Optimization of Neoclassical Transport in Stellarators

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OVERVIEW

- Introduction
- Optimization Target
- Optimization Procedure
- Application to TJ-II
- Summary

INTRODUCTION

- Optimizations of stellarators are planned.

- Optimization tool is developed in real space coordinates (e.g., TJ-II, U-2M).

- Quantity of interest - total stored energy.

  - effective ripple ϵ_{eff}
    - $\mathbf{B}$-field (Biot-Savart)
      - coil currents
      - coil positions
      - coil angles
      - coil geometry

TOTAL STORED ENERGY

Normalized stored energy in plasma:

$$\hat{W} = \int_0^a dr\, r\, \hat{n}(r) \left(\int_r^a \frac{dr'}{r'\, \epsilon_{\mathrm{eff}}^{3/2}(r')} \right)^{2/9} \tag{1}$$

a . . . plasma radius (↑)
r . . . effective radius
$\hat{n}$ . . . normalized plasma density (↑; two profiles)
ϵ_{eff} . . . effective ripple (↓; computed by NEO)

EFFECTIVE RADIUS

- Definition in differential form: $Sdr = dV$ (S area of the magnetic surface, V volume limited by the magnetic surface) see V. V. Nemov et al., Phys. Plasmas **6**, 4622 (1999).

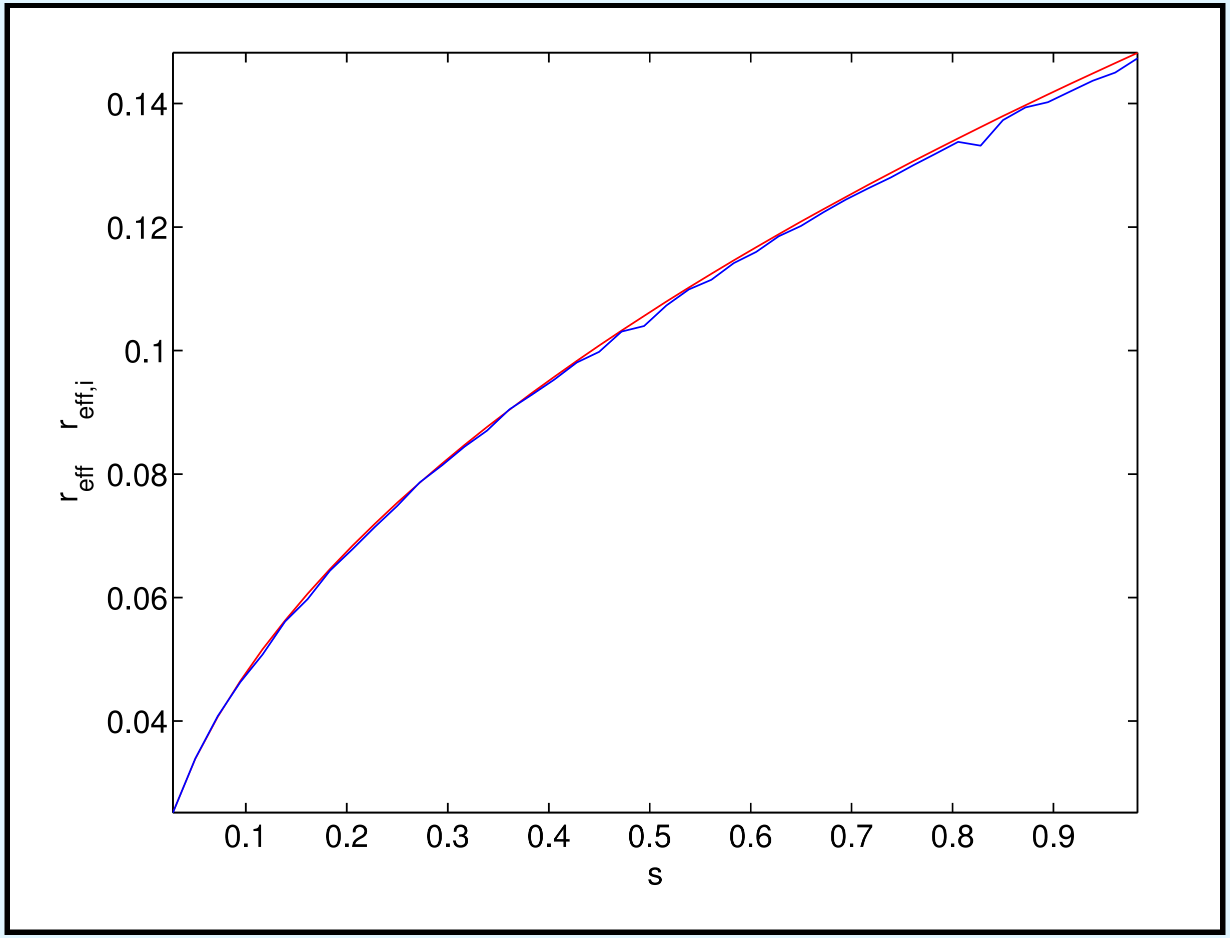
- Desirable: computation of $r_{\rm eff}$ during field line integration.

