Z. Phys. C – Particles and Fields 45, 597–605 (1990)

Zeitschrift für Physik C Particles and Fields

Screening of the confinement potential in the $SU(2)$ lattice gauge theory with scalar matter*

W. Bock[1,2], H.G. Evertz[3], K. Jansen[2], J. Jersák[1,2], K. Kanaya[4], H.A. Kastrup[1], D.P. Landau[5], T. Neuhaus[6], J.L. Xu[5]

[1] Institut für Theoretische Physik, RWTH Aachen, D-5100 Aachen, Federal Republic of Germany
[2] HLRZ c/o KFA Jülich, P.O. Box 1913, D-5170 Jülich, Federal Republic of Germany
[3] II. Physikaliches Institut, Universität Hamburg, D-2000 Hamburg, Federal Republic of Germany
[4] Institute for Physics, University of Tsukuba, Tsukuba 305, Japan
[5] Center for Simulational Physics, University of Georgia, Athens, GA 30602, USA
[6] Institut für Theoretische Physik, Universität Bielefeld, D-4800 Bielefeld, Federal Republic of Germany

Received 4 September 1989

Abstract. Using the $SU(2)$ Higgs system with dynamical scalar matter fields as a model for analyzing screening properties of the confining potential in gauge theories, we examine the Wilson loop $W(T,R)$ and the gauge invariant 2-point function $G(T,R)$ by Monte Carlo simulations on a 16^4 lattice. For small values of the hopping parameter κ these quantities show non-asymptotic area law behavior which changes to asymptotic perimeter law behavior as κ increases. Close to the Higgs phase transition we find an indication that both these asymptotic and non-asymptotic terms are present simultaneously within the lattice at different distances R and T and that the breaking of the confining flux tube by matter pair production occurs within the lattice. Introducing an appropriate ansatz for $W(T,R)$ and $G(T,R)$, respectively, we determine in this complex situation the string tension, the screening energy of the external sources, and the order parameter introduced by Fredenhagen and Marcu.

1 Introduction

Gluonic fields lead to a confining potential between two heavy external quarks which increases forever with increasing distance [1–4]. However, in the presence of dynamical quarks, pair creation leads to a screening of the external charges and to a flattening of the potential [4, 5]. The distance R_{sc} where the flattening sets in corresponds physically to the breaking of the color flux string between the two initial quarks and to the formation of hadrons. As this string breaking (SB) is a non-perturbative mechanism its study in Monte Carlo simulation is of considerable interest. It has been already investigated in lattice QCD and some simpler models with dynamical fermions [6]. However, due to the technical problems when including dynamical quarks this mechanism has not yet been understood as thoroughly as it is necessary in order to analyze the hadronization process.

As dynamical scalar fields in Higgs models are technically much easier to handle, it is of considerable advantage to study the dynamical screening of the gluonic interquark potential in the $SU(2)$ Higgs model with the matter fields in the fundamental representation. This model is known to be confining [4, 5] and, therefore, can be expected to have similar screening properties as QCD; but allows the possibility of using larger lattices and accumulating higher statistics than in QCD.

In addition the study of the confinement in the Higgs models [7–19] suggests the use of the following additional tool for analyzing the screening mechanisms by dynamical matter fields: In QCD one has relied until now mainly on the behavior of the Wilson and Polyakov loops in order to analyze the flattening of the potential due to the screening by dynamical quarks. In Higgs models one has realized that the gauge invariant 2-point function of the matter fields is also sensitive to the dynamical screening properties of those fields and therefore is very useful for constructing order parameters [7] which differentiate between the free charge and the confinement/screening phase in such models. We point out in this paper that

* Supported by the Deutsches Bundesministerium für Forschung und Technologie and by the Deutsche Forschungsgemeinschaft

$G(T, R)$ is suitable also for the analysis of the SB and therefore deserves more attention in lattice QCD.

There is the following basic problem associated with an attempt to analyze the flattening of the potential and the corresponding *screening length* R_{sc} on a finite lattice of a given length L, e.g. $L = 16$ as in our case: The screening length can only be seen if it is smaller than $L/2$. However, R_{sc} is quite different in different parts of the phase diagram of the $SU(2)$ Higgs model. For $L = 16$ it is considerably larger than $L/2$ in the confinement region and much smaller than $L/2$ in the Higgs or screening region. This property is related to the physical reason that the scalar "quarks" are rather heavy in the confinement region but become light in the screening region. On a lattice with $L = 16$ the onset of the screening, i.e. the string breaking, becomes visible around the Higgs phase transition line. It is here that the change from the area law to the perimeter law behavior of the Wilson loop can be observed.

Previously [10, 11, 14, 15] we have analyzed this transition in terms of a static potential parametrized by the string tension σ as the coefficient of the linear part of the confining potential for $R < R_{sc}$ and a Debye screening mass μ which determines the asymptotic value 2μ of the potential for $R > R_{sc}$. In this paper we employ a more refined ansatz which takes the *area* law behavior of the Wilson loop and of the 2-point function at *short* distances and their *perimeter* law properties at *large* distances into account. This form not only contains the parameters σ and μ, but is also sensitive to the relative weights of the area and perimeter law contributions in the different parts of the phase diagram.

Our Monte Carlo data—which were obtained in the different context of our investigating the order of the phase transition in the $SU(2)$ Higgs model [24]—comprise a large number of points in the phase diagram, but the statistics at individual points is rather limited. This implies that the nature of our present analysis is exploratory and rather qualitative. In particular the error analysis suffers from this restriction. Nevertheless we think that the results show our method to be useful for future high statistics investigations of screening effects in Higgs models and in QCD.

The paper is organized as follows: In the next section we summarize the theoretical expectations concerning the form of the Wilson loop and the 2-point function. In Sect. 3 we introduce the ansatz for each quantity and describe the specifics of the MC calculations, the strategy of the fit procedures and the results. The final section contains the conclusions.

