Generalized Spitzer Function with Finite Collisionality in Toroidal Plasmas

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The drift kinetic equation solver NEO-2 [1] which is based on the field line integration technique has been applied to compute the generalized Spitzer function in a tokamak with finite plasma collisionality. The resulting generalized Spitzer function has specific features which are pertinent to the finite plasma collisionality. They are absent in asymptotic (collisionless or highly collisional) regimes or in results drawn from interpolation between asymptotic limits. These features have the potential to improve the overall ECCD efficiency if one optimizes the microwave beam launch scenario accordingly.

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1 Adjoint approach for ECCD computations

The standard method for calculation of ECCD generated current in tokamaks and stellarators is the adjoint approach where the flux surface averaged current is given by a convolution of a quasilinear source term with the adjoint generalized Spitzer function (local current drive efficiency). This function is well studied for high collisionality regimes where it is equivalent to the classical Spitzer function, and in the long mean free path (LMFP) regime where a bounce averaging procedure can be used to reduce the dimensionality of the problem to 2D. For a detailed discussion of various approaches to the LMFP regime see Reference [2]. For benchmarking results of various codes and pertinent models especially for ITER see References [3, 4] and citations within these papers. In the general case of finite plasma collisionality, the kinetic problem to compute the local efficiency remains essentially 3D for tokamaks and 4D for stellarators. For this reason, this general case is not studied as well as cases in the asymptotic limits.

In the linear approximation, generation of steady state plasma current by ECCD is described by the linearized kinetic equation:

\[ v \cdot \nabla f - L_{CL} f = Q_{RF}, \]

(1)

where \( f \) is the perturbation of the electron distribution function, \( v \) is a unit vector along the magnetic field, \( v \parallel \) is parallel velocity, \( L_{CL} \) is a linearized collision integral and \( Q_{RF} \) is a quasilinear particle source in the phase space. Using the adjoint approach (see e.g., Ref. [5]), flux surface averaged parallel current density is expressed through the adjoint generalized Spitzer function \( \bar{g} \) (current drive efficiency) as follows:

\[ \langle j\parallel \rangle = \left\langle e \int d^3p v\parallel f \right\rangle = eL_c \left\langle \int d^3p \bar{g} Q_{RF} \right\rangle, \]

(2)

where \( \langle \ldots \rangle \) denotes "flux surface average" (average over the volume between neighboring flux surfaces), \( e \) is electron charge, \( L_c \) is free path length given by \( L_c = T_e^2/(\pi n_e e^4 \Lambda) \) where \( n_e, T_e, \) and \( \Lambda \) are electron density, temperature and Coulomb logarithm, respectively. The adjoint function \( \bar{g} \) in (2) is expressed through the generalized Spitzer function \( g \) as follows, \( \bar{g}(v\parallel) = g(-v\parallel) \), where \( g(v\parallel) \) is the solution to the conductivity problem.

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\( v_{||} \cdot \nabla f_{MG} - L_{CL} f_M = (1/l_e) v_{||} f_M, \) and \( f_M \) is a Maxwellian. Using the momentum space flux density due to the wave-induced quasilinear diffusion, \( \Gamma_{RF} \), parallel current density is expressed via derivatives of the adjoint Spitzer function

\[
Q_{RF} = -\frac{\partial}{\partial p} \cdot \Gamma_{RF}, \quad (j_p) = e l_e \left( \int d^3 p \frac{\partial g}{\partial p} \cdot \Gamma_{RF} \right). \tag{3}
\]

Within geometrical optics used for calculation of ECRH/ECCD, quasilinear flux density can be described in local approximation. In this approximation \( \Gamma_{RF} \) differs from zero in the velocity space only at the resonance line where the (multiple) cyclotron resonance condition taking into account Doppler shift is fulfilled, \( \omega = n \omega_c + k_{\parallel} v_{||} \), where \( \omega, \omega_c, n \) and \( k_{\parallel} \) are wave frequency, relativistic cyclotron frequency, cyclotron harmonic index and parallel wave vector, respectively. For weakly relativistic electrons these resonance lines are close to circles on the \((p_{\perp}, p_{\parallel})\) plane whose centers are located at \( p_{\perp} = 0 \) axis. In this case, the largest component of the quasilinear flux density is over perpendicular momentum. Therefore, as follows from (3), the behavior of the derivative of \( g \) over perpendicular momentum at the resonance curve is of main importance for ECCD.