Introducing new definition of an effective radius: $r = 2V/S$ (V volume limited by the magnetic surface area S)

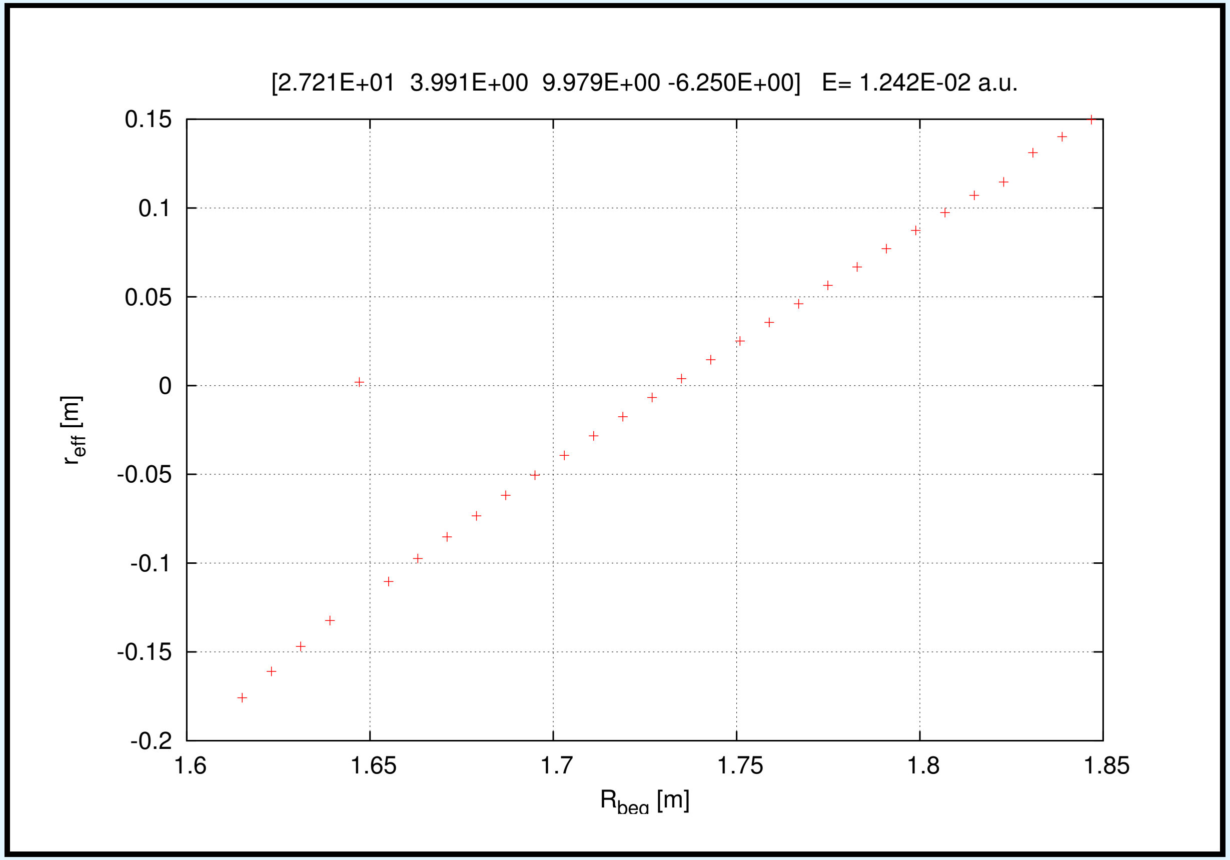
$$r = \frac{2}{3}\frac{\int dS \mathbf{r} \cdot \frac{\nabla\psi}{|\nabla\psi|}}{\int dS} = \frac{2}{3}\frac{< \mathbf{r} \cdot \nabla\psi >}{< |\nabla\psi| >} = \frac{2}{3} \lim_{L_S \to \infty} \frac{\int_0^{L_S} \frac{dl}{B}\mathbf{r} \cdot \nabla\psi}{\int_0^{L_S} \frac{dl}{B}|\nabla\psi|} \qquad (2)$$