2 Theoretical background

2.1 $SU(2)$ Higgs model

The $SU(2)$ Higgs model, which has been analyzed in substantial detail in a number of MC simulations, is defined by the action

$$S(\kappa) = -\frac{\beta}{4}\sum_{P} \mathrm{Tr}(U_P + U_P^\dagger) - \kappa \sum_x \sum_{\mu=1}^{4} \mathrm{Re}(\mathrm{Tr}\, \Phi_x^\dagger U_{x,\mu} \Phi_{x+\mu}) + \lambda \sum_x \frac{1}{2}\mathrm{Tr}(\Phi_x^\dagger \Phi_x - 1)^2 + \sum_x \frac{1}{2}\mathrm{Tr}(\Phi_x^\dagger \Phi_x). \quad (1)$$

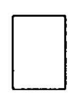
Here $U_{x,\mu}$ and U_P are link and plaquette variables respectively, which are in the fundamental representation of the $SU(2)$ gauge group. The site variables Φ_x can be represented by $\Phi_x = \rho_x \sigma_x$ where σ_x is also in the fundamental representation of $SU(2)$ and ρ_x is a non-negative number, the so-called length of the Φ-field. The model has three coupling constants, namely the gauge field coupling $\beta = 4/g^2$, g being the usual gauge coupling constant in the continuum notation, the "hopping parameter" κ and the quartic self-coupling λ. For $\kappa = 0$ and $\kappa = \infty$ the squared bare mass of the scalar field is $+\infty$ and $-\infty$, respectively. Thus for $\kappa = 0$ the model reduces to the pure $SU(2)$ gauge theory with action $S(0)$ (up to a trivial constant).

The system has only one phase [4, 5, 19–24], the so-called confinement/Higgs phase. Figure 1 shows the phase diagram for the fixed $\lambda = 0.5$ (for this λ-value the properties of the model are very similar to those for $\lambda = \infty$ [20, 21]). There is a Higgs phase transition line with an endpoint *inside* the phase diagram. The region below the phase transition line is called the confinement region, because for $\kappa = 0$ one has the pure gauge theory which is in a confinement phase. The region above the transition line is called the screening or Higgs region, because here the static charges are screened, similarly as in a Debye–Hückel plasma [26]. Both regions are analytically connected [4, 5, 19–24] but—as MC simulations have shown [14, 19, 21, 27, 28]—physical quantities can have quite different numerical behavior in the two regions for a given finite lattice. This applies especially to the size of the screening length R_{sc}.

2.2 *Confinement and screening in gauge theories with matter fields*

For pure lattice gauge theories the existence of a confinement phase in a certain region of the coupling constant space can be demonstrated numerically by means of the asymptotic area law decay for large T and R of the Wilson loop

$$W(R, T) = \left\langle \mathrm{Tr}\left(\prod_{l \in \Gamma_W} U_l \right) \right\rangle, \quad \Gamma_W = \quad (2)$$



T. R

For the static potential

$$V(R) = -\lim_{T \to \infty} \frac{1}{T} \ln W(T, R) \quad (3)$$

between two external gauge charges this means a linear increase, $V(R) \simeq \sigma R$, for sufficiently large R. For example, in the pure $SU(2)$ lattice gauge theory the accumulated numerical evidence for this behavior is impressive [1–3]. If the gauge fields are coupled to matter fields in the fundamental representation, then the SB occurs, leading to an asymptotic perimeter decay of the Wilson loop (2) even in the confinement phase [4, 5].

Thus the Wilson loop alone can no longer serve as an order parameter to distinguish between phases of free or confined gauge charges. However, combining the asymptotic properties of the Wilson loop (2) with those of the gauge invariant 2-point function

$$G(R,T) = \left\langle \mathrm{Tr}\, \Phi_x^\dagger \left(\prod_{l \in \Gamma_G} U_l \right) \Phi_y \right\rangle, \quad \Gamma_G = \;[\text{path from } x \text{ to } y:\ \text{extent } R \text{ and } T], \tag{4}$$

where Φ_x denotes the dynamical matter field, one can construct new order parameters [6–10] which are nonvanishing in a confinement phase and zero in a free charge phase. The parameter proposed by Fredenhagen and Marcu (FM) [7, 8] is

$$\rho_{\mathrm{FM}}^{\infty} = \lim_{T \to \infty} \rho_{\mathrm{FM}}(T,R), \quad R \geqq \text{const.} \times T, \tag{5}$$

where

$$\rho_{\mathrm{FM}}(T,R) = \frac{G(T,R/2)}{W(T,R)^{1/2}}. \tag{6}$$

The parameter $\rho_{\mathrm{FM}}^{\infty}$ is nonzero in a confinement phase and vanishes in a free charge phase [11]. This has been demonstrated analytically [6–9] and numerically [9–18] for several lattice Higgs models. Determining the behavior of both the Wilson loop (2) and of the 2-point function (4) is, therefore, of considerable importance for the analysis of the screening phenomena. The properties of the screening mechanism in different parts of the phase diagram are reflected by the behavior of $W(T,R)$ and $G(T,R)$ at different distances T and R. The following properties of these observables should be valid for $\beta = 2 - 3$, i.e. in the β-region where both the string tension is measurable in pure $SU(2)$ lattice gauge theory [2–4] and where the Higgs phase transition is observed [20, 21].

2.3 Theoretical expectations for the Wilson loop

For any $\kappa > 0$, the Wilson loop $W(T,R)$ decreases asymptotically for T, $R \to \infty$ according to the perimeter law [9]

$$W(T,R) \propto \exp(-\mu 2(R+T)), \tag{7}$$

where μ is the energy of fields around one external charge. As the model (1) has only one, namely the confinement/screening phase [4, 5, 19–24], this external charge must be screened by the matter fields. Therefore μ is called the *screening energy*. In the Higgs region $W(T,R)$ exhibits the behavior (7) at R and T larger than $1/m_W$, the inverse mass of the gauge boson, which is substantially smaller than the lattice size except close to the Higgs phase transition.

