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Relativistic unsteady Petschek-type model of magnetic reconnection

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Abstract

A model of time-dependent Petschek-type reconnection of arbitrary strong magnetic fields is presented where all necessary relativistic effects are taken into account. Reconnection is supposed to be initiated due to a local decrease of the plasma conductivity inside the diffusion region, which results in the appearance of an electric field along the X-line. This electric field is considered as a given arbitrary function of time. Then all MHD parameters as well as the shape of the moving Petschek-type shocks are obtained from the ideal relativistic MHD equations written in terms of 4-magnetic field and 4-velocity vectors as suggested by Lichnerowicz. The analysis is restricted to a symmetric current sheet geometry and to the case of weak reconnection, where the reconnection rate is supposed to be a small parameter. The solution obtained extends the well-known Petschek model for the steady-state case to incorporate relativistic effects of impulsive reconnection. It is shown that the plasma is accelerated at the slow shocks to ultrarelativistic velocities with high Lorentz-factors only for current layers embedded into strong magnetic fields and low-beta plasmas. In this case the plasma is strongly compressed and heated while the normal size of the outflow region with the accelerated plasma becomes very small.

Keywords: Magnetic reconnection; Relativistic MHD; Slow shock

1. Introduction

Reconnection is an energy conversion process which is believed to be of universal importance for astrophysical plasmas (Priest and Forbes, 2000). It is known that the most effective regime of the magnetic energy conversion in non-relativistic plasmas is the Petschek mechanism (Petschek, 1964). Hence, the extension of the classical reconnection model to the relativistic case is very important.

The assumption that reconnection is initiated locally inside a small diffusion region distinguishes the Petschek-type reconnection (Petschek, 1964) from magnetic field annihilation theories such as those introduced by Sweet (1958) and Parker (1963) (for the relativistic generalization, see Blackman and Field, 1994; Lyutikov and Uzdensky, 2003), or the tearing instability (Furth et al., 1963) (for the relativistic generalization, see Komissarov et al., 2006). In astrophysical applications pure magnetic field diffusion is too slow to be of interest. But, as first realized by Petschek, large-amplitude MHD waves or shocks may originate in the dissipative region where reconnection is initiated, and then rapidly convert the magnetic energy into the energy of the plasma.

Magnetic reconnection can occur in all cases when inhomogeneous plasma motion forms strong current layers in space (Priest and Forbes, 2000). In particular, reconnection of superstrong magnetic fields is an important element of models of pulsar winds (Lyubarsky and Kirk, 2001; Kirk et al., 2002), of magnetars (Komissarov et al., 2006), of

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relativistic jets associated with active galactic nuclei (AGN) (Blackman and Field, 1994; Romanova and Lovelace, 1992; Larrabee et al., 2003), and so on.

Steady-state relativistic reconnection was investigated by Blackman and Field (1994), and Lyubarsky (2005). Here, we present an analysis of relativistic reconnection with moving slow shocks (Semenov and Bernikov, 1991; Tolstykh et al., 2005) which extends the famous solution obtained by Petschek for the steady-state case to incorporate the effects of impulsive reconnection.

2. Methodology

We consider magnetic reconnection in a symmetric current sheet separating two uniform and identical plasmas with oppositely directed magnetic fields $\pm B_0$. The background magnetic fields and the total (magnetic + gas) pressure are assumed to be constant. Additionally, we consider an initially stationary plasma, meaning that the background velocities in the inflow regions in the lowest order are zero.

We introduce a coordinate system in Minkowsky space in which $x^0 = ct$, the $x^1 = x$-axis is directed along the current sheet, and the $x^3 = z$-axis is perpendicular to the current sheet. Reconnection starts due to an abrupt drop of the plasma conductivity in a small part of the current sheet, the so-called diffusion region. As a result, an electric field $E^*(x^0)$ is generated and transferred by MHD surface waves from the diffusion region to the current sheet (Fig. 1a) which leads to the decay of the disturbed part of the current sheet into a system of slow shocks of switched-off type. The plasma is highly accelerated and heated at the shock fronts forming outflow regions (OR) with strong plasma flow and weak magnetic field (Fig. 1b). At some stage,