$\nabla\psi$ . . . vector normal to the flux surface
$\mathbf{r}$ . . . radius vector
B . . . module of the magnetic field
dl . . . distance measured along the magnetic fieldline

Effective Radius

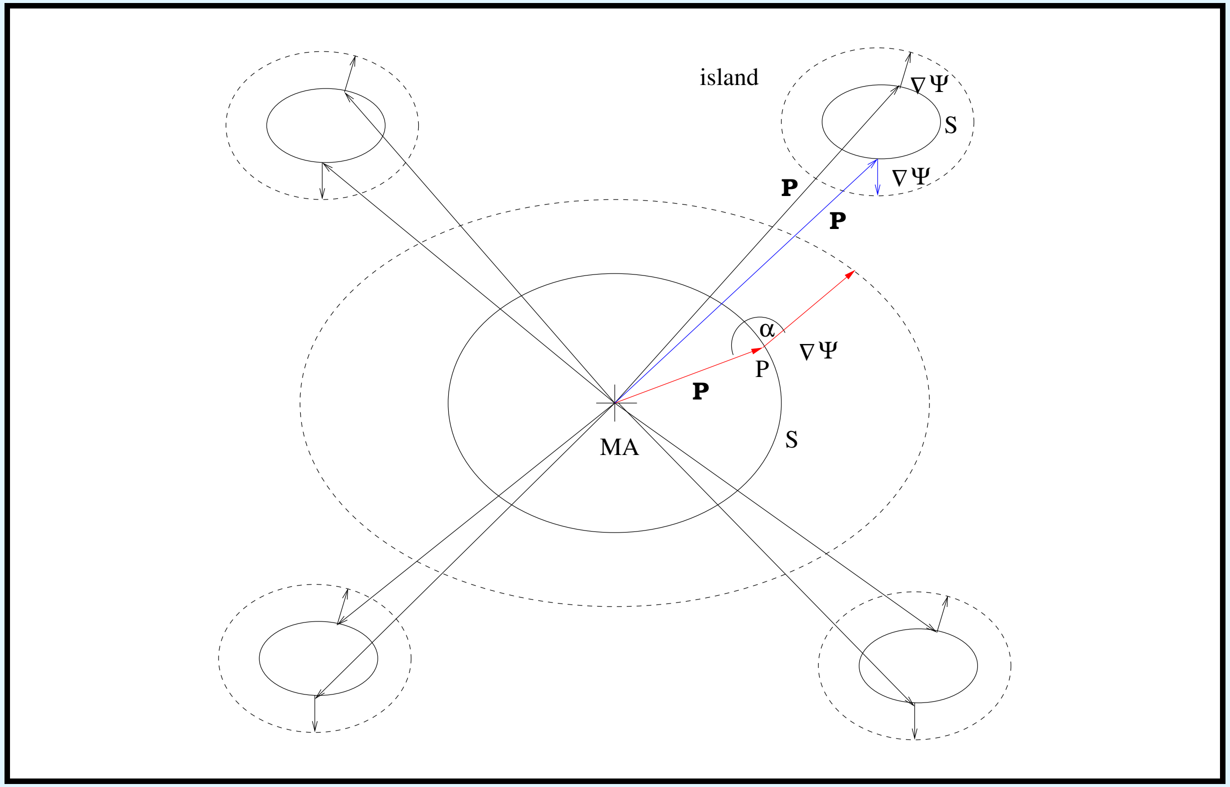


Effective Radius



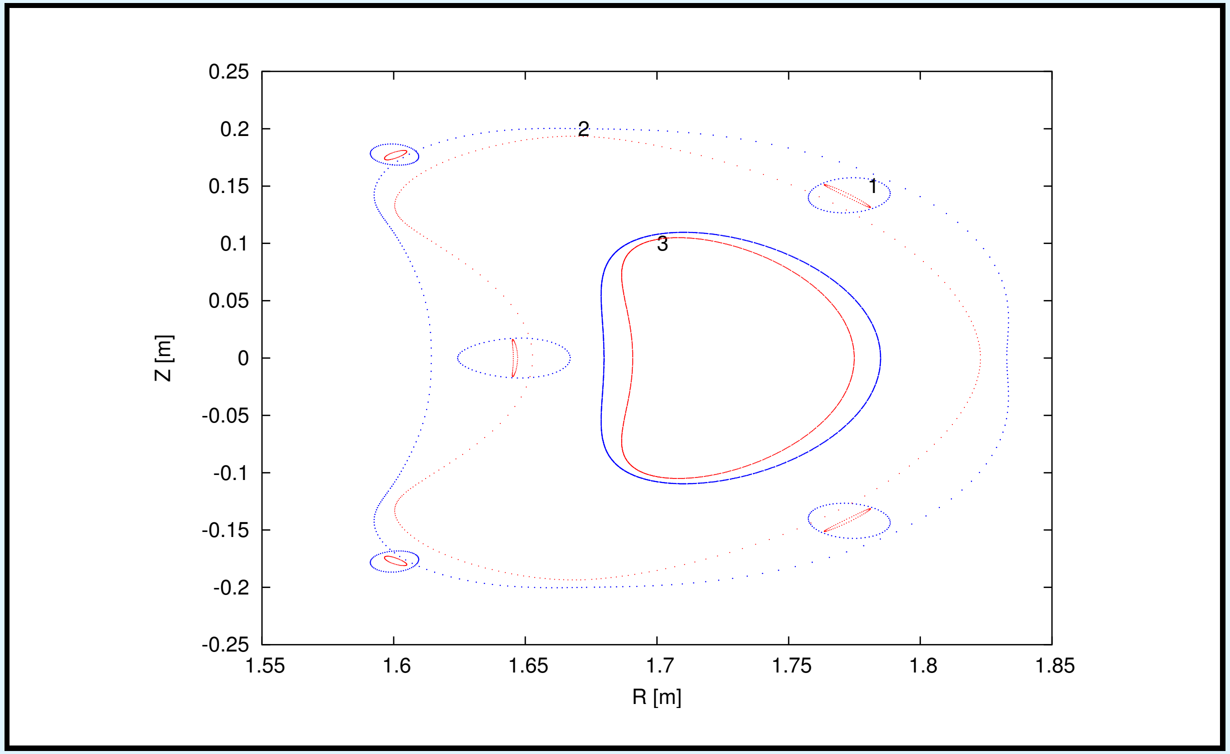
MAGNETIC ISLANDS - DETECTION

Principle of island detection



MA ... magnetic axis, S ... magnetic surface

Normal vector $\nabla\Psi$ is sensitive to close islands



Flux surfaces (red) and the corresponding $\nabla\Psi$ (blue) for an island chain (1) and two flux surfaces (2,3).

Optimization Procedure

Four key "ingredients" for the implementation of simulated annealing:

1. Choice of a starting configuration C_a and computation of the energy $E(C_a)$.
2. Creating a configuration C' out of C using a proper rule and computation of the energy $E(C')$; let $\eta = (E(C') - E(C))/T$, with the artificial temperature T.
3. If $\eta \leq 0$ accept C' as new configuration: $C' \to C$ and go on at step 2 or stop if the path is long enough for the temperature T.
4. If $\eta > 0$ choose a random number r, distributed equally in $0 \leq r < 1$. If $r < \exp(-\eta)$, accept C', $C' \to C$, otherwise reject C' and go on with step 2.

Thermodynamical Equilibrium

Number of accessible neighbors is used.

