Neoclassical Transport Modelling for Impurities in the Tracer Limit

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Neoclassical impurity transport: simple picture

- **neoclassical particle fluxes for species $\alpha = e,\ i,\ I$**

$$\Gamma_\alpha = -n_\alpha D_{11}^{\alpha} \left\{ \frac{n'_\alpha}{n_\alpha} - q_\alpha \frac{E_r}{T_\alpha} + \frac{D_{12}^{\alpha}}{D_{11}^{\alpha}} \frac{T'_\alpha}{T_\alpha} \right\}$$

with the neoclassical transport matrix, $D_{ij}^{\alpha}(r, B, E_r, n_\alpha, T_\alpha, \nu_\alpha)$

- **assume $T_I = T_i$ and $n_I \ll n_e$, i.e. $n = n_e \simeq Z_i n_i$**

$\rightarrow$ ambipolar E_r only from $\Gamma_e \simeq \Gamma_i$

- **standard "ion root" E_r**

if $\Gamma_i(E_r = 0) \gg \Gamma_e \quad \rightarrow \quad \Gamma_i(E_r) = 0$:

$$\rightarrow \quad Z_i \frac{eE_r}{T_i} \simeq \frac{n'}{n} + \frac{D_{12}^{i}}{D_{11}^{i}} \frac{T'_i}{T_i}$$

$$\Gamma_I = -n_I D_{11}^{I} \left\{ \frac{n'_I}{n_I} - \frac{Z_I}{Z_i} \frac{n'}{n} + \left(\frac{D_{12}^{I}}{D_{11}^{I}} - \frac{Z_I}{Z_i} \frac{D_{12}^{i}}{D_{11}^{i}} \right) \frac{T'_i}{T_i} \right\}$$

Neoclassical impurity transport: simple picture (2)

- **For stationary conditions** $\rightarrow$ $\Gamma_I \simeq 0$

$$\frac{n_I'}{n_I} = \frac{Z_I}{Z_i}\left\{\frac{n'}{n} + \frac{D_{12}^{\,i}}{D_{11}^{\,i}}\frac{T_i'}{T_i}\right\} - \frac{D_{12}^{\,I}}{D_{11}^{\,I}}\frac{T_i'}{T_i}$$

