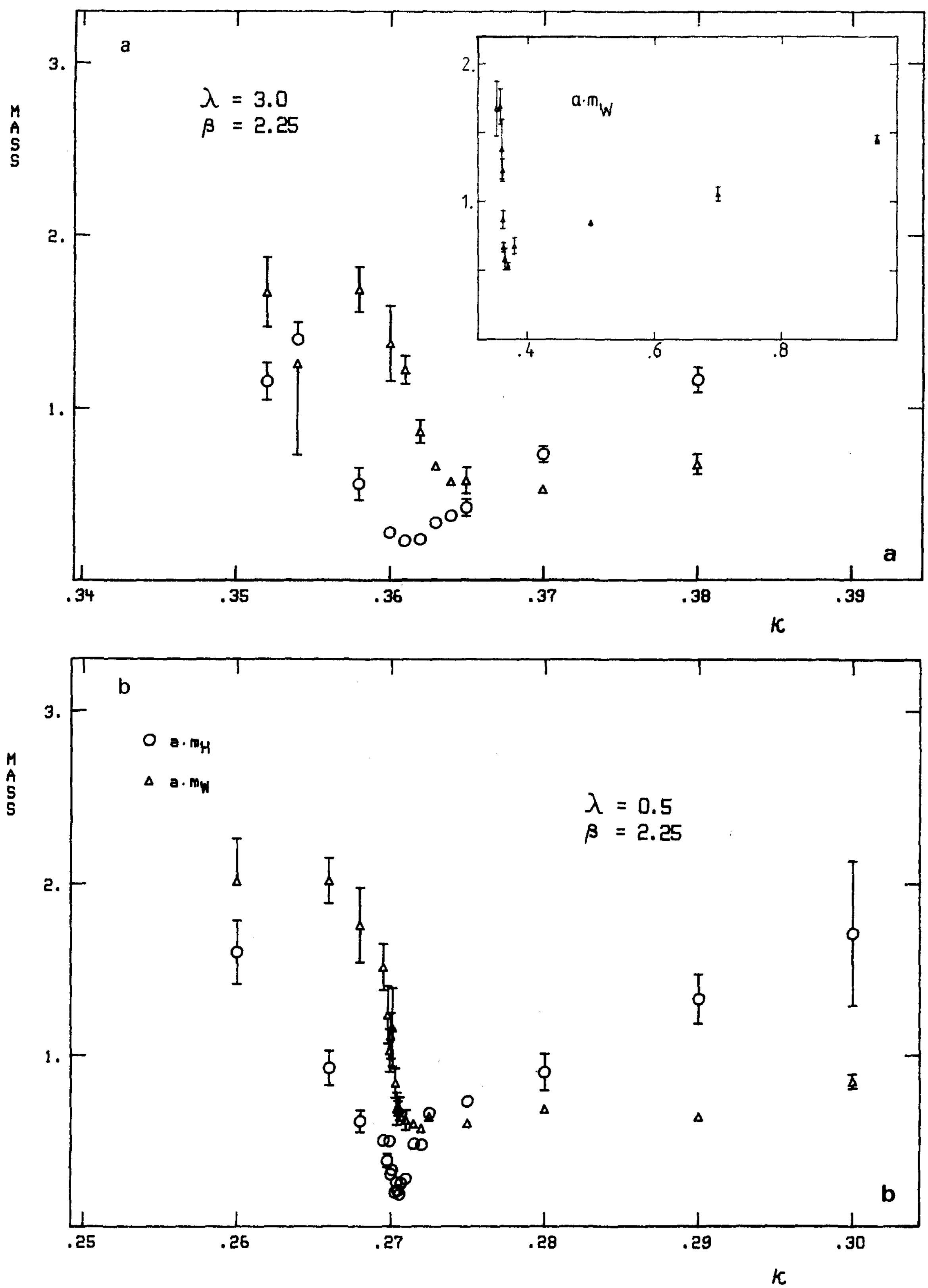


Fig. 1. Logarithms of correlations C_{ii} at $\lambda = 0.5, \beta = 2.4$ for (a) the isoscalar 0^{++} channel (operator H_2), and (b) the isovector 1^{--} channel (operator W_1). Starting from the top, the curves represent correlations at $\Delta t = 0, 1, 2, ..., 8$. Error bars have been omitted here.



a
λ = 3.0
β = 2.25
MASS
3.
2.
1.
a·m_W
2.
1.
.4
.6
.8
.34
.35
.36
.37
.38
.39
κ
b
a·m_H
a·m_W
λ = 0.5
β = 2.25
MASS
3.
2.
1.
.25
.26
.27
.28
.29
.30
κ