Starting Temperature

$$< E >= \frac{1}{L} \sum_k E(k), < E^2 >= \frac{1}{L} \sum_k E^2(k)$$

Heat capacity:

$$C(T) := \frac{d < E >}{dT} = \frac{< (\delta E)^2 >}{T^2} \quad (3)$$

Integration of the heat capacity (Eq. 3) leads to:

$$< E > (T) - < E > (\infty) \approx -\frac{< (\delta E)^2 > (\infty)}{T}$$

Estimation for the starting temperature:

$$T_a = \sqrt{< (\delta E)^2 > (\infty)} \quad (4)$$

Cooling Strategy

Empirical rule: $T_k = T_a q^k$, with $0 < q < 1$ e.g. $q = 0.95$
Disadvantage: Phase transitions

Automatic adaption desired:
Occupation probabilities $w(C, T)$ for two temperatures T_k and T_{k+1} should be close to each other.

Formulated as:

$$\frac{1}{1+\delta} < \frac{w(C,T_k)}{w(C,T_{k+1})} < 1+\delta$$

For $w(C,T)$ the Boltzmann distribution for equilibrium is used:

$$w(C,T) \sim \exp\left(-\frac{E(C)-E(0)}{T}\right)$$

Scheme for cooling:

$$T_{k+1} = \frac{T_k}{1+\frac{T_k}{3\sqrt{<(\delta E)^2>}}\ln\left(1+\delta\right)} \tag{5}$$

Stopping Criterion

Requested, that $< E > (T_e) - E_0$ is small.
Formulated as

$$\frac{< E > (T_e) - E(0)}{< E > (T_a) - < E > (T_e)} < \varepsilon$$

Leads to the criterion

$$\frac{< (\delta E)^2 > (T_e)}{T_e \left(< E > (T_a) - < E > (T_e)\right)} < \varepsilon \qquad (6)$$

Speeding Up . . .

