Neoclassical transport in NCSX

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Introduction

• Helical neoclassical particle and thermal transport is expected to play a small role in NCSX.
This is not usual for hot plasmas in stellarators without a quasi-symmetry direction.

• Helical neoclassical transport should play an important role in determining E_r and flows.
It should be the dominant part of the viscosity and the non-ambipolar part of the particle flux.

Neoclassical calculations for NCSX are more difficult than usual:
• Configuration flexibility is multi-dimensional: iota magnitude, shear, shape, current profile change.
Need a cheap, accurate 'prediction' of the coefficients in new configurations
• Usual collision operator doesn't conserve momentum; a problem in quasi-symmetric configurations.
Correction procedure is messy, but appears to add no fundamental difficulties.

Progress toward a practical method of computing neoclassical transport in NCSX is described.

Configuration Flexibility

Independently power modular coils provide a large degree of configuration flexibility.
Neil Pomphrey's configuration flexibility studies (see figures below) show NCSX can:

- Change magnitude of rotational transform with little change in magnetic shear.
  Note the changes in boundary shape!
- Change shear while holding central (or edge) rotational transform steady.
- Change internal currents, magnitude of iota, magnetic shear with small shape change.

Configuration-space is multi-dimensional: iota magnitude, shear, shape, current profile change.
It's impractical to pre-compute monoenergetic coefficients for all parts of configuration space.

- Need a cheap, accurate 'prediction' of the coefficients in new configurations

Several very different alternatives are being investigated:

1) Purely numerical interpolation scheme
   Too many configuration-space dimensions to be tractable and accurate?
   No apparent trends in $\square_{eff}$ for a simple scans in I_p and $\square$.

2) Use a semi-analytic fit to monoenergetic coefficients?
   Effective helical ripple calculated quickly by NEO, and can be included within a transport co
   Boozer components, B_{mn}, are easily computed; are they easily related to other fit parameters?

3) Use 'stochastic mapping technique' for very fast orbit following?
   Fast enough for a 'first principles' calculation within a transport code??

Monoenergetic Diffusion Coefficients

Matrix formulation of neoclassical transport is based on convolutions of monoenergetic coefficients.

- DKES: Drift Kinetic Equation Solver; uses moments representation of distribution function.
  Low ν: need many moments to represent the boundary layer at the passing-trapped boundary.
- DCOM/MOCA follow drift-orbits of a sample population using Monte Carlo collision algorithm.
  Low ν: need very long integration times to fully sample phase space; time*ν_{90}~1.
- MOCA used here because it can deal with very low collisionalities more easily than DKES.
- Good benchmark agreement between DKES and orbit codes for wide range of ν and E_r.

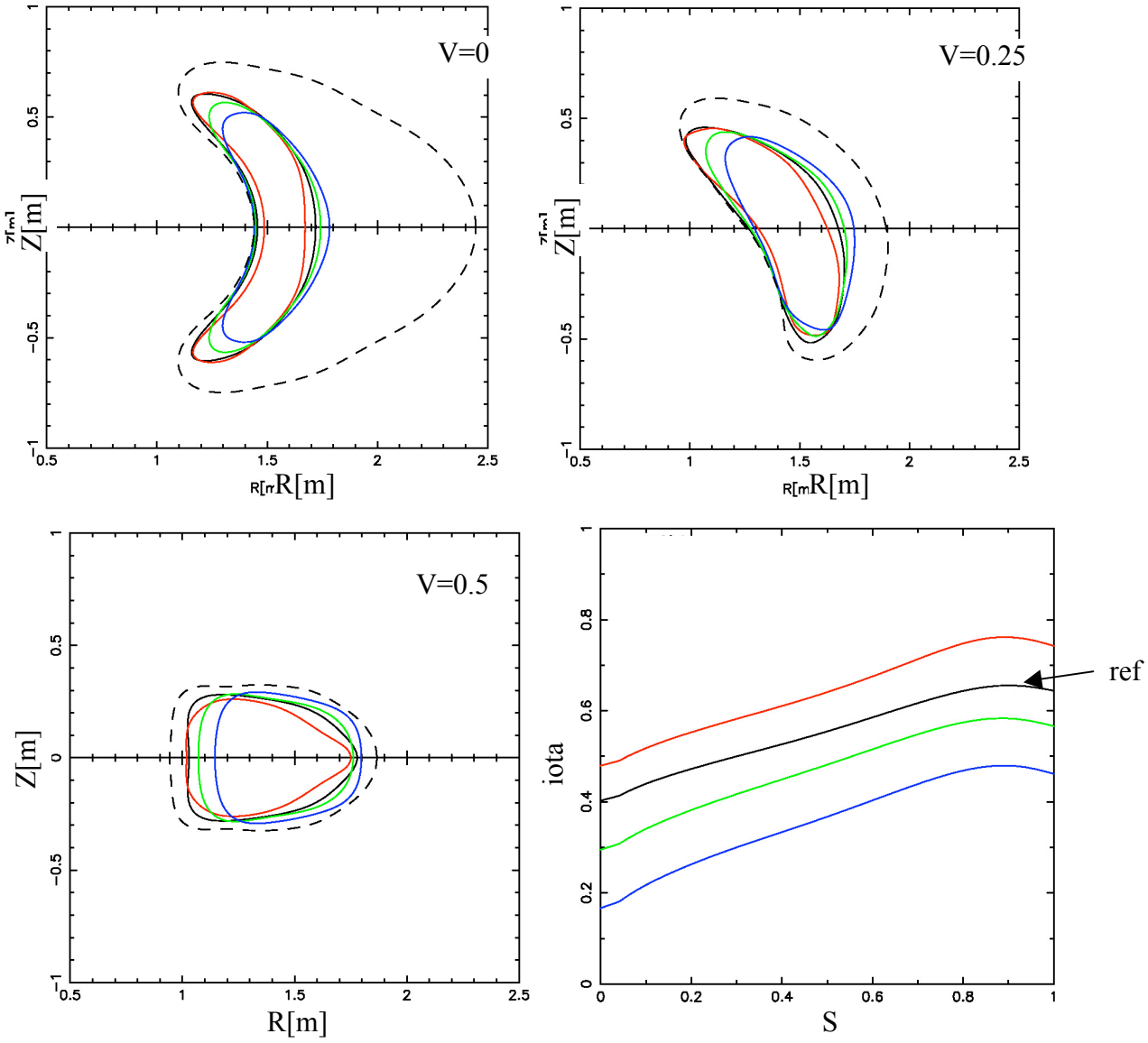
$$\begin{pmatrix} \Gamma_j \\ Q_j \end{pmatrix} = D_{11} e^{-x} \begin{pmatrix} n \\ \frac{n}{} \end{pmatrix} \frac{Z_j e E_r}{T_j} + (x - \frac{3}{2}) \frac{T_j}{} \begin{pmatrix} x^2 \\ x^3 \end{pmatrix} \sqrt{x} dx$$

Conventional matrix formulation needs only D_{11} for particle and heat transport, but

- Sugama's correction for momentum non-conservation needs all three coefficients.
- D_{11} can be calculated by DKES or Monte Carlo orbit code; D_{33} and D_{31} come from DKES.
- D_{33} (conductivity) and D_{31} (bootstrap) have negligible dependence on E_r.
- D_{11} is still the only computationally expensive part of the calculation.

For more detail on the Sugama procedure see Don Spong's poster RP1.023 (this session).

Figure 8-13. Plasma boundaries and iota profiles for an iota-scan where ι(s) is raised/lowered but the shear is preserved.



N. Pomphrey, NCSX Conceptual Design Review

Figure 8-14. Plasma boundaries and iota profiles for a magnetic-shear scan with fixed central iota.