For small κ in the confining region the screening energy μ is large as it includes the bare mass of the dynamical field which diverges at $\kappa = 0$. In the lowest order of the κ expansion of $W(T,R)$ one can easily derive for $\lambda = \infty$ in analogy to the calculation in [5], that

$$\mu \simeq \ln \frac{2}{\kappa}. \tag{8}$$

A large value of μ implies that the perimeter law (7) starts to dominate only at some large distances when the area law terms in $W(T,R)$ are suppressed (see inequality (10) below) and R_{sc} is very large. At the distances smaller than R_{sc} the Wilson loop is expected to be only slightly affected by the matter fields and to have qualitatively the same R- and T-dependence as in the pure $SU(2)$ gauge theory. This can be seen again from the low orders of the κ expansion. Thus it is appropriate to parametrize $W(T,R)$ in a way similar to the pure gauge theory. For R and T large enough, so that the short range (Coulombic) behavior is no longer seen, this amounts to the ansatz

$$W(T,R) \propto \exp(-\sigma TR - E_{\mathrm{ext}} 2(R+T)), \tag{9}$$

where σ is the *string tension* and E_{ext} is the *self-energy of the (confined) external charge*. Comparing the expressions (7) and (9) we see that the perimeter decay dominates for T and R satisfying roughly

$$\sigma TR > (\mu - E_{\mathrm{ext}}) 2(R+T). \tag{10}$$

For $T \gg R$ we obtain an estimate for the *screening length*, R_{sc}, at which the area law behavior changes into the perimeter law behavior:

$$R_{sc} \simeq \frac{2(\mu - E_{\mathrm{ext}})}{\sigma}. \tag{11}$$

This estimate for R_{sc} is reasonable only as long as the string tension σ can be properly defined, which is not the case for large κ.

These expectations can be also expressed in terms of the static potential $V(R)$, whose short distance properties can be taken into account, too. In the Higgs region $V(R)$ is of Yukawa type, whereas in the confinement region the form of $V(R)$ is quite complex: with increasing R the short distance Coulombic form changes into a linear form up to $R \simeq R_{sc}$ and then turns into a constant at even larger distances.

For a given finite lattice size we call the region in the coupling parameter space, where the area law behavior changes to the perimeter law one at a distance smaller than $L/2$, a "string breaking" or *SB region.* We denote its approximate boundaries by κ_A and κ_P with $\kappa_P > \kappa_A$.

2.4. Theoretical expectations for the gauge invariant 2-point function $G(T, R)$

$G(T, R)$ also decreases asymptotically for $T, R \to \infty$ at any $\kappa > 0$ according to the perimeter law [6–8]

$$G(T, R) \propto \exp(-\mu(T + 2R)). \qquad (12)$$

In the model we are investigating μ is the same screening energy that also determines the perimeter decrease of the Wilson loop (7). The asymptotic behavior (5) of $\rho_{FM}(T, R)$ follows from the relations (7) and (12). Again taking the hopping parameter expansion as a guide, we expect in analogy to (9) the following behavior outside the short distance region:

$$G(T, R) \propto \exp(-\sigma TR - E_{ext}(T + 2R) - \varepsilon_c T). \qquad (13)$$

The parameter ε_c is to be interpreted as the sum of the bare mass and the self-energy of the confined scalar particle. The self-energy is due to the interaction with the gauge field and with the scalar field. In analogy to (8) one obtains in the lowest order of the κ-expansion $\varepsilon_c = \ln(2/\kappa) + E_{ext}$.

For small κ, $G(T, R)$ is dominated by the area law (13) up to some large but finite distances T and R. Only for even larger distances does the perimeter law (12) start to dominate: As μ and ε_c diverge for $\kappa \to 0$ (see (8)), at small κ the perimeter law (12) is suppressed with respect to the area law (13) by the factor $\exp(-\mu 2R)$ as long as $\exp(-\sigma TR)$ is not too small for large T and R. Apparently it is difficult to estimate the relative importance of the area and perimeter contributions, (13) and (12), a priori at finite T and R. We shall obtain some information from the analysis of our data, however.

3 Analysis of the string breaking by means of the superposition formulae

3.1 Superposition formulae

Apart from a traditional analysis of our data for the Wilson loops by means of some formula for the static potential [10] we have looked for an analytic form directly for the Wilson loops. As we want to take into account the perimeter law decay and the area law decay of $W(T, R)$ simultaneously, we have chosen a superposition of two terms with the corresponding decay properties. Combining this with a simple ansatz for the short distance behavior we arrive at the following *superposition formula for* $W(T, R)$:

$$\begin{aligned} W(T, R) &= \{\bar{W}_P \exp[-\mu 2(T + R)] \\ &\quad + \bar{W}_A \exp[-\sigma TR - E_{ext} 2(T + R)]\} \\ &\quad \times \exp\left[\frac{3}{4}\alpha\left(\frac{T}{\hat{R}}\exp(-m_Y R) + \frac{R}{\hat{T}}\exp(-m_Y T)\right)\right] \\ &\equiv W_P(T, R) + W_A(T, R). \end{aligned} \qquad (14)$$

The first term is the perimeter law term which is dominant for large distances, and the second one is the area law term which might dominante at intermediate distances. The common Yukawa potential factor represents the short distance properties of $W(T, R)$. As the perimeter and area law terms are symmetric with respect to an interchange of T and R, and as both variables T and R are equally important, we have symmetrized also the short distance part. The list of the 7 parameters defining this formula is:

μ: screening energy of an external charge for $\kappa > \kappa_P$
E_{ext}: selfenergy of an external charge for $\kappa < \kappa_A$
σ: string tension for $\kappa < \kappa_A$
α: renormalized fine structure constant
m_Y: Yukawa mass
$\bar{W}_P$: coefficient of the perimeter law term
$\bar{W}_A$: coefficient of the area law term.

We have indicated the standard physical interpretation of the parameters μ, E_{ext} and σ within the corresponding regions. An extension of this interpretation to the SB region will be discussed later. The ansatz (14) is meant to be valid for $W(T, R)$ for finite T and R, and not intended for determining $V(R)$ in the limit $T \to \infty$. Such an extrapolation from the data obtained on a finite lattice would be too unreliable, as a small T-dependence of $\ln W(T, R)/T$ cannot be excluded.

The symbols $1/\hat{R}$ and $1/\hat{T}$ denote the lattice Coulomb potential. We do not use the complete lattice Yukawa potential since it depends on m_Y and would have to be recalculated numerically many times during the fits with m_Y being a free parameter.

Our ansatz for $G(T, R)$ as a superposition of a perimeter (12) and an area (13) law term is analogous to that for the Wilson loops. The *superposition formula for* $G(T, R)$ is

$$\begin{aligned} G(T, R) &= \{\bar{G}_P \exp[-\mu(T + 2R)] \\ &\quad + \bar{G}_A \exp[-\sigma TR - E_{ext}(T + 2R) - \varepsilon_c T]\} \\ &\quad \times \exp\left[\frac{3}{4}\alpha\frac{R}{\hat{T}}\exp(-m_Y T)\right] \\ &\equiv G_P(T, R) + G_A(T, R). \end{aligned} \qquad (15)$$

Most parameters appear already in the formula (14) for $W(T, R)$. The new parameters are

ε_c: bare mass + selfenergy of the constituent particle for $\kappa < \kappa_A$
$\bar{G}_P$: coefficient of the perimeter law term
$\bar{G}_A$: coefficient of the area law term.