- Computing on a grid
- Storing computed configurations
- Parallelization

ALGORITHM

```
MPI_INIT()
MPI_COMM_RANK(myid)
IF (myid .EQ. 0) THEN     ! M A S T E R
 C1 := SetInitialSolution()
 T := WarmingUp()
 do
       do
             C2 := Neighbor(C1)
             (E1 = Energy(C1))
             MPI_Bcast()
             MPI_Send()
             MPI_Irecv()
             E2 = Energy(C2)
             Δ Energy := E2 - E1
             if Δ Energy < 0 or
             Accept(Δ Energy, T)
                   C1 := C2
       until Equilibrium()
       T := DecrementT()
      until Frozen()
ELSE        ! S L A V E
      MPI_Bcast()
      MPI_Irecv()
      Field Line Integration()
      MPI_Send()
END IF
```

Message Passing Interface MPI

algorithm structure (solver)

problem specific functionality (user implemented)

temperature scheduling (scheduler)

Application to TJ-II

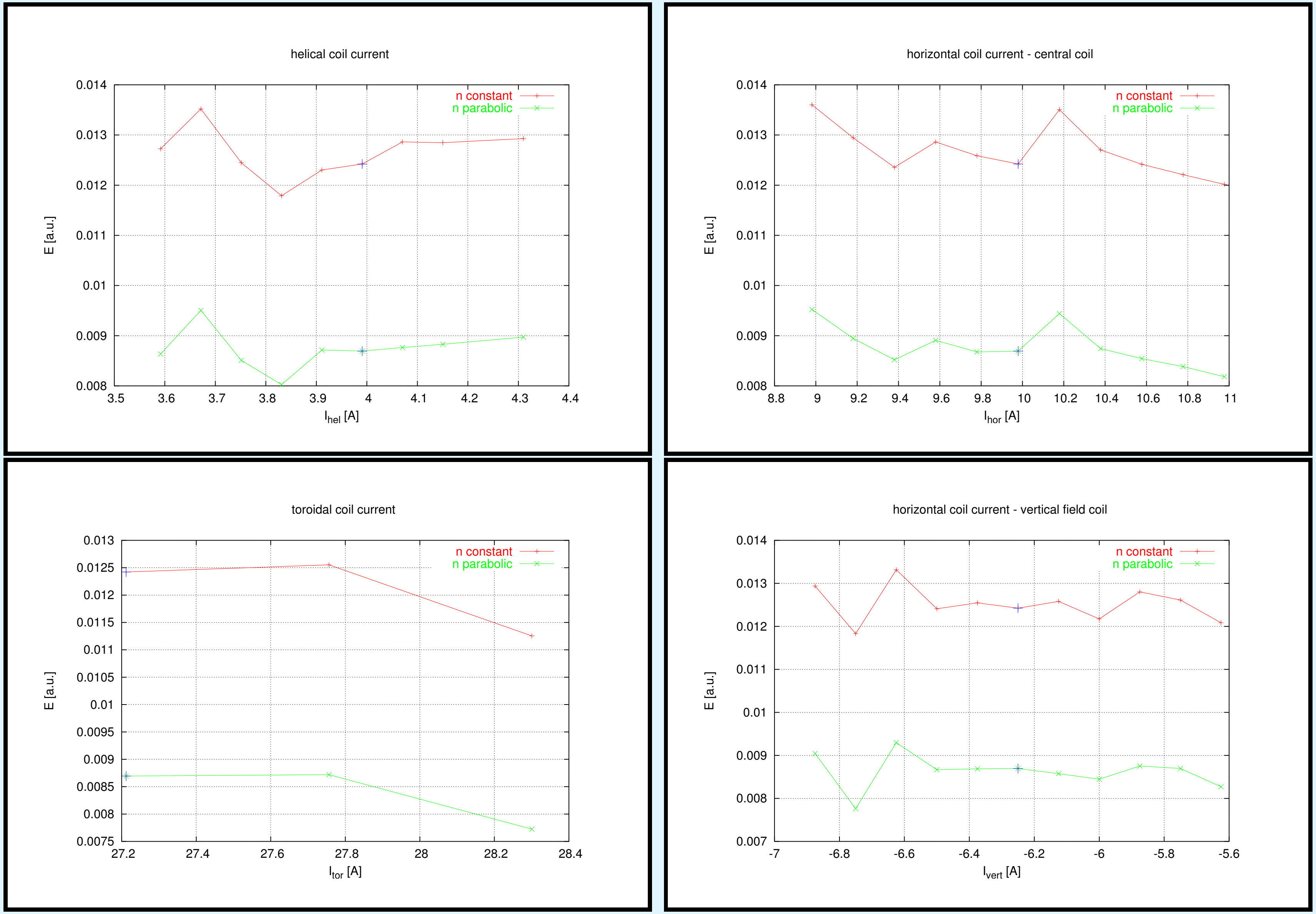
- Variation of four currents:

  I_{tor} toroidal coil current
  I_{hel} helical coil current
  I_{hor} horizontal coil current corresponding to the central coil
  I_{vert} horizontal coil current corresponding to the vertical field coils

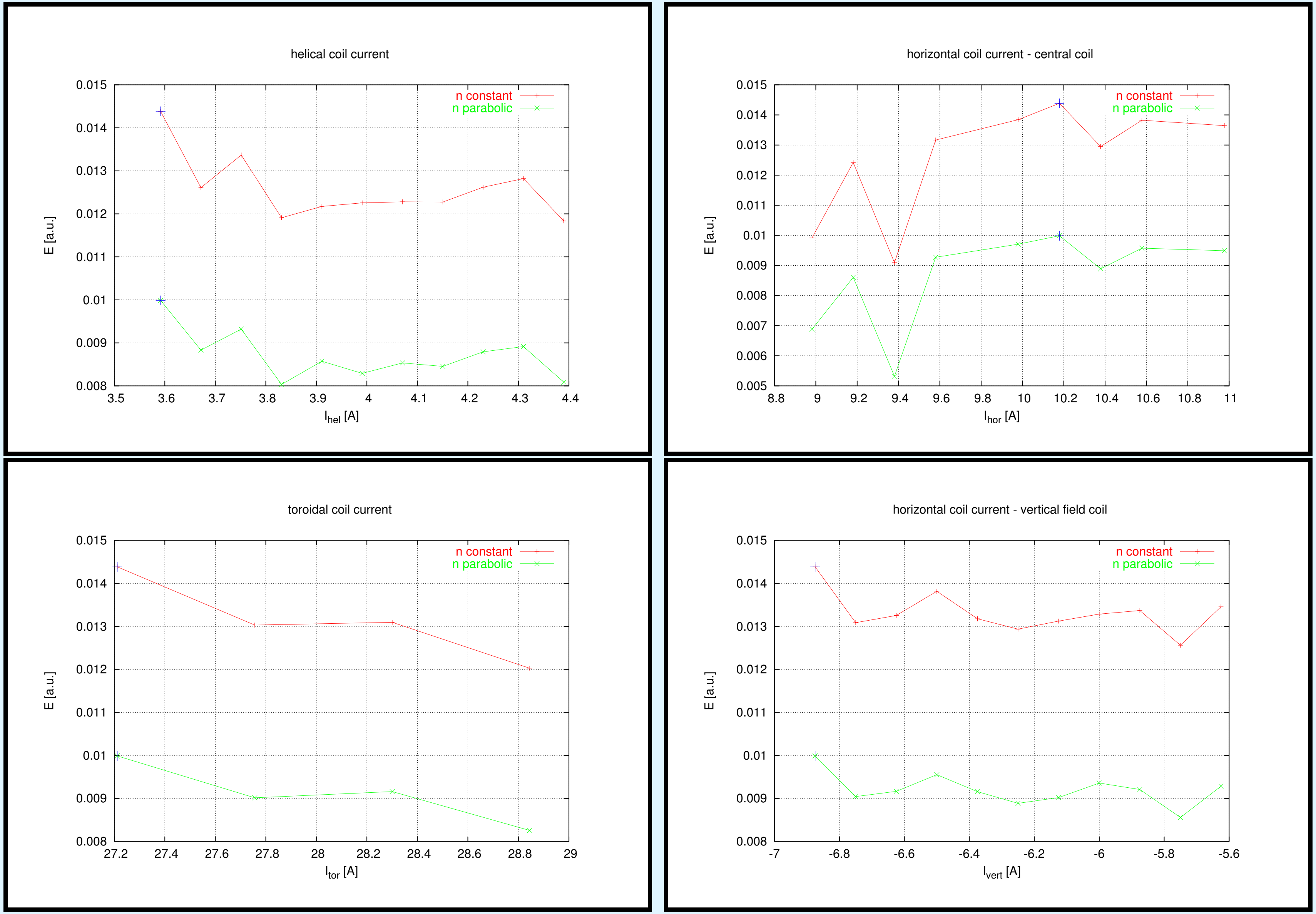
- Two models for the particle density have been applied