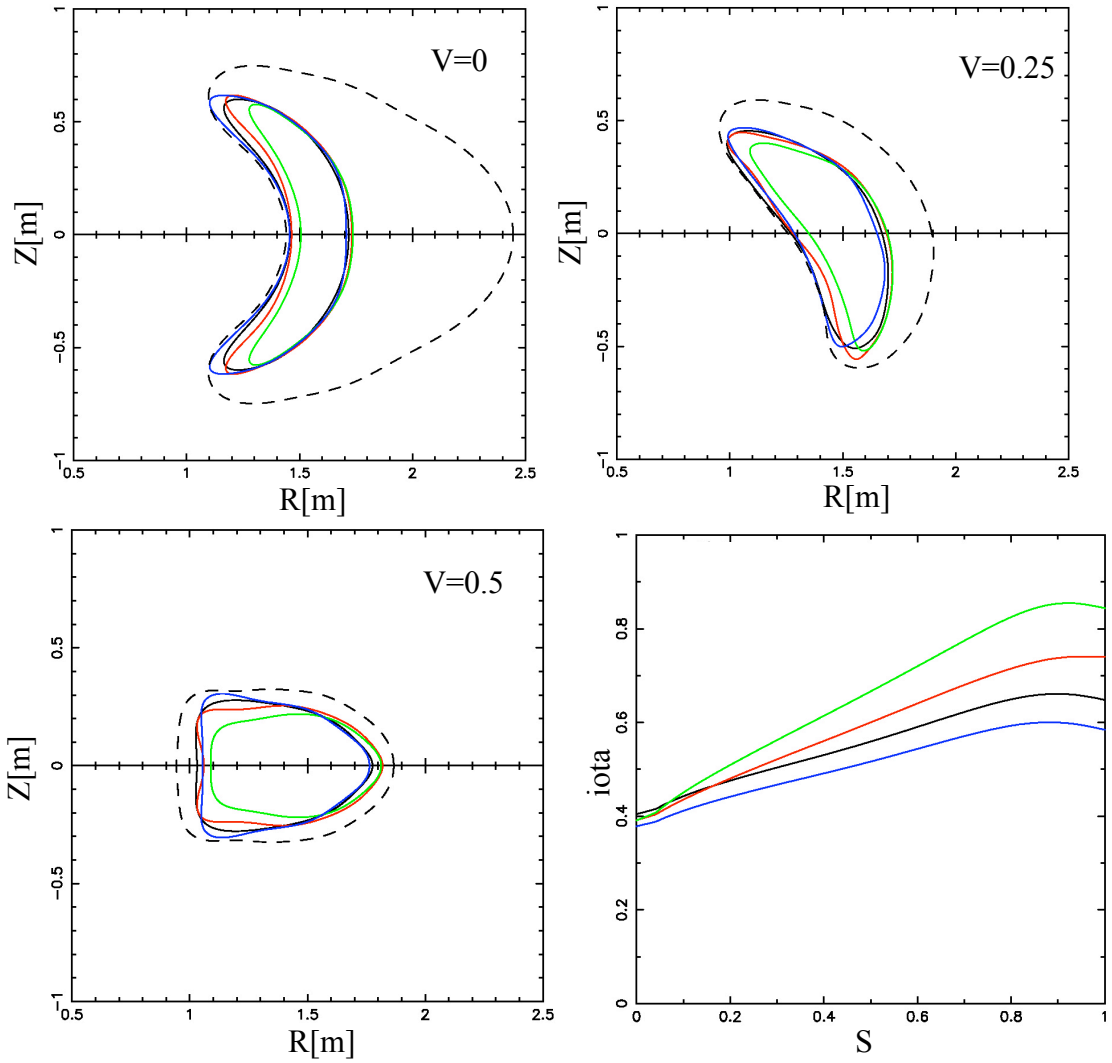


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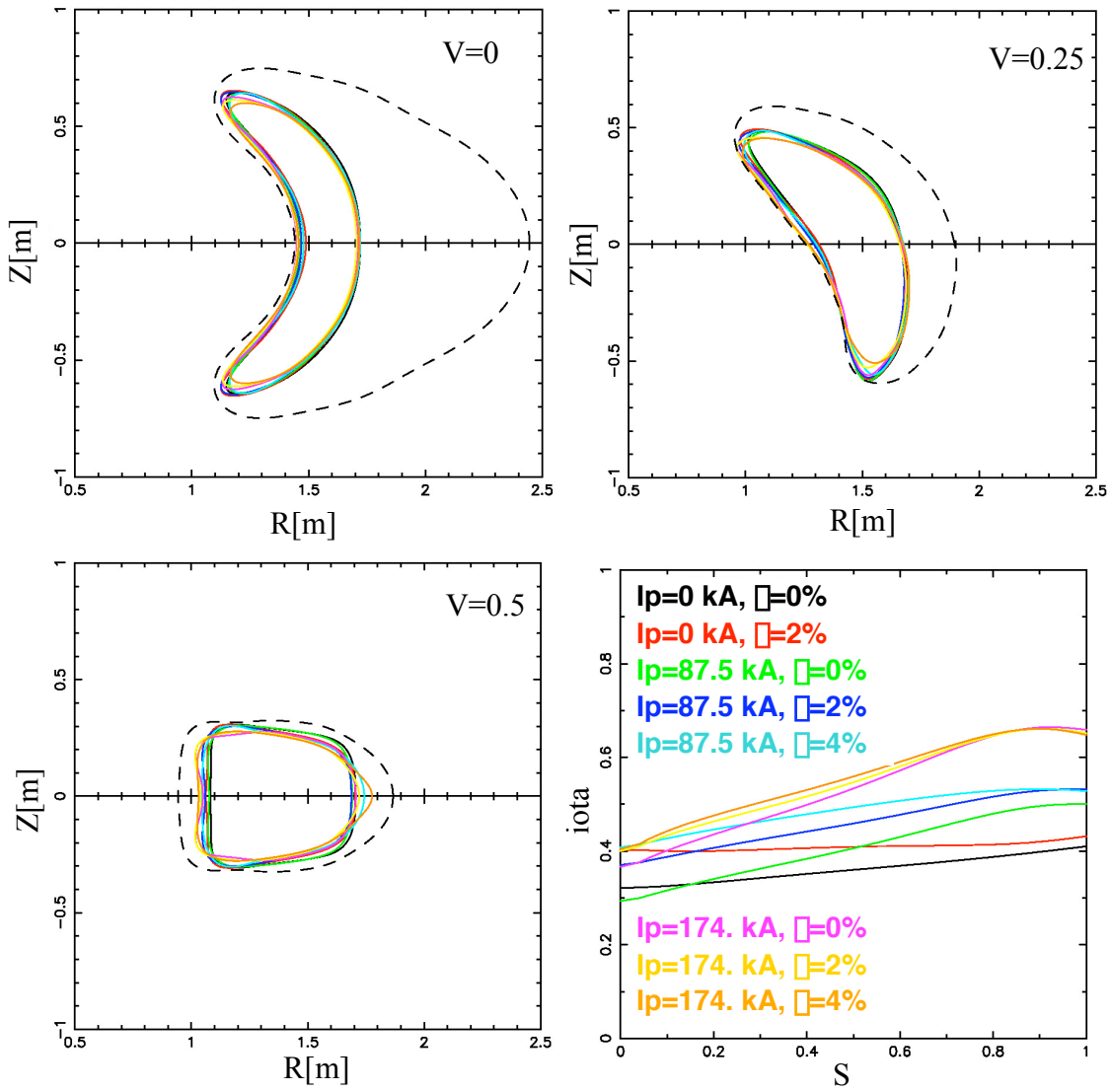
Figure 8-6. Overlay of plasma boundaries and iota profiles for stable optimized equilibria from an I_p - □ scan.