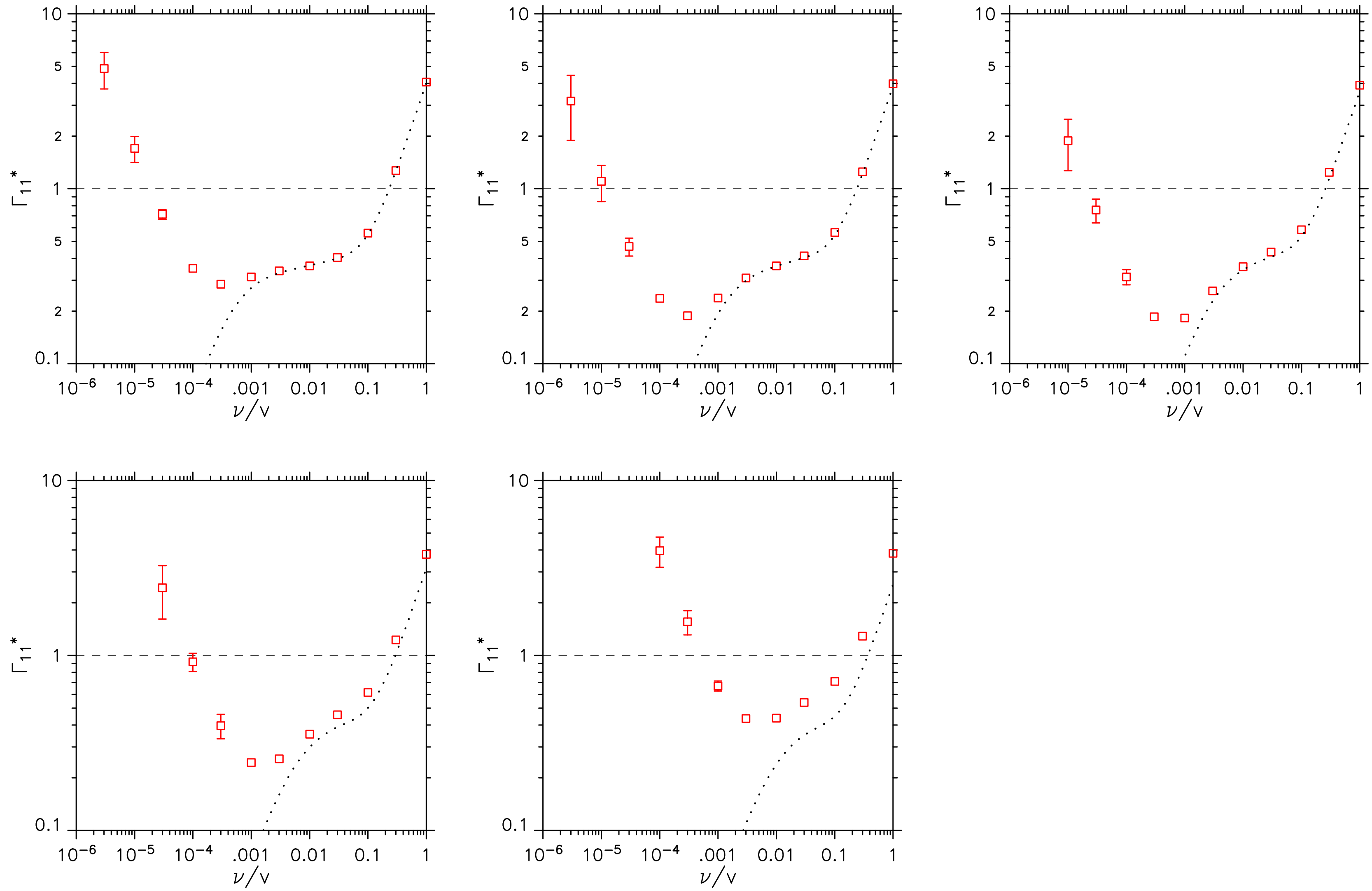
- **For "pure" collisionality regimes:**

$$\frac{D_{12}^{\alpha}}{D_{11}^{\alpha}} = \frac{3}{2} \quad \text{plateau regime} \qquad = -\frac{1}{2} \quad \text{tokamak banana regime}$$

$$= \frac{7}{2} \quad \frac{1}{\nu}\text{-regime} \qquad = \frac{5}{4} \quad \sqrt{\nu}\text{-regime}$$

$$= \frac{1}{2} \quad \nu\text{-regime}$$

NCSX-2: no tokamak-like banana regime



Neoclassical impurity transport: simple picture (3)

- **Tokamak:** large Z_I, peaked $n(r)$, flat $T_i(r)$

$$\rightarrow \quad n_I \propto n^{\frac{Z_I}{Z_i}} \qquad \text{highly peaked}$$

- large Z_I, flat $n(r)$, peaked $T_i(r)$ $\quad \rightarrow \quad$ "temperature screening"

$$\text{if} \quad \frac{n'}{n} - \frac{1}{2}\frac{T_i'}{T_i} \geq 0 \quad \rightarrow \quad \text{no accumulation}$$

- **Stellarator:** for flat $n(r)$, peaked $T_i(r)$

$$\rightarrow \quad n_I \propto T_i^{\frac{Z_I}{Z_i}\frac{D_{12}^i}{D_{11}^i} - \frac{D_{12}^I}{D_{11}^I}} \qquad \text{accumulation}$$

- for ambipolar $E_r \geq 0$ with $n' \simeq T_i' \simeq 0$ (positive "ion root")

$$Z_i \frac{eE_r}{T_i} \simeq -\frac{D_{12}^e}{D_{11}^i}\frac{T_e'}{T_e} \quad \rightarrow \quad n_I \propto T_e^{-\frac{Z_I}{Z_i}\frac{D_{12}^e}{D_{11}^i}} \qquad \text{no accumulation}$$

Neoclassical impurity transport with ambipolar E_r

- calculate ambipolar E_r from root-finding or from diffusion eq.