  $n = constant$ (red)
  $n = 1 - \alpha * (r/a)^2$ with $\alpha = 0.8$ (green)

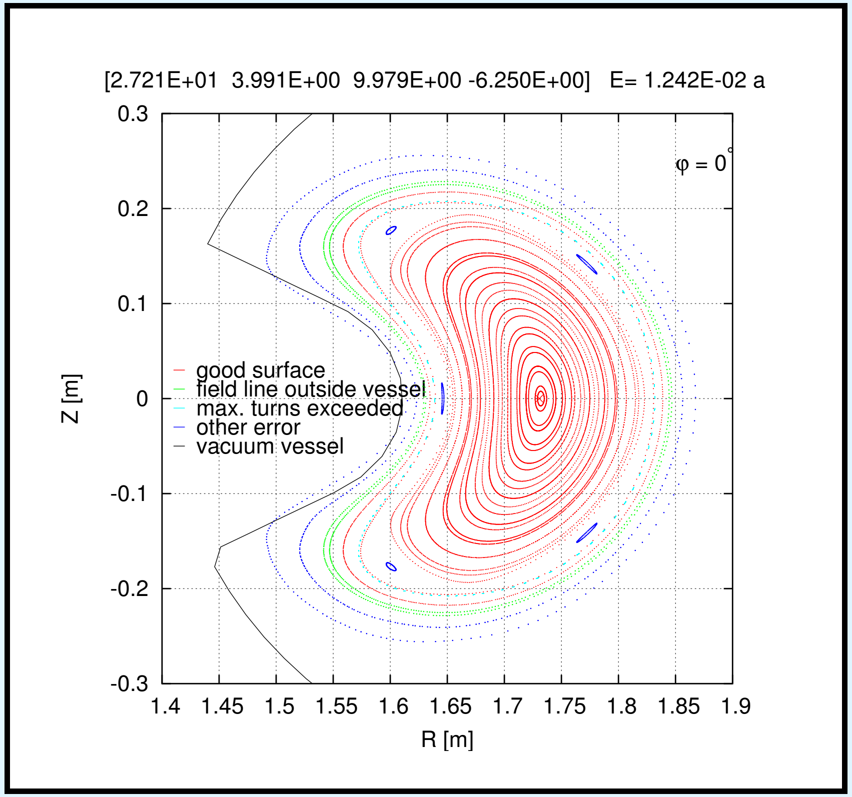
Normalized stored energy $\hat{W}$ - ”standard” configuration