The formula (15) reproduces again the expected behavior both at asymptotic and at intermediate distances. The choice of the short range term has some ambiguities [31]. We have kept it as simple as possible.

3.2 Specifics about the Monte Carlo calculations

Our data have been obtained during the investigation

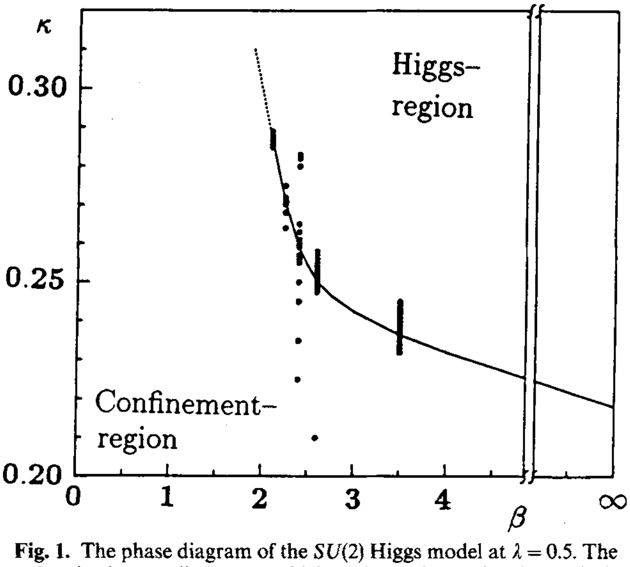


Fig. 1. The phase diagram of the $SU(2)$ Higgs model at $\lambda = 0.5$. The points in the coupling space which we have chosen for the analysis cluster around the phase transition line and are represented in this figure by large dots. The positions of the phase transition points are listed in Table 1 for $\beta = 2.1$, 2.25, 2.4, 2.6 and 3.5. The critical end point lies on the dotted line but its exact location is not yet known [24]

of the nature of the Higgs phase transition at $\lambda = 0.5$ [24]. For this purpose it was necessary to perform the calculations at many points (about 80) clustered in a narrow interval around the phase transition for several different values of β. These points are indicated in Fig. 1.

The simulations were carried out on the 16^4 lattice by means of a vectorized code on a CYBER 205 computer using a Metropolis algorithm. While most of our runs were performed with a one link update, we also performed runs with 3 hits, allowing for an improved acceptance rate. We simulated the icosahedral finite subgroup of $SU(2)$ which is a very good approximation for the couplings we have considered [21]. Typically we discarded 10,000 sweeps at the beginning, allowing for equilibration, and then retained up to 10^5 sweeps close to the phase transition, while far away 10,000–30,000 sweeps were available for measurement. The total computer time used was about 1000 CPU hours.

A large number of points allowed us to obtain information about the properties of $W(T, R)$ and $G(T, R)$ in various regions of the coupling space. At individual points the statistics was correspondingly limited, however.

3.3 Strategy of the fit procedures

We fit $W(T, R)$ and $G(T, R)$ *simulateneously*. Thus the total number of free parameters is 10. Even with some necessary cuts for small R and T, the number of data points to be fitted, allowed by the restriction $T, R \leq 8$ on our 16^4 lattice, is about 90. The ratio of the number of data points to the number of free parameters is therefore quite high. On the other hand, we may expect

Table 1. Positions of the phase transitions [24] and the boundaries of the SB region for $\lambda = 0.5$

β	κ_{PT}	κ_A	κ_P
2.1	0.28675 ± 0.00010	0.284 ± 0.002	0.291 ± 0.002
2.25	0.27065 ± 0.00005	0.267 ± 0.002	0.274 ± 0.002
2.4	0.25900 ± 0.00020	0.256 ± 0.002	0.263 ± 0.002
2.6	0.25060 ± 0.00030	0.247 ± 0.002	0.254 ± 0.002
3.5	0.23650 ± 0.00050	—	—

that the data for different R and T are not completely statistically independent, as they are obtained from the same ensemble of configurations. This may effect the errors of the fit parameters.

The χ^2 function is a quadratic form of the 4 fit parameters $\bar{W}_P$, $\bar{W}_A$, $\bar{G}_P$ and $\bar{G}_A$. Therefore it is possible to determine the minimum with respect to these 4 quantities analytically. Doing this one gets an expression for χ^2, which depends only on the remaining 6 parameters, the data and their statistical errors. Thus for the actual MINUIT procedure a tolerable number of 6 free parameters remains. The values of the fit parameters are quite stable with respect to various changes of the fit procedures and so we believe that they are at least semiquantitatively reliable. However, the error bars produced by the MINUIT program sometimes seem to be unrealistic. In the case of complex fits by means of the superposition formulae the statistics at individual points was not sufficient for performing a block analysis of the errors. The best estimate of the errors can be obtained from the variation of the values of the parameters at close κ-points.

The most important result of the superposition fits is the determination of the κ-values for which both perimeter and area law terms in the superposition formulae are *approximately of the same importance* for the description of the data. This we take as a *signal that the system is in the SB region.* The relative importance of the perimeter and area law terms changes with T and R and it is necessary to devise some "global" comparison of both terms. There are numerous ways to do that, and several methods we have tried give consistent results.

Of course, the fits always produce some values of the parameters for the perimeter and area terms even in the cases when these terms are unimportant, i.e. for $\kappa < \kappa_A$ and $\kappa > \kappa_P$. *These values of the parameters are not to be believed.* The reason is that the non-dominant terms must be expected to simulate systematic uncertainties in the ansatz for the dominant term, which are unknown. On the other hand, we expect that the physical interpretation of various parameters outside the SB region can be extended into the SB region.