2. Codes

NEO-2 is a solver for the drift kinetic equation (DKE) which is based on the method of field line tracing. Originally, it was developed to compute monoenergetic transport coefficients with the special aim of good convergence in low collisionality regimes. This is accomplished through adaptive level placement over the normalized magnetic moment \( \eta = p_{\perp}^2 / (p^2 b) \) with \( b \) being the normalized magnetic field module. With this adaptive placement NEO-2 effectively resolves steep behavior of the distribution function \( f \) at the trapped passing boundary.

The usefulness for ECCD comes from the fact that NEO-2 can also use the full linearized collision operator including energy diffusion and (momentum conserving) integral response of the background particles. This is in contrast to most DKE solvers where momentum conservation is completed with momentum correction techniques applied to flux surface averaged quantities. Thus local information within a flux surface is lost. This makes NEO-2 especially suited for ECCD computations where power deposition is highly localized.

In the code NEO-2, the dependence of the generalized Spitzer function on kinetic energy is presented in the form of expansion over the associated Laguerre polynomials of the order 3/2 (Sonine polynomials),

\[
g(r, p) = \sum_{m=0}^{M} g_m(r, \eta) L_{3/2}^{(m)} \left( \frac{p^2}{2 m_e T_e} \right). \tag{4}
\]

Expansion coefficients \( g_m \) are discretized on the adaptive grid over \( \eta \) which reduces the kinetic equation to a set of coupled ordinary differential equations with the independent variable being the distance counted along the field line. This set of equations is solved by numerical ODE integration (see [1] for details).

In NEO-2 the perpendicular (cross field) rotation of the plasma within the flux surface (which is mainly in the poloidal direction), in particular the rotation due to the radial electric field is ignored in the kinetic equation. This limits its usage in the computation of neoclassical transport data base entries for monoenergetic transport coefficients. For ECCD computations, however, radial electric fields play practically no role and therefore NEO-2 is not limited to certain regimes. At the moment, a non-relativistic collision operator is used, but this is not an intrinsic limitation and can be improved during further development. The main limitation at the moment is the speed of the code which restricts practical usage to tokamak problems. This limitation is caused by the stiffness of the ODE set resulting from the discretization of the kinetic equation over normalized magnetic moment \( \eta \) and momentum module.

The code SYNCH [6] computes the generalized Spitzer function and its derivatives in the long mean free path regime in general toroidal geometry and all types of flux coordinates. Therefore it is suitable for tokamaks as well as stellarators. Originally, this fully relativistic code was developed for studies of passive cyclotron current drive in tokamaks [6] and has recently been upgraded to general geometry. In the context of this paper it is used to compute the reference cases for the collisionless limit.
3 Results

To illustrate the difference between collisional and collisionless cases, the generalized Spitzer function $g$ and its derivatives $\partial g/\partial \nu_{\perp}$. $\partial g/\partial \nu_{\nu}$ are presented as functions of the pitch parameter $\lambda = \nu_{\nu}/\nu$. All computations were done for a circular tokamak with major radius $R_0 = 100$ cm, minor radius $r = 25$ cm ($A = 4$), rotational transform $\kappa = 0.62$, electron density $n_e = 6.65 \cdot 10^{13}$ cm$^{-3}$, electron temperature $T_e = 1 \cdot 10^4$ eV. These parameters result in collisionality $2\pi q R / (L_{ae}^{1/2}) = 0.257$. For NEO-2 computations the number of Laguerre polynomials was 6. To highlight the different aspects of the influence of collisions on $g$, results are presented for four different positions on the flux surface, namely $B_{min}$ (outer side), $B_{max}$ (inner side), top and bottom. Velocities correspond approximately to the following important transport regimes: (i) Pfirsch-Schlüter regime, $\nu = 0.5\nu_{\nu}$ (sub-thermal); (ii) plateau regime, $\nu = \nu_{\nu}$ (thermal); banana regime, $\nu = 2\nu_{\nu}$ (intermediate); and deep banana regime, $\nu = 3\nu_{\nu}$ (fast). Here, $\nu_{\nu} = \sqrt{2T_e/m_e}$ is the thermal velocity. Overall one sees good convergence to asymptotic limits and various collisional results which are mainly caused by a combination of the magnetic mirroring force and collisional detrapping processes. Since the adjoint generalized Spitzer function has a simple physical meaning - this is the amount of parallel current produced by a point particle source at given location in the momentum space - further on the function $g$ is discussed in terms of particle motion in the phase space.