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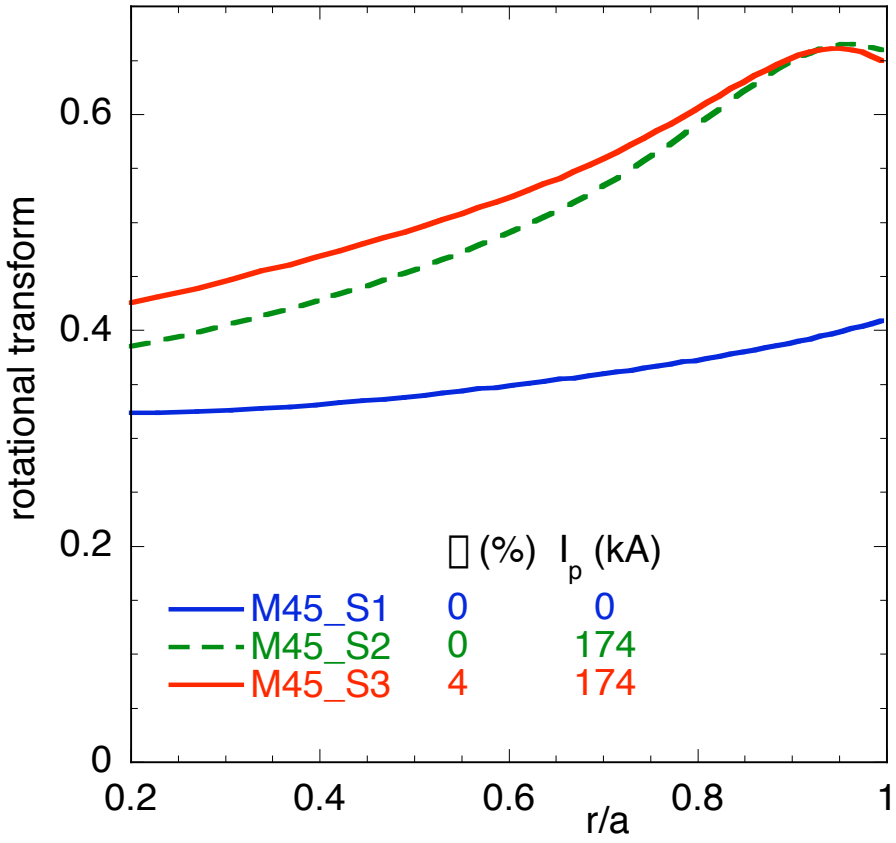
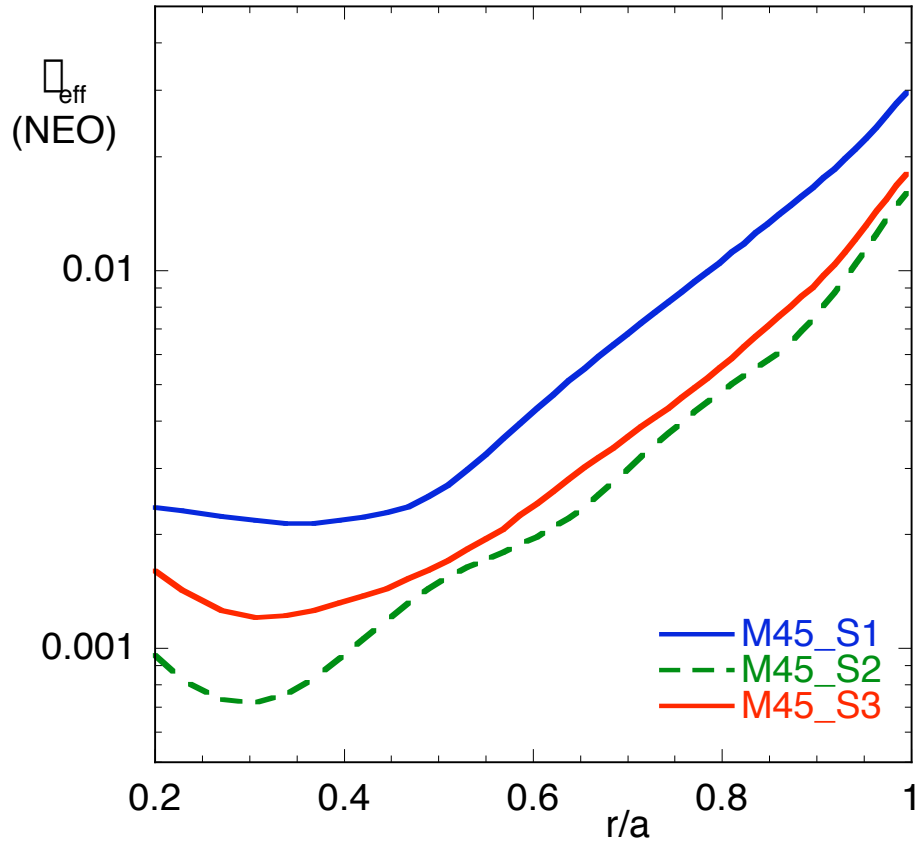
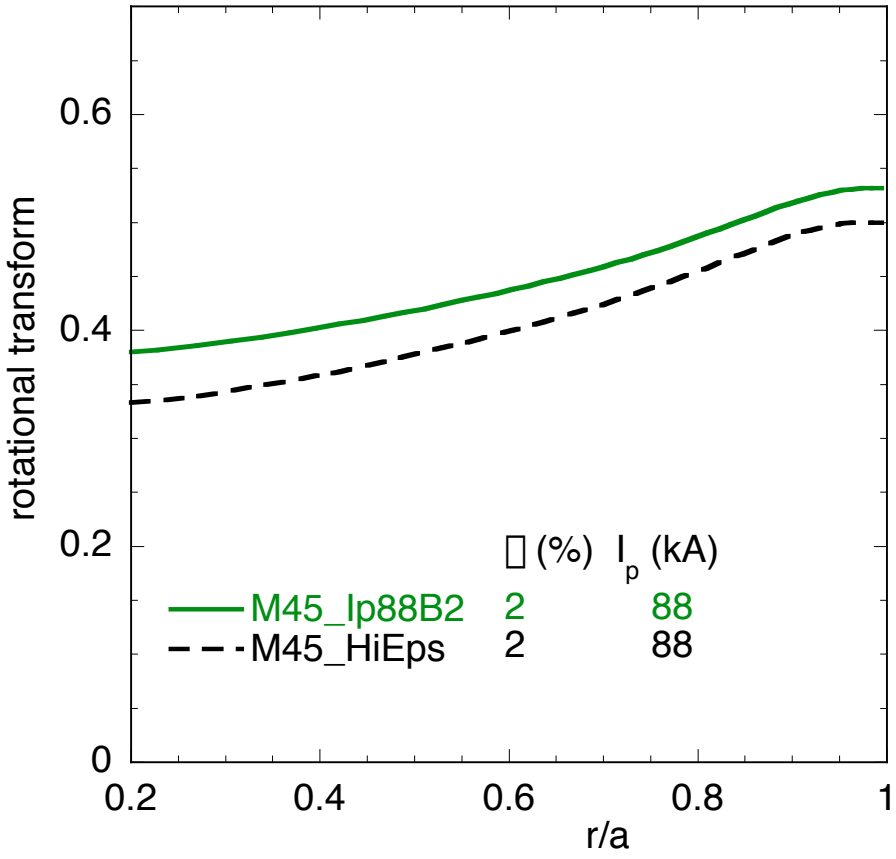
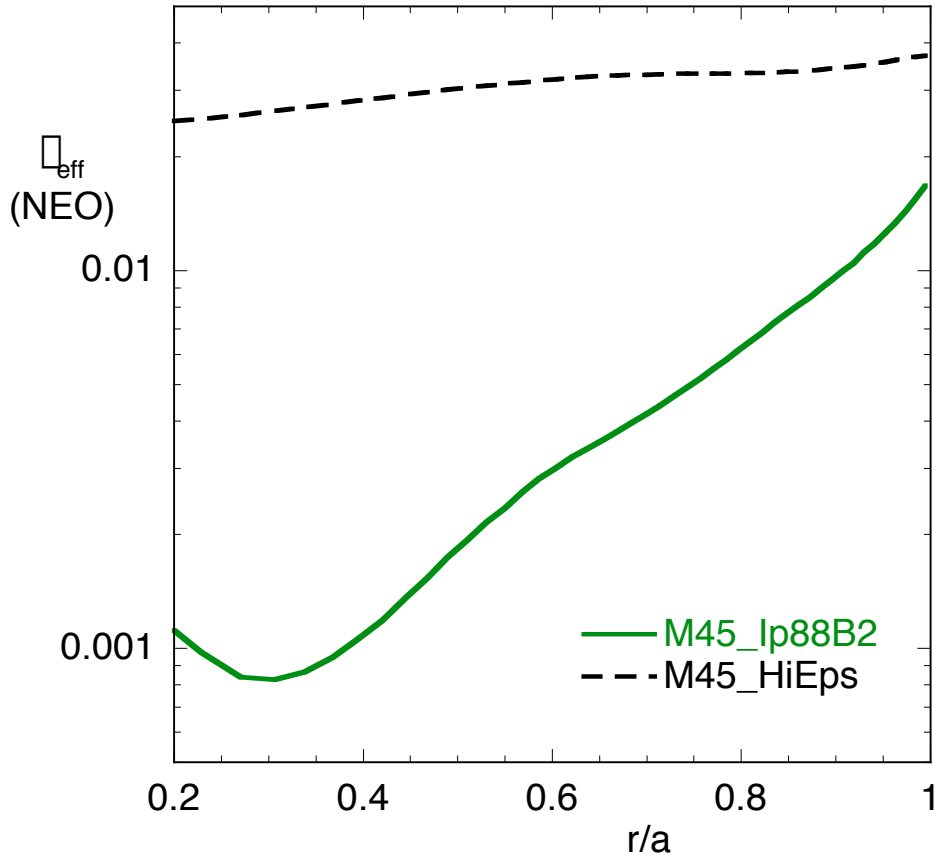




Magnitude of rotational transform and shear have wide ranges.