3.4 Results

The formulae describe the data in the SB region very

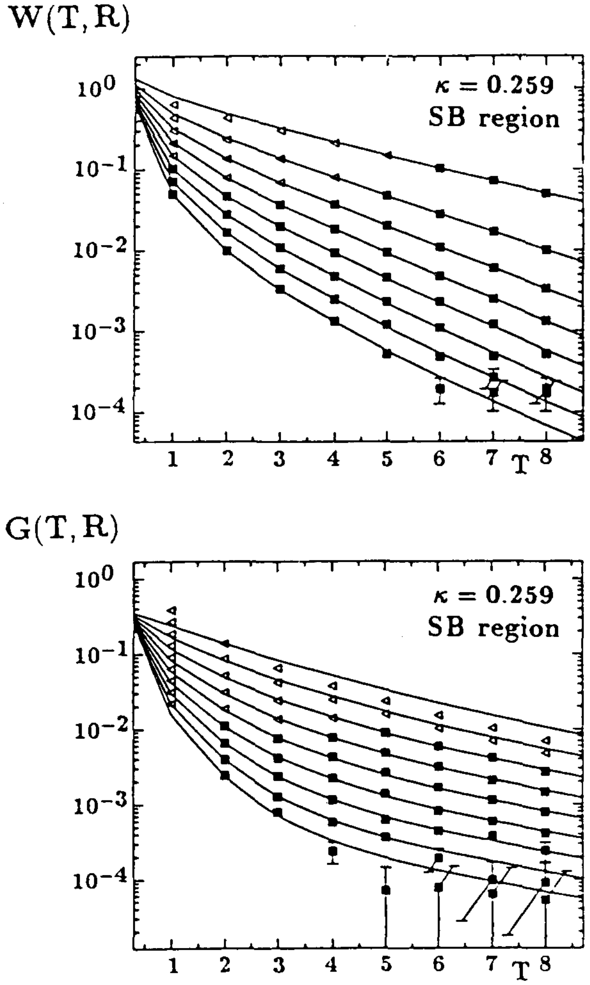


Fig. 2. Data for $W(T,R)$ and $G(T,R)$ for $\beta = 2.4$ fitted by the superposition formulae (14) and (15) in the SB region. The open symbols denote the data which were exluded from the fit procedure by the choice of the cuts. Different lines correspond to different R. The upper line corresponds to $R = 1$ for a $W(T,R)$ and $R = 0$ for $G(T,R)$, etc.

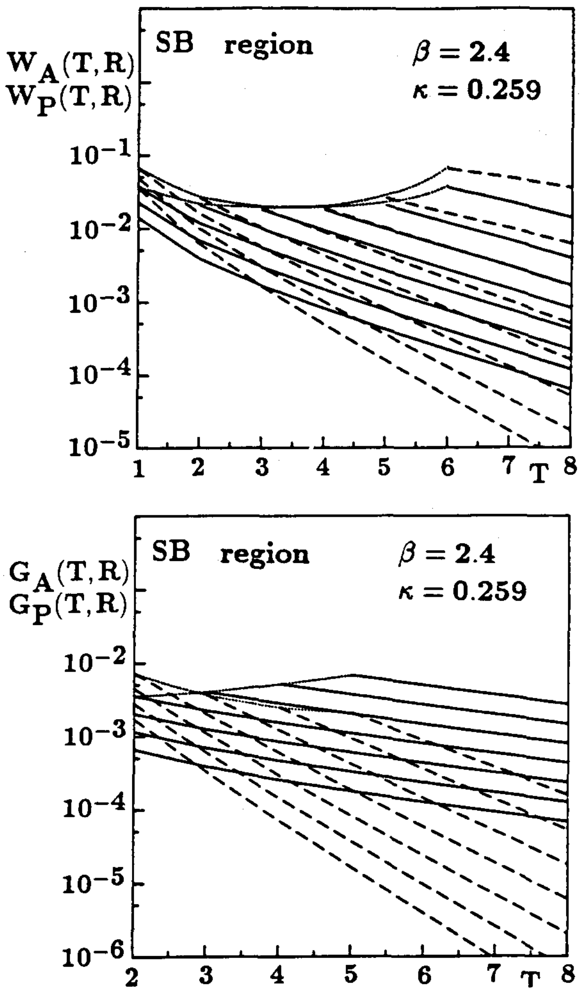


Fig. 3. Comparison of the relative importance of the perimeter and area law terms in (14) and (15) in the SB region. Only the R, T points included in the fits are shown. Different lines correspond to different R. The upper line coresponds to $R = 1$ for $W(T,R)$ and $R = 2$ for $G(T,R)$, etc. ———: $W_P(T,R)$, $G_P(T,R)$; ----: $W_A(T,R)$, $G_A(T,R)$

well. An example is shown in Fig. 2. For $\beta = 2.1$, 2.25, 2.4 and 2.6 we have been able to determine the boundaries of the SB region on a 16^4 lattice. This has been achieved by using several methods of comparison of the relative importance of the terms $W_P(T,R)$ and $W_A(T,R)$ in (14), and $G_P(T,R)$ and $G_A(T,R)$ in (15) for the description of the data. The resulting values of κ_A and κ_P are given in Table 1. An illustration of the equal importance of both terms for a good description of the data in the SB region is illustrated in Fig. 3.

The SB region forms a very narrow strip with the width of about 0.007 in κ, the Higgs phase transition at $\kappa = \kappa_{PT}$ being approximately in the middle of this strip. Below the SB region it is not possible to determine the asymptotic properties of $W(T,R)$ and $G(T,R)$ whereas this is easy to do so above the SB region. The superposition formulae allow us also to extract at least semi-quantitative values of the parameters σ, E_{ext} and μ from the data in the SB region determining both the perimeter and the area law behavior of these observables.

The results of the superposition fits for the ratios $\bar{W}_A/\bar{W}_P$ and $\bar{G}_A/\bar{G}_P$ are shown in Fig. 4. Three of the parameters with a physical interpretation, namely σ, E_{ext}, and μ are displayed in Fig. 5. In this figure we have shadowed for each of the parameters that κ-region in which the values of the parameter have been obtained from the smaller of the two terms in the superposition formulae. The smaller terms might be strongly influenced by theoretical uncertainties of the ansatz and the values of their parameters are spurious. Therefore *we attribute no physical significance to the values of the parameters in the shadowed regions* and show them only for completeness.