$$\Gamma_i - \Gamma_e = 0 \qquad \rightarrow \qquad E_r$$

- neoclassical impurity flux with $T_I \simeq T_i$:

$$\Gamma_I = -n_I \, D_{11}^{\,I} \left\{ \frac{n_I'}{n_I} - Z_I \frac{E_r}{T_i} + \frac{D_{12}^{\,I}}{D_{11}^{\,I}} \frac{T_i'}{T_i} \right\} \simeq 0$$

- rewrite (used by experimentalists)

$$\Gamma_I = -D_I \, n_I' + v_I \, n_I$$

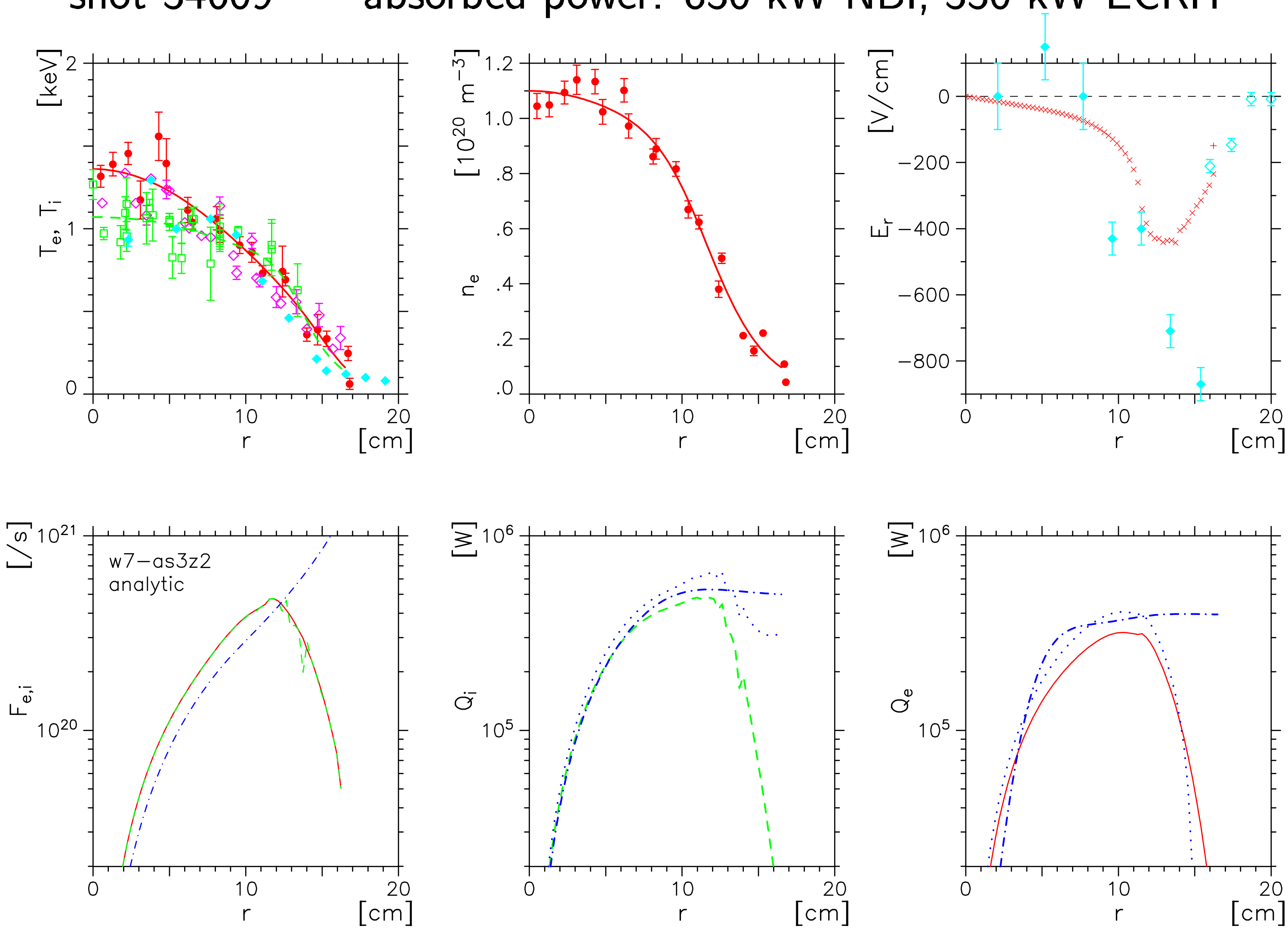
$$\text{with} \quad D_I = D_{11}^{\,I} \quad \text{and} \quad v_I = Z_I \frac{E_r}{T_i} D_{11}^{\,I} - \frac{T_i'}{T_i} D_{12}^{\,I}$$