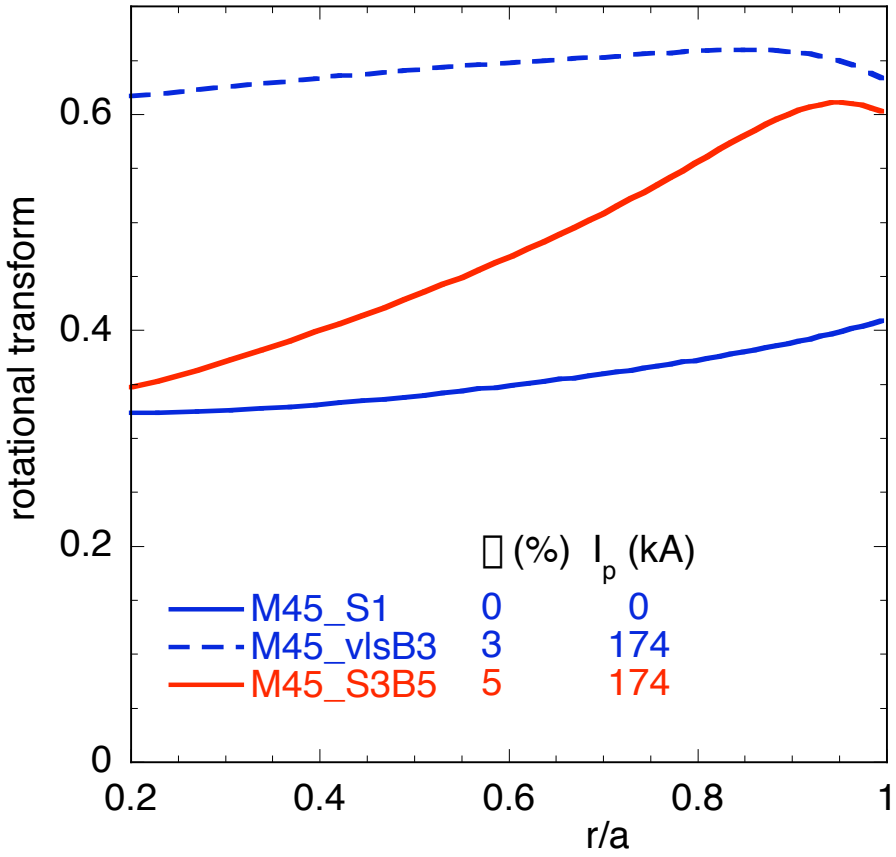
Want neoclassical predictions for all configurations.

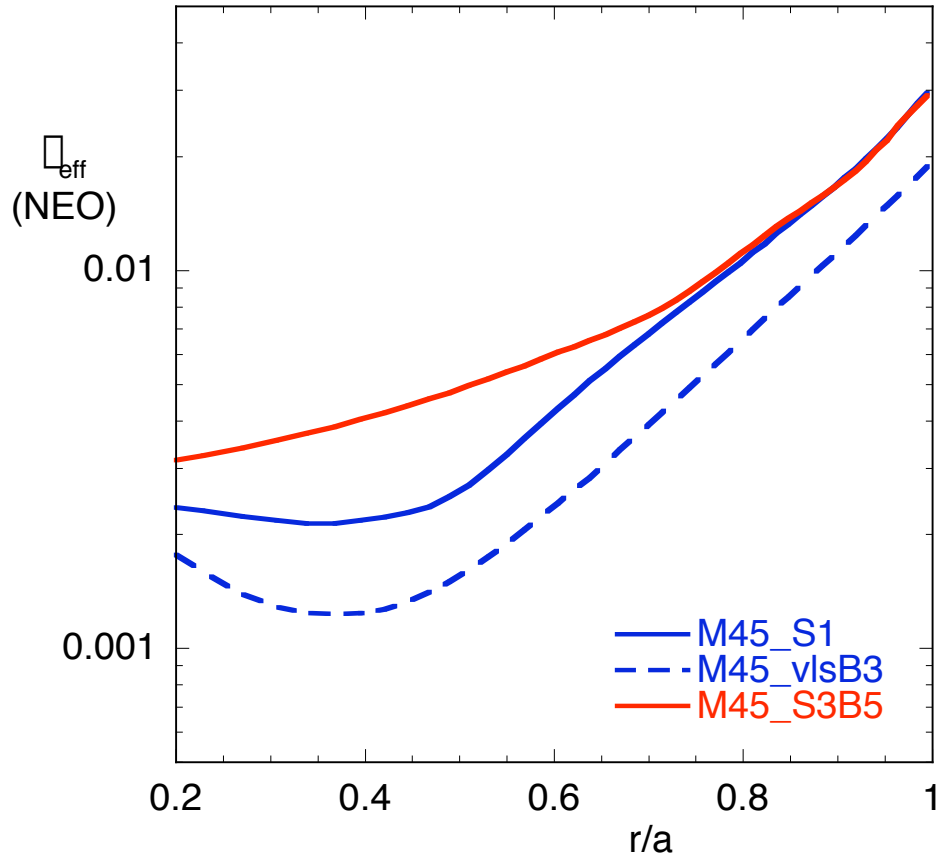




Effective ripple varies greatly, nearly constant rotational transform.

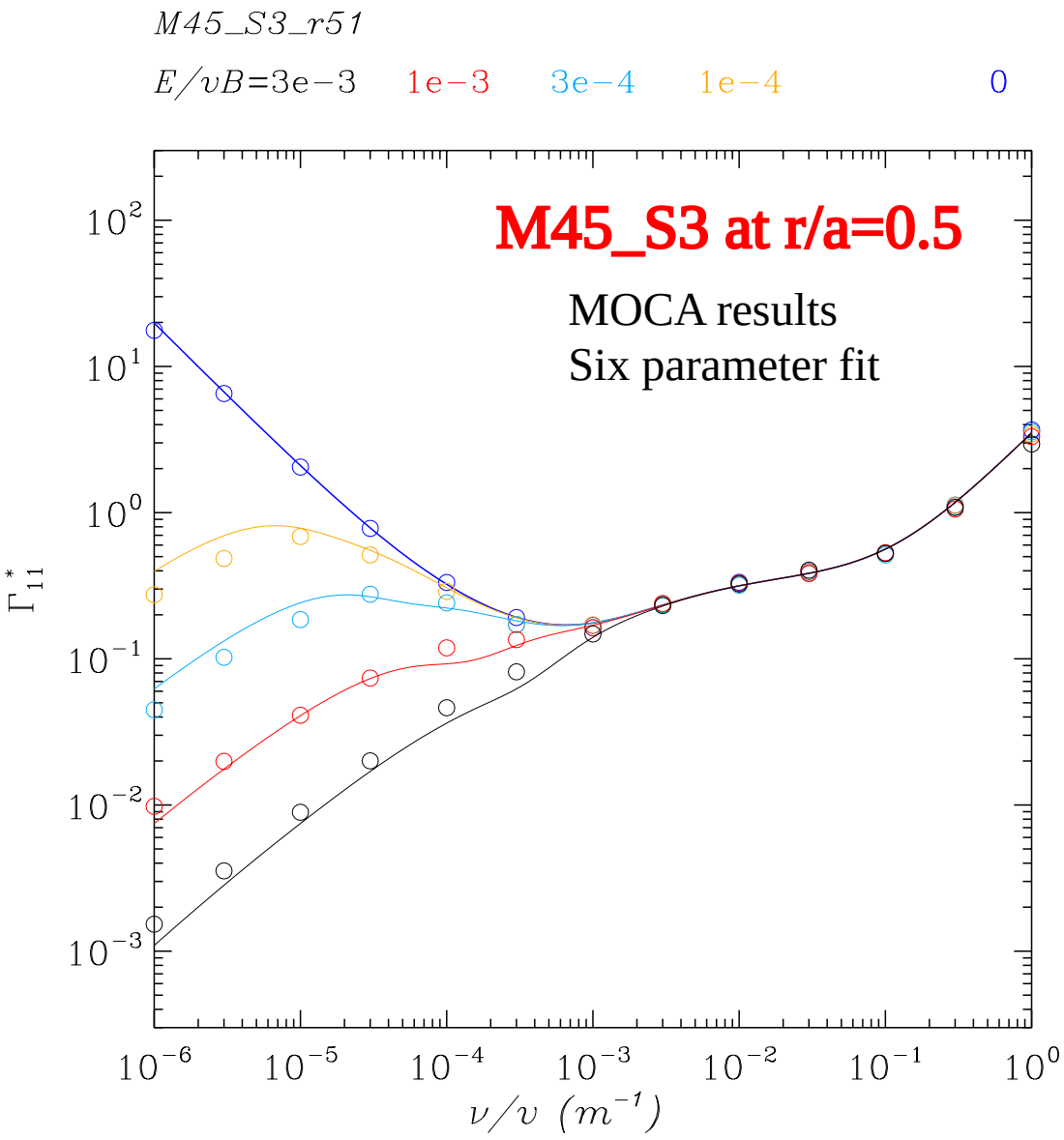
No simple relationship between rotational transform and effective ripple.