The most remarkable aspect of the Figs. 4 and 5 is that the ratios of the coefficients of the area and perimeter terms change strongly when passing through the SB region, whereas the values of the three

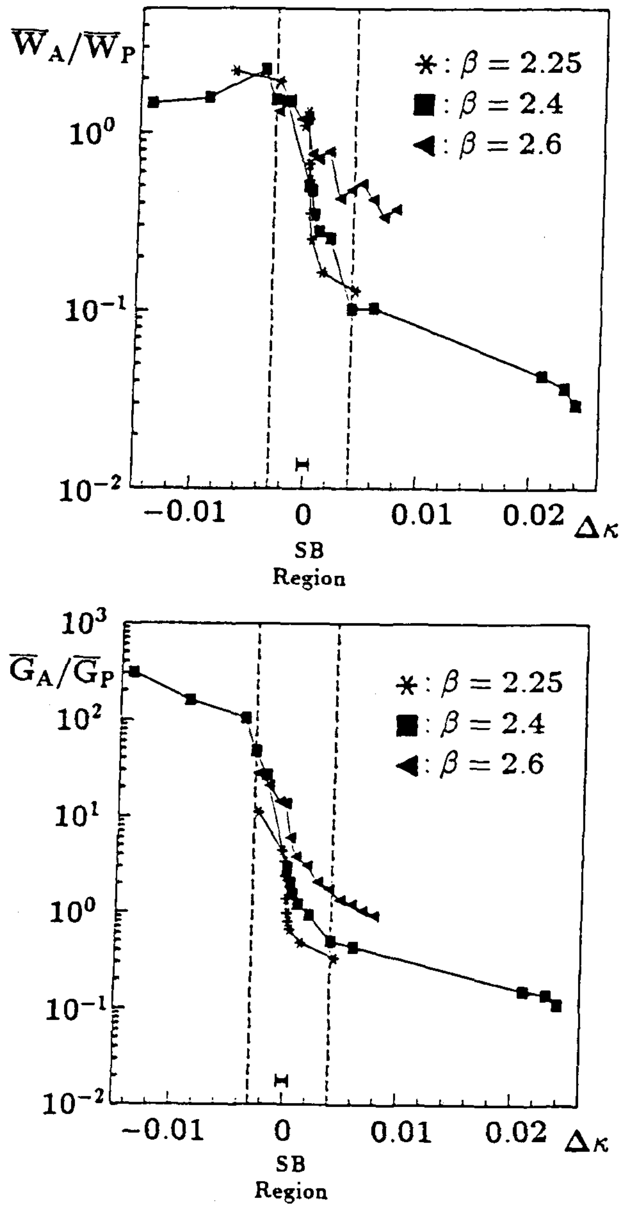


Fig. 4. Dependence of the ratios $\bar{W}_A/\bar{W}_P$ and $\bar{G}_A/\bar{G}_P$ in the superposition formulae (14) and (15) on $\Delta\kappa = \kappa - \kappa_{PT}$. The approximate position of the SB region is indicated by the dashed lines. Notice the fast decrease of both ratios in this region

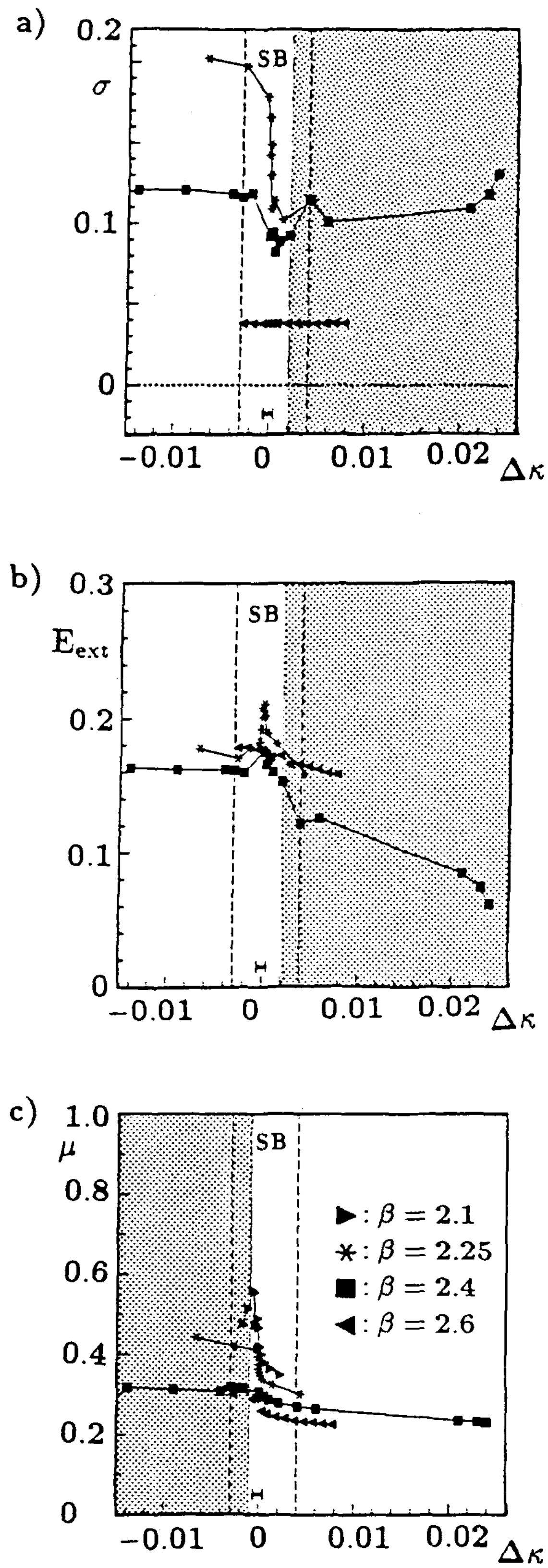


Fig. 5 a–c. Values of the parameters σ, E_{ext} and μ. The approximate position of the SB region is indicated by the dashed lines. No physical meaning can be attributed to the values of these parameters in the shaded regions

parameters displayed change only moderately. Also the values of the parameter ε_c are κ-independent, being approximately $\varepsilon_c \simeq 1.5$ for all β-values. Thus the change of the relative importance of both terms in the superposition formulae is mainly due to the change of the coefficients without much change of the physical parameters.

The values of α determined at $\beta = 2.4$ and 2.6 by means of the superposition formulae are quite κ-independent and therefore we just give their values $\alpha = 0.2 \pm 0.02\,(\beta = 2.4)$ and $\alpha = 0.19 \pm 0.01 (\beta = 2.6)$ (for $\beta = 2.1$ and 2.25 a reliable determination of α-values from our data was not possible). A determination of the Yukawa mass from $W(T,R)$ and $G(T,R)$ turned out to be very unprecise.