- stationary impurity density profile ($\Gamma_I = 0$)

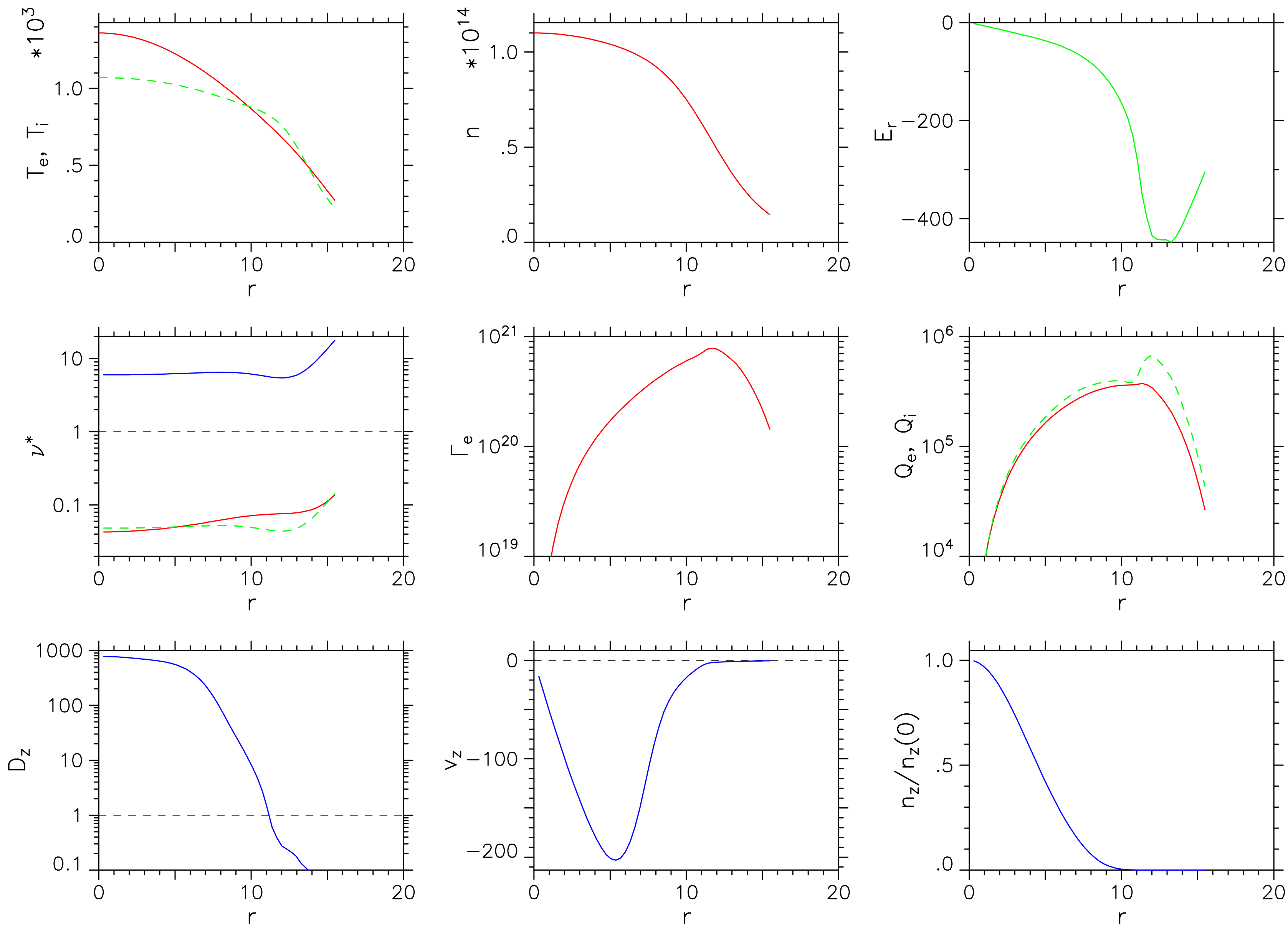
$$n_I(r) = n_I(0) \, \exp \left\{ \int_0^r \frac{v_I}{D_I} \, dr \right\}$$