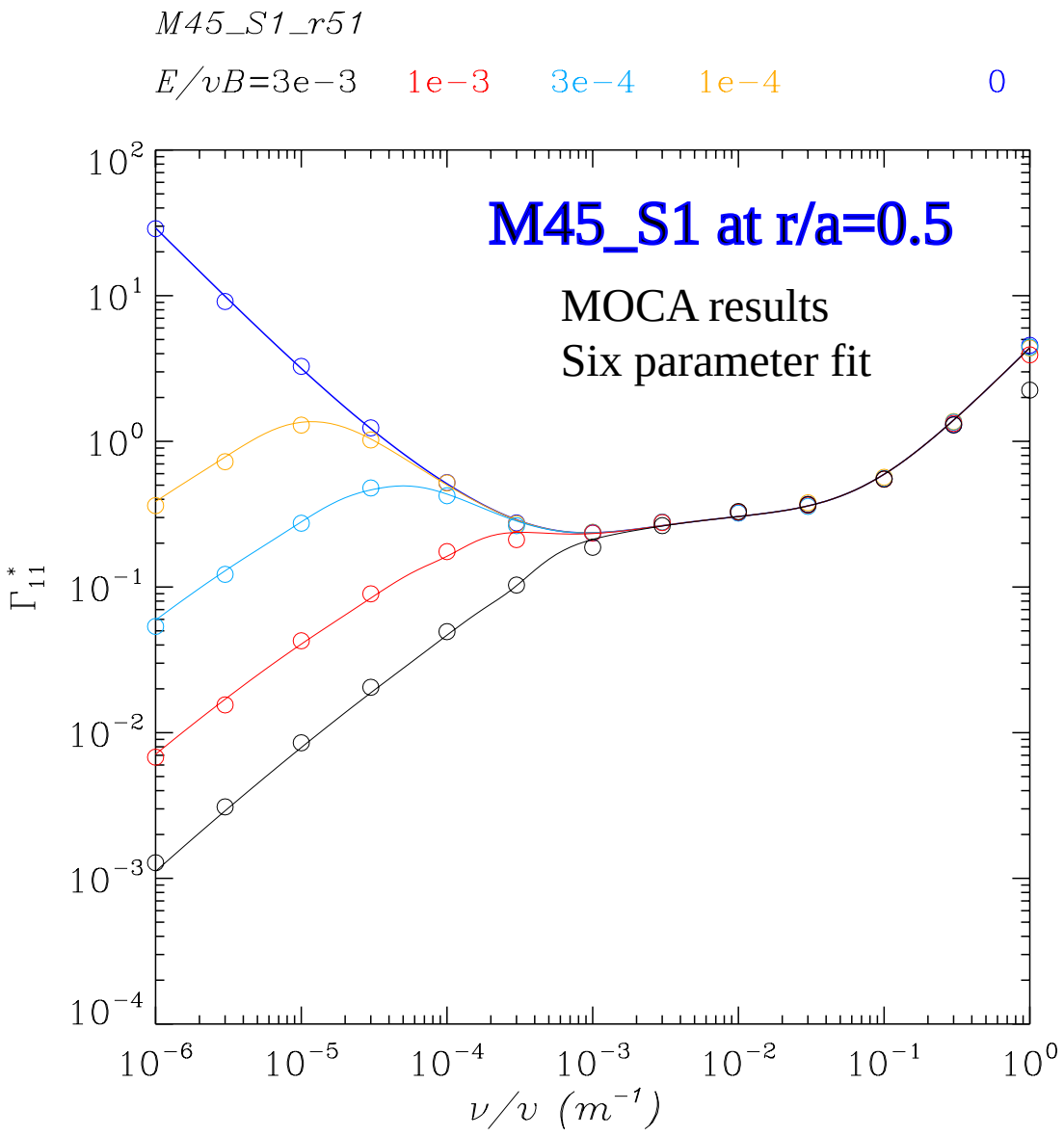


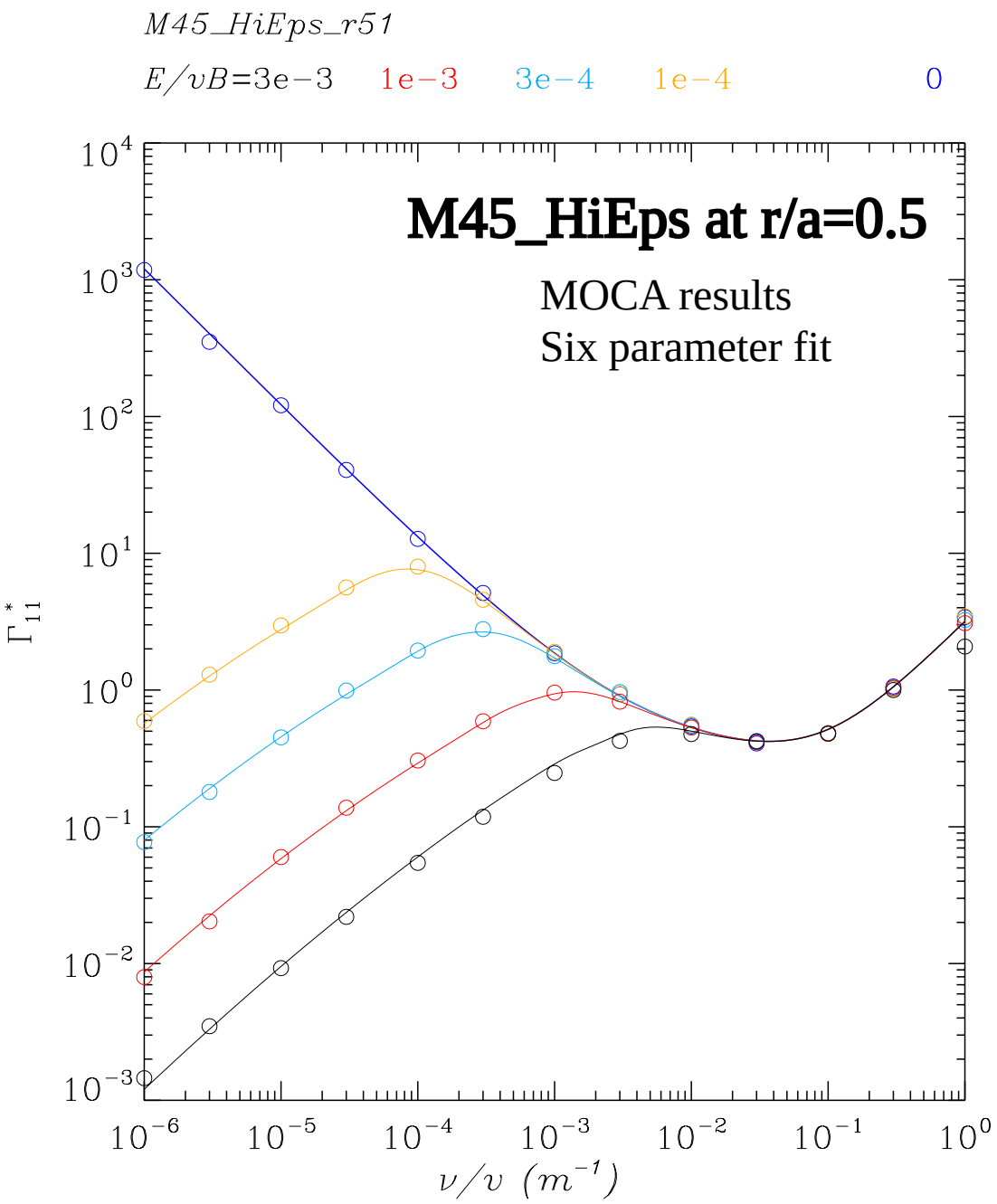


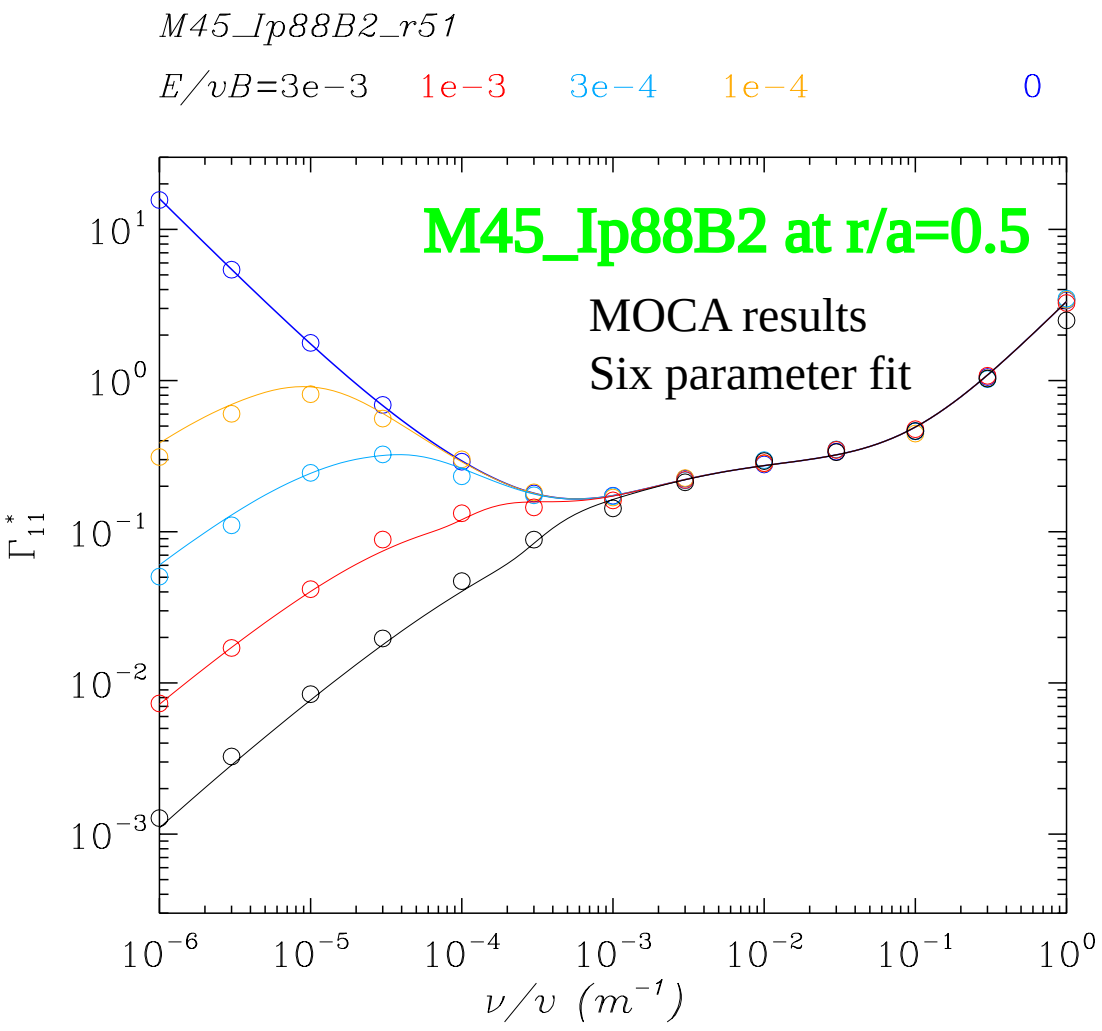
Can have both high magnetic shear and high effective ripple.

Can have both low magnetic shear and low effective ripple.


M45_S3_r51
E/vB=3e-3 1e-3 3e-4 1e-4 0
M45_S3 at r/a=0.5
MOCA results
Six parameter fit
Γ_{11}^*
10^2
10^1
10^0
10^{-1}
10^{-2}
10^{-3}
10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^0
$\nu/v\ (m^{-1})$


M45_S1_r51
E/vB=3e-3 1e-3 3e-4 1e-4 0
M45_S1 at r/a=0.5
MOCA results
Six parameter fit
Γ_{11}^*
$\nu/v\ (m^{-1})$
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-4}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}


M45_HiEps_r51
E/vB=3e-3 1e-3 3e-4 1e-4 0
M45_HiEps at r/a=0.5
MOCA results
Six parameter fit
Γ_{11}^*
$\nu/v\ (m^{-1})$
10^{4}
10^{3}
10^{2}
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-6}
10^{-5}
10^{-4}
10^{-3}
10^{-2}
10^{-1}
10^{0}


M45_Ip88B2_r51
E/υB=3e−3 1e−3 3e−4 1e−4 0
M45_Ip88B2 at r/a=0.5
MOCA results
Six parameter fit
Γ_{11}^*
10^{1}
10^{0}
10^{-1}
10^{-2}
10^{-3}
10^{-6} 10^{-5} 10^{-4} 10^{-3} 10^{-2} 10^{-1} 10^{0}