Minimum $B$ point: Figure 1 presents the transition from sub-thermal to fast particles at the $B_{min}$ point. One can clearly observe how collisional detrapping of particles results in current generation even from particles originally situated in the trapped region. These phenomena gradually disappear with increasing velocity and results finally converge to the collisionless limit. Since the derivative of $g$ with respect to the perpendicular velocity $\nu_{\perp}$ is most important for ECCD, examples for $\partial g/\partial \nu_{\perp}$ are also presented.

![Fig. 1 Generalized Spitzer function, $g$ (left), and its perpendicular derivative, $\partial g/\partial \nu_{\perp}$ (right), vs. pitch parameter, $\lambda$, at the $B_{min}$ point for $\nu = 0.5\nu_{\nu}$ (black), $\nu = \nu_{\nu}$ (red) and $\nu = 2\nu_{\nu}$ (blue), respectively. Results from NEO-2 (dashed) are compared to the collisionless limit computed by SYNCH (solid). Case $\nu = 0.5\nu_{\nu}$ is not shown for derivatives. (Color figure: www.cpp-journal.org).](image1)

![Fig. 2 Generalized Spitzer function, $g$ (left), and its perpendicular derivative, $\partial g/\partial \nu_{\perp}$ (right), vs. pitch parameter, $\lambda$, at the $B_{max}$ point for $\nu = 0.5\nu_{\nu}$ (black), $\nu = \nu_{\nu}$ (red) and $\nu = 2\nu_{\nu}$ (blue), respectively. Results from NEO-2 (dashed) are compared to the collisionless limit computed by SYNCH (solid). Case $\nu = 0.5\nu_{\nu}$ is not shown for derivatives. (Color figure: www.cpp-journal.org).](image2)

Maximum $B$ point: Figure 2 shows a fundamental difference between collisional and collisionless cases in the range of small values of the pitch parameter $\lambda$. In results from NEO-2 the functional dependence of $g$ at...
small pitch values is similar to a cubic root. This is in contrast to results in the collisionless case where this dependence is closer to a cubic parabola. This rapid increase of $g$ from NEO-2 at small pitch values is connected with acceleration of electrons by the magnetic mirror force. Very slow electrons starting from the top of the hill at $B_{max}$ are accelerated by the magnetic mirroring force towards $B_{min}$. The velocity at $B_{min}$ is almost independent on differences in small starting velocities, but is mainly determined by the change of the magnetic potential energy thus resulting in roughly the same final values. During this process of acceleration, collisions move half of those electrons deeper into the passing region so that they are not decelerated back to the starting velocity when approaching the field maximum for the next time, thus producing a finite time averaged net current before their distribution becomes a Maxwellian. Of course, this behavior is more pronounced in sub-thermal and thermal regimes but it is still present in LMFP regimes in a small vicinity of $v_{||} = 0$. Further away from $v_{||} = 0$, NEO-2 curves tend to a cubic parabola similar to the collisionless approach (SYNCH). Because of the importance for ECCD computations, again examples for $\partial g/\partial v_{\perp}$ are given.

**Top and bottom points:** It can be seen from Figure 3 that antisymmetry of the generalized Spitzer function $g$ pertinent to asymptotic regimes and to magnetic field extrema is not existing anymore since particles starting from the trapped region tend to produce the current flow in the direction of the magnetic field minimum. The sign of this current depends on the position of the source (the sign of such a current produced by a source at the top of the flux surface is opposite to the sign of the current from a source at the bottom). As pointed out in Ref. [7], this feature allows to generate currents by waves with a symmetric spectrum introducing up-down asymmetry of the microwave radiation. This behavior is again the result of the magnetic mirroring force. Particles starting within the loss cone are accelerated (or decelerated) by this force so that, independent on their initial velocity, all of them finally have the same velocity sign at the magnetic field minimum point. They, however, might not reach the next maximum as trapped ones because they can be detrapped by collisions and will continue as passing ones, thus producing a net time averaged current. At higher velocities where electrons are in the banana regime, such a behavior is preserved only for trapped electrons close to the trapped passing boundary. Actually, the above feature is responsible also for the bootstrap effect [7] where the source term (gradient drive) possesses a natural up-down antisymmetry.