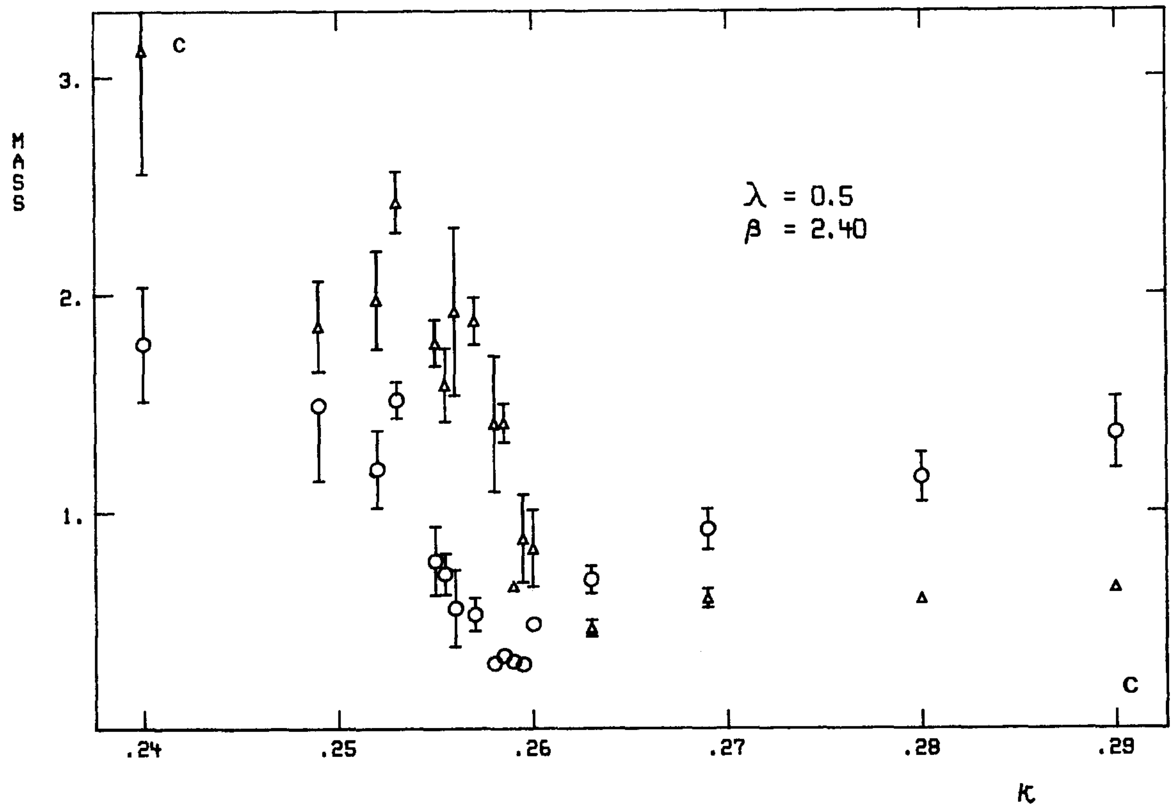


Fig. 2. Isovector (triangles) and isoscalar masses (circles) for (a) $\lambda = 3.0, \beta = 2.25$, (b) $\lambda = 0.5, \beta = 2.25$, and (c) $\lambda = 0.5, \beta = 2.40$.

patible with the size of our lattice. On the other hand m_W does *not* show critical behaviour, though it drops very fast at κ_{PT}. In the Higgs region both m_H and m_W increase steadily, m_W with a small constant slope (see the inset in fig. 2c).

Fig. 3 gives the ratio m_H/m_W for $(\lambda, \beta) = (0.5, 2.4)$, again exhibiting a sharp dip at the transition. Corresponding pictures for (0.5, 2.25) and (3.0, 2.25) are statistically compatible with fig. 3.

4. Conclusions.

(1) Mass curves at different λ are compatible with each other (except for the phase transition regions which have different widths). In scalar Φ^4 theory λ appears to be an irrelevant coupling [7] and it may well be irrelevant in the gauge-Higgs system too [4]. It is remarkable that this might be realized in such a simple way, i.e. that it suffices to renormalize κ for constant β in order to get λ independent results.

(2) Outside the immediate PT region mass curves at different β are compatible with each other, too. In the Higgs region the values of m_W might be slightly lower at $\beta = 2.4$ than at $\beta = 2.25$ at the same distance from the Higgs PT. The very weak β-dependence of the masses on lines parallel to the PT sheet is a considerable surprise when compared with the decrease of the lattice constant by a factor of $\sqrt{2}$ in pure SU(2) for the same change in β. Larger steps in β will be necessary in order to see a substantial β-dependence. The small β-dependence observed may also explain why curves for different λ look similar at constant β.