ν/υ (m^{-1})

Fitting first Step: $E_r = 0$

$$D^* = D^*_{1/\nu} + D^*_{axi} + D^*_{add}$$

$$D^*_{1/\nu} = \frac{16}{9\pi^2}\left(2\epsilon_{eff}\right)^{3/2}\frac{\iota}{R_0}\left(\frac{v}{\nu}\right)$$

$$D^*_{axi} = \left(\left(D^*_{bp}\right)^{3/2} + \left(D^*_{PS}\right)^{3/2}\right)^{2/3}$$

$$D^*_{bp} = \frac{D^*_b D^*_p}{(D^*_b + D^*_p)} \qquad D^*_b = \frac{8}{\pi}|b_{\mathrm{T}}|^{1/2}\frac{R_0}{\epsilon_t^2\iota}\left(\frac{\nu}{v}\right)$$

$$D^*_p = \left(\frac{b_{\mathrm{T}}}{\epsilon_t}\right)^2 \qquad D^*_{PS} = \frac{32}{3\pi}\left(\frac{b_{\mathrm{T}}}{\epsilon_t}\right)^2\frac{R_0}{\iota}\left(\frac{\nu}{v}\right)$$

$$D^*_{add} = \frac{\mathcal{A}(b_{\mathrm{T}}/\epsilon_t)^2}{\left((R_0/\iota)(\nu/v) + \mathcal{B}b_{\mathrm{T}}^2\right)^{1/2}}$$

Least-squares method finds the "best" values of $\mathcal{A}$, $\mathcal{B}$ and b_{T}.

Second Step: Non-zero E_r

$$D^* = D^*_{LMFP} + D^*_{axi} + D^*_{add}$$

$$D^*_{LMFP} = D^*_{an}\frac{1+\gamma\hat{\nu}^2}{1+\hat{\nu}^2}$$

where

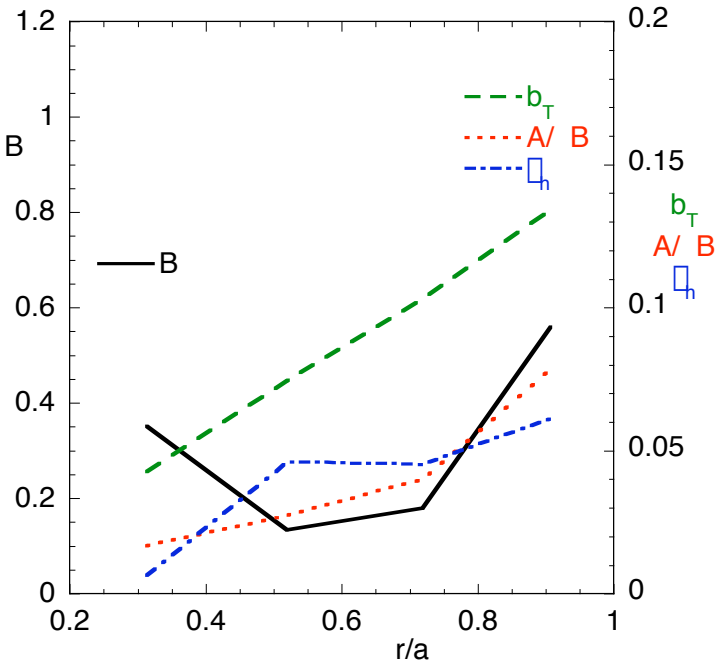
$$\gamma = \lim_{\hat{\nu}\to\infty}\left(\frac{D^*_{1/\nu}}{D^*_{an}}\right)$$