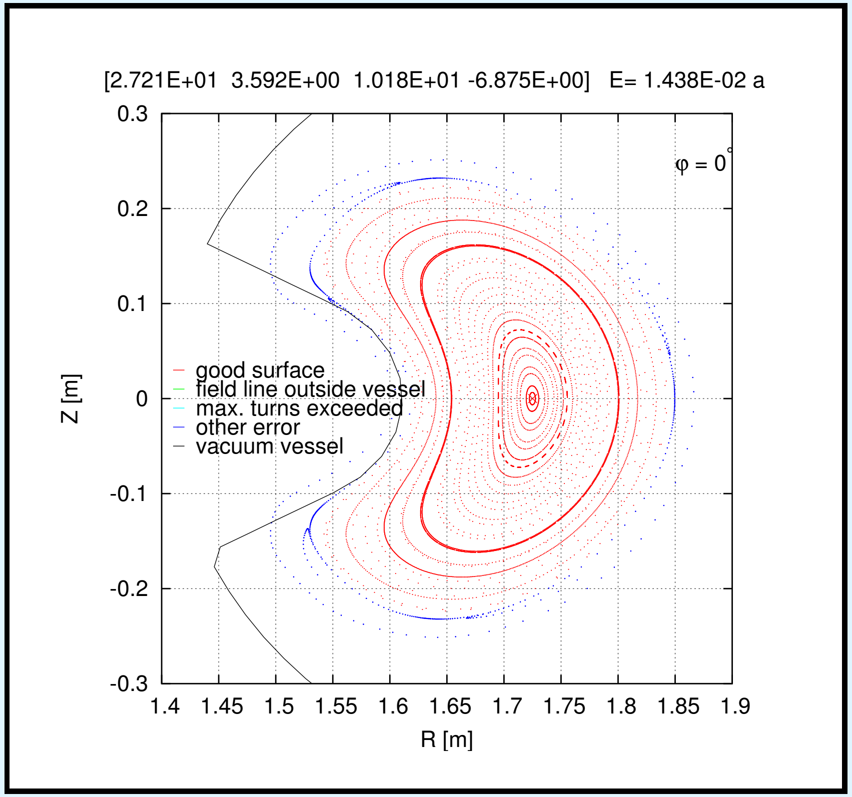


Normalized stored energy $\hat{W}$ - "best" configuration



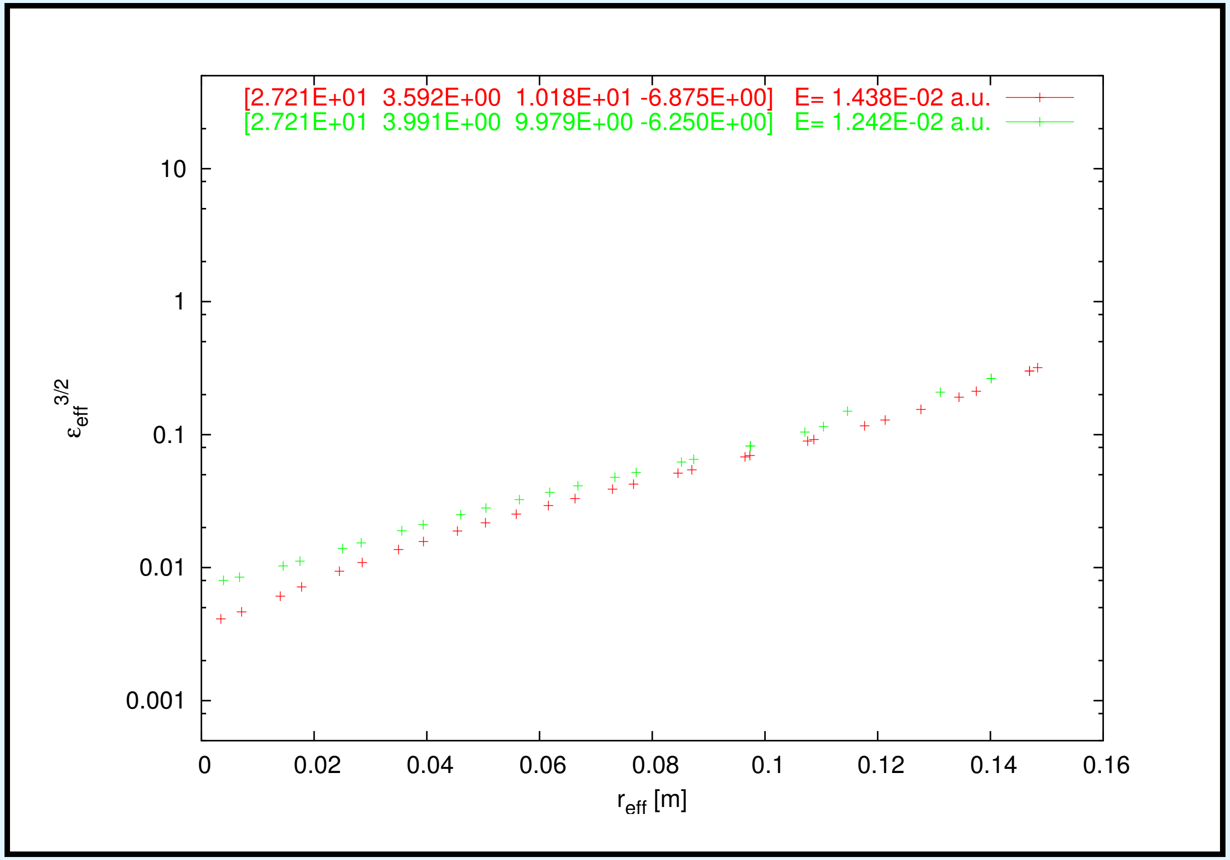
Cross-sections





Cross-sections at $\varphi = 0^\circ$ for the TJ-II "standard" configuration (left) and for the configuration with enhanced energy confinement (right).

Effective Ripple ϵ_{eff} vs. Effective Radius r_{eff}.



Plots for the TJ-II "standard" configuration (green) and for the configuration with enhanced energy confinement (red).

SUMMARY

- A new tool for optimizing existing stellarators, based on the technique for evaluating the effective ripple is presented and has been applied to TJ-II.
- The magnetic field computed directly from the coil currents is used for the computation of the effective ripple.
- Configurations with enhanced total stored energy in plasma have been found. Comparing the best configuration with the ”standard” configuration it can be seen that neoclassical transport across the flux surfaces is diminished.

- Experiments?
- Stability analysis?
- Interesting region - operation point?

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Thank you for your attention!