(3) At the Higgs PT the Higgs boson and vector boson masses behave in a qualitatively different way. The Higgs mass has a dip indicating criticality. On the other hand the vector boson mass does not decrease below the value $0.5/a$, though it drops rapidly from

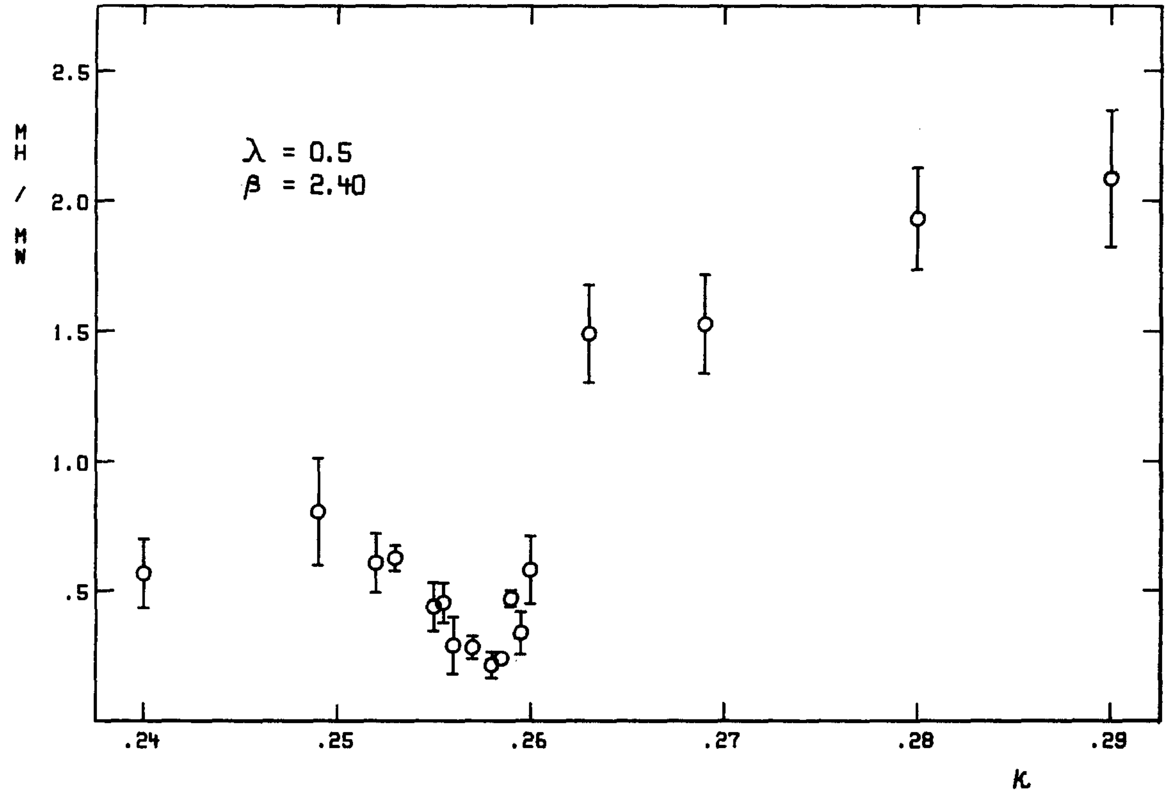


Fig. 3. Ratios m_H/m_W of the masses shown in fig. 2c.

the large values in the confinement region. It might even be discontinuous. The simultaneous occurrence of signals for criticality and for first order discontinuity is usually encountered in the vicinity of tricritical points. It remains an open question whether the Higgs PT is of first order for all β or whether there is a line of tricritical points within the PT sheet behind which the transition is of second order. In the second case the continuum limit taken on the tricritical line would correspond to a theory with one more relevant parameter.

(4) The lattice constant a cancels in the ratio am_H/am_W which can therefore in principle be compared with experiment. Interestingly, this ratio shows a minimum at the PT. Its value lies between 0.2 and 2 in the κ ranges studied. The m_H/m_W curves at all three (λ, β) points are very similar. Thus curves of "constant physics" for all these values of m_H/m_W run approximately parallel to the Higgs PT sheet in the range of couplings we examined, both in the confinement and in the Higgs phase. (The indicated slight decrease of m_W would mean that these lines slowly approach the PT sheet from above with growing β.)

(5) The fact that m_W does not show any sign of criticality means that a continuum limit with finite m_W would not be possible in the region of coupling parameters we have studied. Presumably it will be necessary to increase β in order to find a qualitative different picture.

The calculations have been performed on the CYBER 205 in Bochum and we want to thank the computer centers of the University of Bochum and of the Technische Hochschule Aachen for their support. We also thank G. Vones for many discussions in the early stage of our work.

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