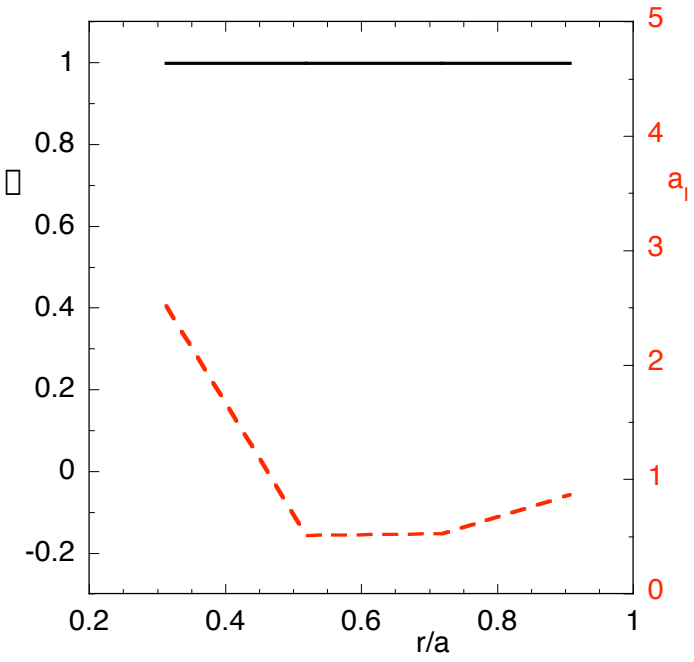
D^*_{an} is obtained from an analytic solution of the bounce-averaged kinetic equation for the simplest model field which can describe strong drift optimization:

$$B/B_0 = 1 + b_{\mathrm{T}}\cos\theta - \epsilon_h(r)(1-\sigma(r)\cos\theta)\,\cos\eta$$

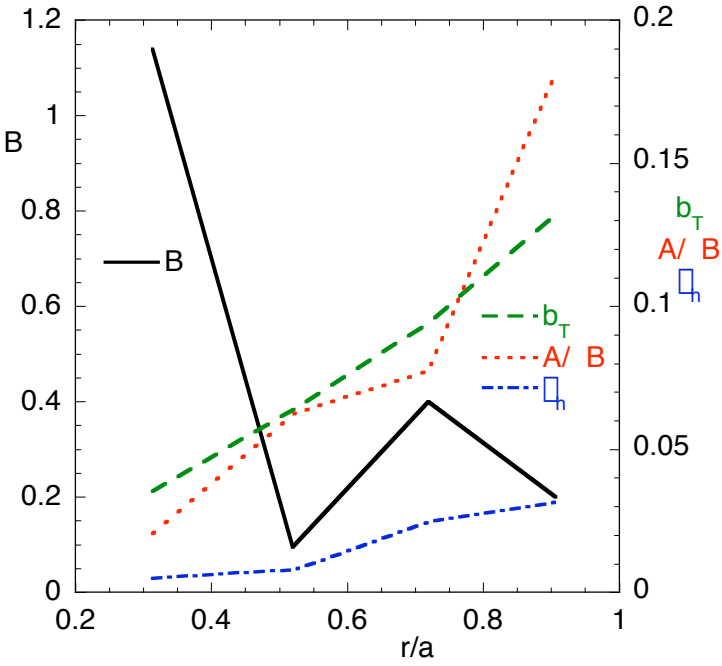
Least-squares method finds the "best" values of ϵ_h, σ and a_I (which appears in the boundary conditions and controls the importance of collisionless trapping and detrapping in the local ripples of B.