3.5 *Order parameter for confinement*

The close relationship between the perimeter laws for $G(T,R)$ and $W(T,R)$ in confining theories with dynamical matter was a motivation for the construction of certain "order parameters" distinguishing between confinement and free charge phases of various models. The parameter proposed by Fredenhagen and Marcu [7, 8] is defined in (5) and (6). The parameter $\rho_{\mathrm{FM}}^{\infty}$ is nonzero in a confinement phase and vanishes in a free charge phase [11]. Therefore $\rho_{\mathrm{FM}}^{\infty}$ should be nonzero in our model for any $\kappa > 0$.

From our previous discussion of the behavior of $W(T,R)$ and $G(T,R)$ at finite distances we conclude that the asymptotic value $\rho_{\mathrm{FM}}^{\infty}$ cannot be determined on a finite lattice below the SB region. The SB occurs at distances which are larger than our lattice. The virtue of our analysis is that we have been able to determine the boundary κ_A of this region in the coupling space rather precisely (see Table 1).

Both above the SB and in the SB region the extraction of the asymptotic behavior of the function ρ_{FM} from the data should be possible. In the SB region this requires some assumption for its non-asymptotic form for which we use here the superposition formula. It provides the following analytic expression for $\rho_{\mathrm{FM}}^{\infty}$:

$$\rho_{\mathrm{FM}}^{\infty} = \frac{\bar{G}_P}{\bar{W}_P^{1/2}}. \tag{16}$$

The values of $\rho_{\mathrm{FM}}^{\infty}$ are displayed in Fig. 6. They

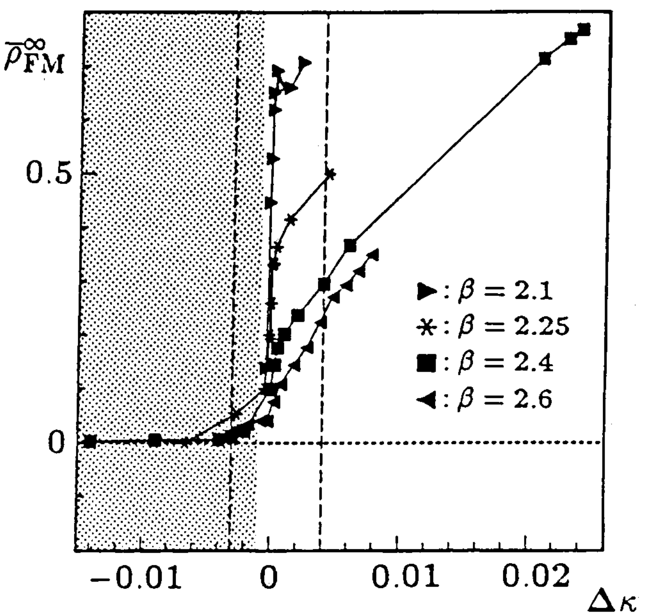


Fig. 6. The asymptotic values $\rho_{\mathrm{FM}}^{\infty}$ of $\rho_{\mathrm{FM}}(T,R)$ calculated by means of (16). The values in the shaded part are not trustworthy because $W(T,R)$ and $G(T,R)$ are not asymptotic below the SB region

represent an extension of our previous results obtained only for $\beta = 2.4$ [11], where we have used the ansatz

$$\rho_{\mathrm{FM}} = \rho_{\mathrm{FM}}^{\infty} + B\exp(-CP), \tag{17}$$

$P = T + 2R$ being the perimeter of $G(T,R)$. The values of $\rho_{\mathrm{FM}}^{\infty}$ obtained by the two methods are consistent both in the SB region and above it. This is an idication that those values can be trusted and one really sees the asymptotic property of the function $\rho_{\mathrm{FM}}(T,R)$.

For completeness in Fig. 6 we also display the values of the parameters below the SB region. It should be clear, however, that they do not represent the true asymptotic values of the function $\rho_{\mathrm{FM}}(T,R)$. Therefore the corresponding κ-region is shaded.

4 Summary and conclusions

Our investigation of the SB region by means of the superposition formula contributes to the study of the hadronisation in gauge theories with dynamical matter fields.

As the numerical simulation of scalar matter fields is much simpler than that of dynamical fermions, the observation and the study of the string breaking in the lattice Higgs models is able to provide helpful clues for the analysis of the corresponding data in lattice QCD [6]. In particular, the gauge invariant 2-point function $G(T,R)$ seems to be similarly useful as the Wilson loop $W(T,R)$.

The investigation of the complex phenomenon of string breaking requires the study of gauge theories with dynamical matter on several different characteristic length scales simultaneously. Our experience with the superposition formula indicates that such a study begins to be possible, at least qualitatively, already on a hypercubic lattice of the size $L = 16$. But it is clear that reliable quantitative results will require the use of even larger lattices and enormous statistical efforts.

Analyzing the data for the Wilson loop $W(T,R)$ and the gauge invariant 2-point function $G(T,R)$ in the $SU(2)$ Higgs model with a doublet scalar field on an L^4 lattice with $L = 16$, we have localized the SB (string breaking) region, $\kappa_A < \kappa < \kappa_P$, where both the non-asymptotic area law and the asymptotic perimeter law behaviors are observed on the lattice simultaneously at intermediate and large distances, respectively. Here the breaking of the confining flux tube is seen. We have determined the approximate positions of the boundaries κ_A and $\kappa_P\,(> \kappa_A)$ for $\lambda = 0.5$ and $2.1 \leqq \beta \leqq 2.6$ (Table 1). The Higgs phase transition lies between these boundaries, i.e. in the SB region.

Because of the complex physical process of the string breaking, the SB region requires an elaborate analysis which we have performed using a superposition of the area and perimeter terms for $W(T,R)$ and $G(T,R)$. This allows us to distinguish the non-asymptotic and asymptotic properties of these functions and to determine the values of some observables which characterize their behavior at different scales.