Fig. 3 Generalized Spitzer function, $g$ (left), its perpendicular derivative, $\partial g/\partial v_{\perp}$ (middle), and its parallel derivative, $\partial g/\partial v_{\parallel}$ (right), vs. pitch parameter, $\lambda$, at the top (NEO-2, dashed) and bottom points (NEO-2, dotted) for $v = 0.5v_{t}$ (black), $v = v_{t}$ (red) and $v = 2v_{t}$ (blue), respectively. SYNCH (solid). Case $v = 0.5v_{t}$ is not shown for derivatives. (Color figure: www.cpp-journal.org).

Formally, the loss of asymmetry of the Spitzer function in regimes with finite plasma collisionality can be seen in the conductivity problem where the dynamic operator and the collision operator have different parity properties with respect to the parallel velocity. While the collision operator conserves parity of the solution, the dynamic operator changes parity to the opposite. The sum of these operators has no definite parity which results in solutions without a definite parity. However, for either very high or very low collisionality the dynamic operator is either small or plays no role for the largest, bounce averaged part of the solution. In these limiting cases the solution is antisymmetric. In a tokamak with up-down symmetry a more general parity with respect to a simultaneous change of the sign of parallel velocity and of the poloidal angle is preserved. (see Figure 3)

Such a behavior of the generalized Spitzer function $g$ suggests a "naive" recommendation for the choice between the upper and the lower deposition points: One should choose the position of the source at the flux surface in such a way that the desired direction of the electron flow velocity (which determines the sign of
parallel phase velocity of the microwaves in the case of ECCD) at the position of the source is towards the magnetic field minimum. In such a case the mirror force would serve to increase the current. However, this recommendation would be correct for a beam-like source in velocity space which does not change the sign. In the case of a ECCD source where there is a change in sign (its velocity space integral is actually zero), mainly the perpendicular derivative of \( g \) determines the effect, and, as it can be seen from Figure 3, there is no general trend for this derivative. For slow particles this trend is the same as for \( g \) but for fast particles the trend changes to the opposite. Since the resonant line in velocity space goes through regions with different values of the velocity module, the conclusion about the role of the mirror force in such cases can be drawn only from direct calculations of the ECCD current with a quasilinear source computed by a ray-tracing code. (In Ref. [7] where the basic idea of driving the current by waves with a symmetric spectrum has been presented, a model expression for this source has been used where the velocity dependence of the quasi-linear diffusion coefficient has been only due to the finite Larmor radius effect, while the resonance condition has been omitted.) These calculations must naturally include also the parallel derivative of \( g \) (see Figure 3) which usually makes a smaller contribution than the perpendicular one, but its role increases if the Ohkawa effect becomes significant.

4 Conclusions

It has been found that in regimes where the collisional detrapping time is comparable to the bounce time, particle acceleration due to the magnetic mirroring force plays an important role in the generation of plasma current from ECCD. In particular, due to this mirroring force, there is a significant difference between upper and lower electron cyclotron resonance zones on a given flux surface in the efficiency of current drive in given toroidal direction. In tokamaks, effects of finite collisionality are important only in the plateau regime and at the beginning of the LMFP regime. For stellarators however, these effects extend further into the LMFP regime because the length of a trapped orbit becomes rather large (many toroidal turns) when approaching the trapped-passing boundary. The reflection points of such an orbit in a stellarator are close to the position of the global magnetic field maximum which is located in one point on the flux surface. In contrast, in a tokamak only one poloidal turn is needed to connect field maxima resulting in much shorter trapped orbits.

The code NEO-2 turns out to be a valuable DKE-solver for ECCD problems because of the unique feature that the full linearized collision operator can be used locally. Thus the full 3D (4D) problem of local current drive efficiency can be tackled in tokamaks (stellarators). At the moment however, usage is only possible for tokamak problems due to limited speed of the code. A substantial speed-up of the code is possible with improvements of the ODE-solver and code parallelization. A parallel version is possible since the DKE is actually solved in NEO-2 on portions of the field line which are treated independently from each other and can be distributed between the processors. Such improvements are in development. Any usage for stellarators is only possible after such a speed-up. At the moment, such a first principle solver can be used to check approximate models and identify improvement possibilities for ECCD efficiency. Thorough studies including ray tracing simulations have to follow this study to provide a better insight into the topic. For this purpose, the coupling of the kinetic equation solver NEO-2 and ray-tracing code TRAVIS has been performed. Preliminary results on this topic have been presented in Ref. [2].

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