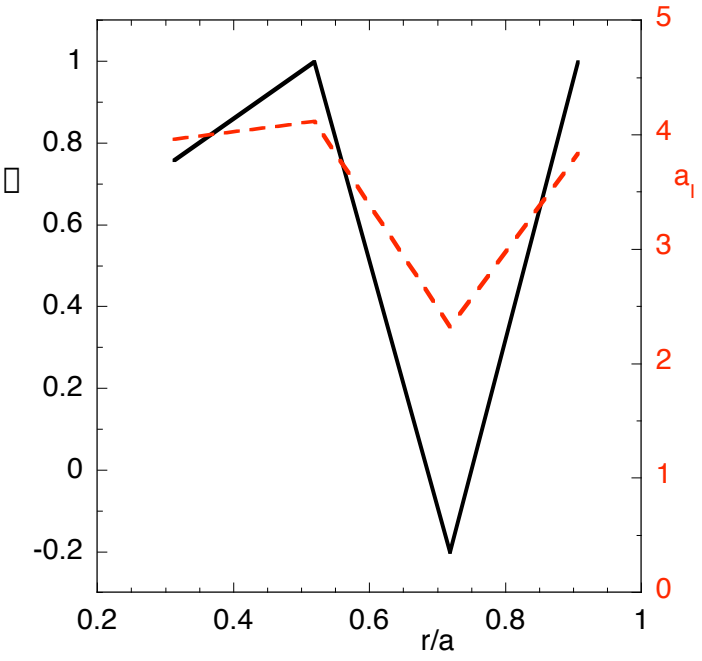
Fit parameters for M45_S3



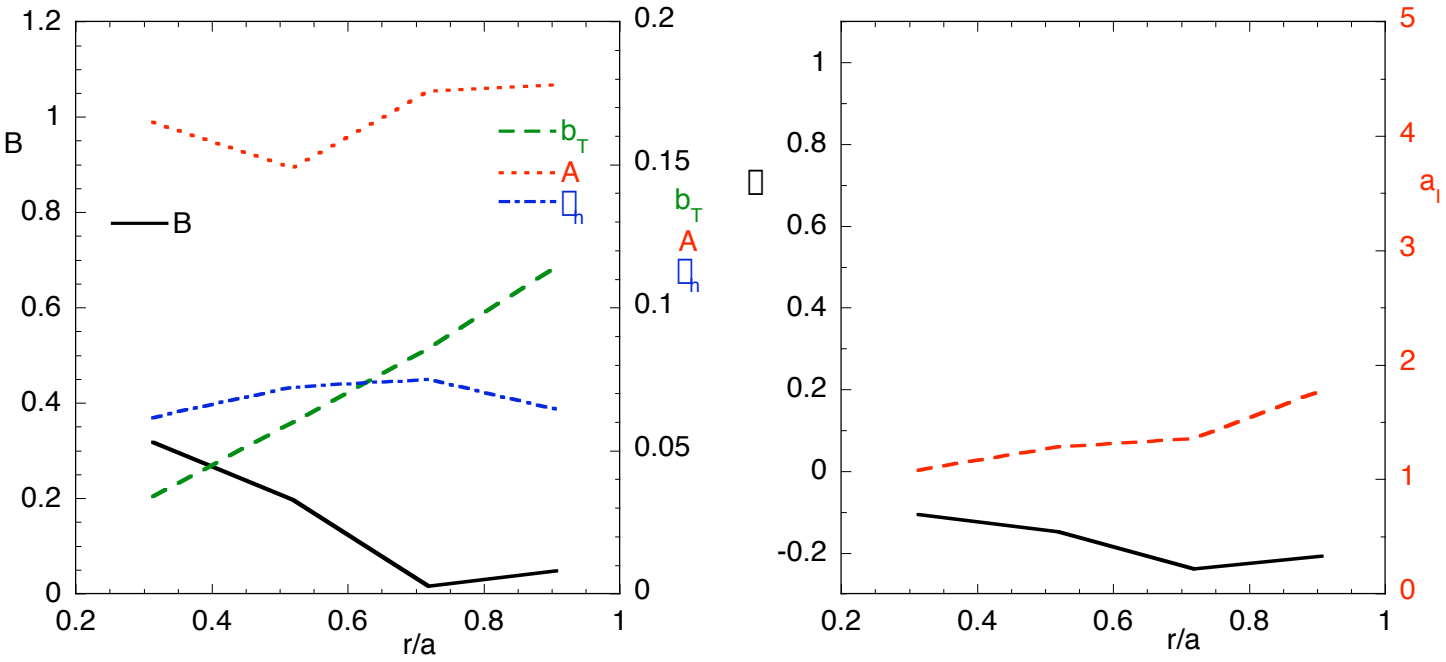


M45_S1

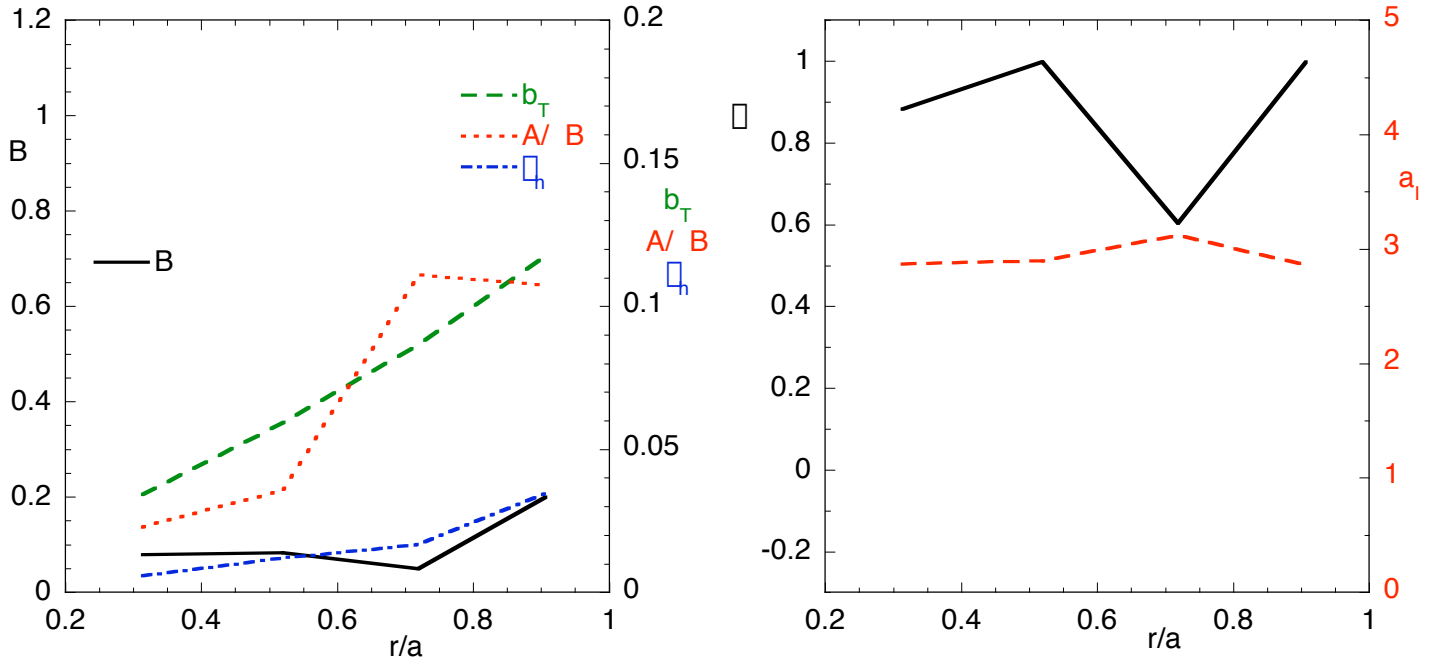




Fit parameters for M45_HiEps



M45_Ip88B2



Outstanding Issues

Fit parameters for the Beidler representation often do not have 'smooth' radial dependence.

- Perhaps this is a sign of too much flexibility in the representation? (faint hope)
- I will try constraining some parameters; reduced flexibility will help??

D_{11} are quite similar for very low ν/v, how to exploit this?

Evaluate the feasibility of 'brute force' diffusivity calculation.

- inside a transport code, one might do a minimal set of MOCA runs (plus NEO – small cost), then interpolate with Beidler fits for the Maxwellian convolutions.
- Perhaps 1 low, but finite, E_r and 1 high E_r is enough? (tried one case so far)
- minimum ν/v is the most expensive calculation; 1.E-5 is low enough? (ν_{eff} from NEO).
- what is a tolerable error on ν_{11} ? This sets the number of Monte Carlo particles.
- For 4000 particles, ν/v = 1.E-5, 20 CPU hours per E_r. takes 1 hour wall clock on 40 CPUs.
- Do 5 surfaces and interpolate, takes 1 hour on 200 CPUs per equilibrium calculation.
- Need a few equilibrium calculations to simulate a time dependent shot.
- NCSX needs 3-D neoclassical to calculate viscosity and E_r, not energy transport, so these calculations might be needed only for a portion of a simulation.
- The magnitude of calculation outlined here is (barely) tolerable for simulations of a few shots.
- Must have something much faster for routine calculations.

Stochastic mapping instead of orbit following?

Stochastic mapping would speed up the orbit averaging for neoclassical transport.
Might also work for fast-ion thermalization - that is expected to be the most costly simulation.

• Constructing a map takes a few hours on one 'typical' CPU. (for one magnetic surface?)
Questions to be investigated:
• How many maps are needed in NCSX?
There are many tiny ripples. Some do not form local wells, they only spoil the axisymmetry.
Do we need a map for every local well??!
• After magnetic equilibrium is recalculated how to map orbits from old maps to new maps?
Some local wells will disappear, and some new local wells will appear. What to do?

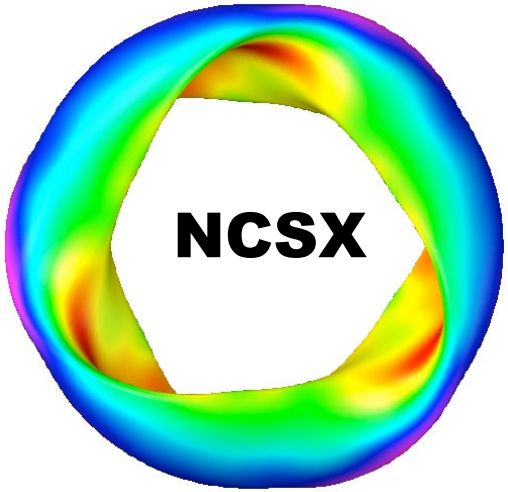
• Current applications of SMT involve electron orbits, will 'fat' orbits be accurately modeled?
Thermal ion orbit widths will not be a large fraction of the minor radius.
Full energy neutral beam ions will have radial excursions of ~a/4?
Compact representation of the maps uses a Taylor series; is this accurate for wide orbits?
Need to test the accuracy. Could test this in a tokamak configuration first.

• For simulation of fast-ion heating, fusion reactions, charge-exchange loss, … we need the time spe
in each radial zone – not just the orbit locations at each 'cut'.
This requires a significant amount of memory, problematic if the number of 'cuts' is large.
• Fast-ions begin with an anisotropic distribution, but pitch-angle scattering broadens the distribution

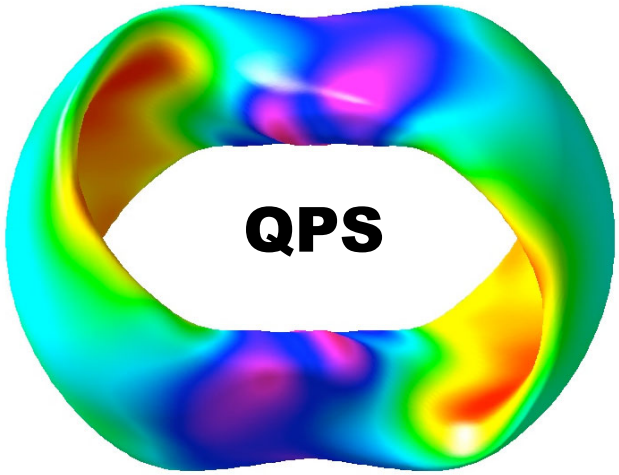
• All ion orbits in W7-X are small fractions of minor radius, so SMT works there?

Devices that test the 3 possible forms of quasi-symmetry are either operating or planned in the U.S.

Univ. of Wisconsin



Princeton Plasma Physics Lab.



Oak Ridge National Lab.