One of these observables is the string tension σ. It decreases slightly in the SB region when κ increases above κ_A but remains finite even at $\kappa = \kappa_P$. An interpretation of this behavior is possible if one identifies σ with the slope of the linear part in the static potential. As the extension l of this linear part decreases with increasing κ monotonically, we can consider σ as a function of l instead of κ. Our results suggest that σ does not vanish as $l \to 0$ in the $SU(2)$ Higgs model. This means that the area law behavior of $W(T, R)$ and $G(T, R)$ at intermediate distances disappears because the R- and T-interval of its validity shrinks to zero near κ_P and not because the string tension vanishes at κ_P. A similar question could be asked also in lattice QCD.

Another interesting observable is the order parameter ρ_{FM}^{∞}, which can be obtained from the asymptotic behavior of $W(T, R)$ and $G(T, R)$ by means of the superposition fits. Its values can be determined not only above but also in the SB region.

Acknowledgments. We have very much benefited from the discussions with T. DeGrand, K. Fredenhagen, C.B. Lang and M. Marcu. H.G. Evertz and T. Neuhaus wish to thank the Center of Simulational Physics at the University of Georgia for its hospitality and support. We thank the computer centers of the University of Georgia and the University of Bochum for providing the Cyber 205 computer time. Technical support of the computer centers of the Technische Hochschule Aachen and of the HLRZ Jülich is also gratefully acknowledged.

References

1. M. Creutz: Phys. Rev. D21 (1980) 2308; G. Bhanot, C. Rebbi: Nucl. Phys. B180 [FS2] (1981) 469; E. Kovacs: Phys. Rev. D25 (1982) 3312
2. J.D. Stack: Phys. Rev. D27 (1983) 412
3. I. Montvay, F. Gutbrod: Phys. Lett. 136B (1984) 411; F. Gutbrod: Z. Phys. C—Particles and Fields 37 (1987) 143
4. K. Osterwalder, E. Seiler: Ann. Phys. 110 (1978) 440
5. E. Fradkin, S. Shenker: Phys. Rev. D19 (1979) 3682
6. H.J. Rothe, K.D. Rothe, J.A. Swieca: Phys. Rev. D19 (1979) 3020; H.G. Dosch, V.F. Müller: Nucl. Phys. B158 (1979) 419; I. Bender, H.J. Rothe, K.D. Rothe: Nucl. Phys. B251 [FS13] (1985) 745; J. Potvin: Phys. Rev. D32 (1985) 2070; Phys. Rev. D33 (1986) 2388; H. Markum: Phys. Lett. B173 (1986) 337; E. Laermann, F. Langhammer, I. Schmitt, P.M. Zerwas: Phys. Lett. B173 (1986) 437; M. Fukugita, Y. Oyanagi, A. Ukawa: Phys. Rev. D36 (1987) 824; M. Campostrini, K.J.M. Moriarty, J. Potvin, C. Rebbi: Phys. Lett. B193 (1987) 78; Ph. de Forcrand, V. Linke, I.O. Stamatescu: Nucl. Phys. B304 (1988) 645; M. Fukugita: Nucl. Phys. B (Proc. Suppl.) 4 (1988) 105; J. Potvin: Nucl. Phys. B (Proc. Suppl.) 4 (1988) 140; M. Faber et al.: Phys. Lett. B200 (1988) 348; K.D. Born et al.: Proceedings of the XXIV Int. Conference on High Energy Physics, R. Kotthaus, J.H. Kühn (eds.). Munich, 1988
7. K. Fredenhagen, M. Marcu: Commun. Math. Phys. 92 (1983) 81; Phys. Rev. Lett. 56 (1986) 223
8. M. Marcu, Lattice gauge theory—A challenge in large-scale computing, B. Bunk, K.H. Mütter, K. Schilling (eds) New York: Plenum Press 1986
9. H. G. Evertz et al.: Phys. Lett. 194B (1987) 277
10. H.A. Kastrup: Nucl. Phys. B (Proc. Suppl.) 390
11. H.G. Evertz et al.: Nucl. Phys. B285 [FS19] (1987) 559
12. T. Filk, K. Fredenhagen, M. Marcu: Phys. Lett. 169B (1986) 405
13. M. Marcu: Lattice gauge theory 1986, H. Satz, I. Harrity, J. Potivin (eds) New York: Plenum Press 1986
14. H.G. Evertz et al.: Phys. Lett. 175B (1986) 335
15. H.A. Kastrup: Proceedings of the 23rd International Conference on High Energy Physics, S.C. Loken (ed.) p. 573. Singapore: World Scientific 1986
16. V. Azcoiti, A. Tarancón: Phys. Lett. 176B (1986) 153
17. J. Jersák: Lattice gauge theory 1986, H. Satz, I. Harrity, J. Potivin (eds) New York: Plenum Press 1986
18. V. Alessandrini, J.L. Alonso, A. Cruz, A. Tarancón: Nucl. Phys. B281 (1987) 445
19. E. Katznelson, P. Lauwers, M. Marcu: Nucl. Phys. B (Proc. Suppl.) 4 (1988) 427
20. C.B. Lang, C. Rebbi, M. Virasoro: Phys. Lett. 104B (1981) 294; H. Kühnelt, C.B. Lang, G. Vones: Nucl. Phys. B230 [FS10] (1984) 16
21. J. Jersák, C.B. Lang, T. Neuhaus, G. Vones: Phys. Rev. D32 (1985) 2761
22. V.P. Gerdt, A.S. Ilchev, V.K. Mitrjushkin, A.M. Zadorozhny: Z. Phys. C—Particles and Fields 29 (1985) 363
23. V.P. Gerdt et al.: Nucl. Phys. B265 [FS15] (1986) 145
24. W. Bock et al.: HLRZ Jülich preprint 89-42
25. J. Jersák: Lattice gauge theory—a challenge in large-scale computing, B. Bunk, K.H. Mütter, K. Schilling (eds.). New York: Plenum Press 1986
26. P. Debye, E. Hückel: Z. Phys. 24 (1923) 185
27. I. Montvay: Nucl. Phys. B269 (1986) 170; W. Langguth, I. Montvay: Phys. Lett. 165B (1985) 135; W. Langguth, I. Montvay, P. Weisz: Nucl. Phys. B277 (1986) 11
28. H.G. Evertz, J. Jersák, C.B. Lang, T. Neuhaus: Phys. Lett. 171B (1986) 271
29. A. Hasenfratz, T. Neuhaus: Nucl. Phys. B297 (1988) 205
30. H.G. Evertz et al.: Nucl. Phys. B285 [FS19] (1987) 590
31. J. Kogut: Rev. Mod. Phys. 51 (1979) 659