W7-AS: neoclassical transport analysis

shot 34609 absorbed power: 830 kW NBI, 330 kW ECRH



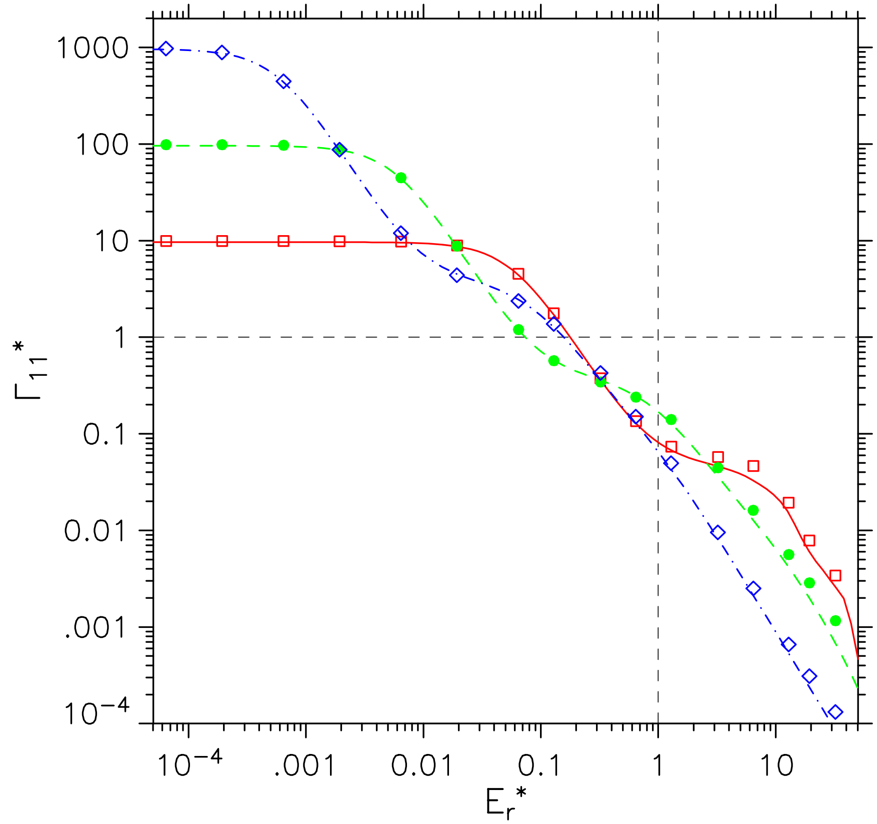
W7-AS: Al^{12} density simulation for shot 34609



W7-AS: neoclassical transport coeff. in PS-regime with E_r

W7-AS: $\iota \simeq 0.34$ at $r = 0.5 \cdot a$ superposition of main b_{mn} contributions

radial transport, D_{11}^*



Neoclassical impurity transport: coupling of E_r and Φ_1

- high Z impurities (in PS-regime) cannot balance $\underline{E} \times \underline{B}$ compression by parallel flow

$$\underline{\nabla}(n_I\, \underline{v}_{E\times B}) = \underline{\nabla} n_I \frac{\underline{B} \times \underline{\nabla}\Phi_0}{B^2} = 0 \qquad \rightarrow \qquad \frac{n_I}{\langle n_I \rangle} = \frac{B^2}{\langle B^2 \rangle}$$

- impurities in tracer limit: no effect of n_I on quasi-neutrality
- assume: $1st$ order potential, Φ_1 (e.g. PS-like) (only determined by quasi-neutrality of background plasma)
- impurity fluxes from coupling both $\underline{E} \times \underline{B}$ drifts:

$$\left\langle n_I \frac{\underline{B} \times \underline{\nabla}\Phi_1}{B^2} \right\rangle = \frac{\langle n_I \rangle}{\langle B^2 \rangle} \left\langle \underline{B} \times \underline{\nabla}\Phi_1 \right\rangle = \langle n_I \rangle \frac{\left\langle \underline{\nabla} \times (\underline{B}\, \Phi_1) \right\rangle}{\langle B^2 \rangle} \equiv 0$$

$\Rightarrow$ no effect on impurity accumulation

- for "strong" toroidal rotation: $n_I \propto B^{-\alpha}$

$\Rightarrow$ effect on impurity fluxes

Neoclassical transport: 1st order potential, Φ_1

- assume $f = f_0 + f_1$ with density variation on flux surfaces

$$\begin{bmatrix} \Gamma^0 \\ \frac{1}{T}\, q^0 \end{bmatrix} = \left\langle \iiint_{-\infty}^{\infty} d^3v \begin{bmatrix} 1 \\ \frac{mv^2}{2T} \end{bmatrix} \left(\underline{v}_d\right)_r f_0 \right\rangle \qquad \text{with} \qquad f_0 = e^{-\frac{q\Phi_1}{T}} f_M(r, v^2)$$

$$\underline{v}_d = \frac{mv^2}{2q}\left(1 + p^2\right) \frac{\underline{B} \times \underline{\nabla} B}{B^3} + \frac{\underline{B} \times \underline{\nabla}(\Phi_0 + \Phi_1)}{B^2}; \qquad x = \frac{v}{v_{th}}$$

$$\iiint_{-\infty}^{\infty} \begin{bmatrix} 1 \\ x^2 \end{bmatrix} \left(\underline{v}_d\right)_r f_0 \, d^3v = \frac{4n}{\sqrt{\pi}} \int_0^{\infty} \begin{bmatrix} 1 \\ x^2 \end{bmatrix} \left\{ \frac{\underline{B} \times \underline{\nabla}\Phi_1}{B^2} - \frac{4}{3}\Phi_1 x^2 \frac{\underline{B} \times \underline{\nabla} B}{B^3} \right\} x^2 e^{-x^2} \, dx$$

$$= n \begin{bmatrix} 1 \\ \frac{3}{2} \end{bmatrix} \frac{\underline{B} \times \underline{\nabla}\Phi_1}{B^2} - n \begin{bmatrix} 2 \\ 5 \end{bmatrix} \frac{\underline{B} \times \underline{\nabla} B}{B^3} \Phi_1$$

Neoclassical transport: 1st order potential, Φ_1 (2)

- **for the particle flux (with Stokes theorem)**

$$\left\langle \underline{B} \times \underline{\nabla} \frac{\Phi_1}{B^2} \right\rangle = \left\langle \underline{\nabla} \times \underline{B} \frac{\Phi_1}{B^2} \right\rangle \equiv 0 \quad \Rightarrow \quad \Gamma^0 \equiv 0 \qquad \text{q.e.d.}$$

- **and for the energy flux**

$$B = B_0 \sum_{n,m} b_{n,m} \cos(m\theta - Nn\phi) \qquad \Phi_1 = \sum_{n,m} \phi_{n,m}\, e^{i\,(m\theta - Nn\phi)}$$

$$\Rightarrow \quad q^0 = \frac{7\, nT}{4\, rB} \sum_{n,m} m\, b_{n,m}\, \Im(\phi_{n,m})$$

- **no particle flux driven by the coupling of $\underline{\nabla} B$-drift and potential variations on the flux surfaces**

Neoclassical impurity transport: preliminary conclusions

- for $T_e \simeq T_i$ with $E_r < 0$: impurity accumulation in 0th order (density variations on flux-surfaces omitted)
  $\Rightarrow$ consistent with experiments (at least at W7-AS)
- no "temperature screening" in stellarator *lmfp*-regimes
- strong accumulation obtained for improved confinement (strongly negative E_r)
- no low order effect of 1st order potentials on impurity flux
- question: strongly negative E_r must be avoided?
- "electron-root" feature (strongly positive E_r):
  $\Rightarrow$ hard to obtain in strongly optimized stellarator configurations

$\Rightarrow$ does stellarator optimisation support impurity accumulation?