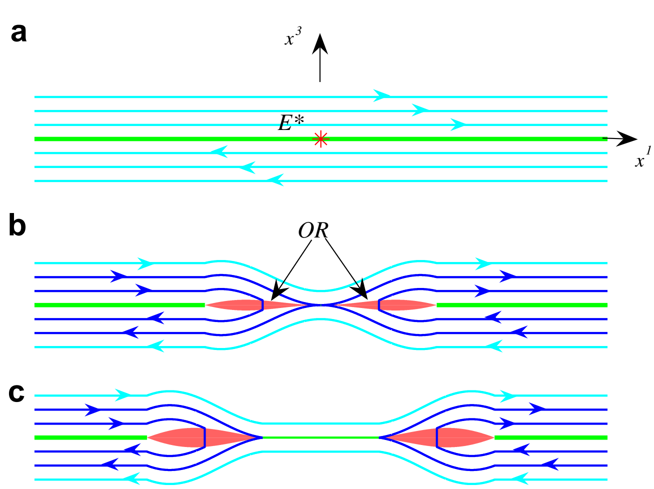

Fig. 1. Schematic illustrating effects of reconnection in a symmetric current sheet. (a) Beginning of reconnection. The process in initiated due to the generation of a dissipative electric field E^* in a small part of the current layer. (b) Switch-on or active phase when the electric field is acting in the diffusion region. (c) Switch-off phase when the electric field vanishes and then the outflow regions propagate along the current sheet as solitary objects in both directions from the former reconnection line.

the reconnection process stops, $E^*(x^0) = 0$ and the outflow regions detach from the site where the electric field originally occurred and propagate along the current sheet as solitary waves (Fig. 1c).

In order to progress with the mathematical modelling of the time-varying reconnection, several assumptions are needed. First of all, the exact mechanism of local dissipation inside the diffusion region is still unknown. It is possible, however, to keep the analysis generally applicable simply by imposing the reconnection electric field $E^*(x^0)$ as an arbitrary function of time and then using this function as an initial-boundary condition at the X-line. Secondly, it is hardly possible to find an exact solution to the nonlinear set of relativistic MHD equations (Lichnerowicz, 1967; Sibgatullin, 1984), but we can develop a perturbation method by introducing the restriction of weak reconnection

$$\varepsilon = \frac{cE^*}{B_0 v_A} \ll 1, \tag{1}$$

with B_0, v_A the initial magnetic field and Alfvén velocity, respectively.

Details of the perturbation analysis can be found in Semenov and Bernikov (1991), and Tolstykh et al. (2005), here we summarize the method. The analysis of unsteady weak reconnection can be separated into three parts.

2.1. The Riemann problem

The zero-order analysis is non-linear and corresponds to the so-called Riemann problem, which determines the appropriate combination of non-linear MHD waves (shocks or rarefaction waves) and the tangential components of the magnetic field and plasma velocity in the outflow regions in terms of initial parameter values. The solution of the non-relativistic Riemann problem can be found in Heyn and Semenov (1996). For the 2-D symmetric configuration shown in Fig. 1 the outflow region is bounded by pairs of slow switch-off shocks which propagate with Alfvén speed (Biernat et al., 1987; Semenov et al., 1983). Solution of the Riemann problem then gives the following results for the gas pressure p, plasma density ρ, the 4-velocity u^k, the Lorentz-factor u^0, and the 3D velocity v_x:

$$p = p_0 + \frac{B_0^2}{8\pi} \tag{2}$$

$$\rho = \frac{4\pi\rho_0^2}{\gamma - 1}\,\frac{c_s^2 + \gamma v_a^2/2}{\sqrt{4\pi Q(4\pi Q - B_0^2)} - 4\pi\rho_0 c^2} \tag{3}$$

$$u^k = \left(\left(\frac{4\pi Q}{4\pi Q - B_0^2}\right)^{1/2}; \left(\frac{B_0^2}{4\pi Q - B_0^2}\right)^{1/2}; 0; 0\right) \tag{4}$$

$$u^0 = \left(\frac{4\pi Q}{4\pi Q - B_0^2}\right)^{1/2} \tag{5}$$

$$v_x = \frac{cB_0}{\sqrt{4\pi Q}} = \frac{cB_0}{\sqrt{4\pi\rho_0\left(c^2 + \frac{1}{\gamma-1}c_s^2 + v_a^2\right)}} \equiv U_A \tag{6}$$

where $c_s^2 = \gamma\frac{p_0}{\rho_0}$, $v_a^2 = \frac{B_0^2}{4\pi\rho_0}$, $Q = \rho_0(c^2 + \frac{1}{\gamma-1}c_s^2 + v_a^2)$, U_A is the relativistic Alfvén velocity, and γ is the polytropic index. It is important that the results of this part of the analysis, do not depend on the specific behaviour of the reconnection rate.

2.2. Surface wave analysis

The shape of the outflow region and the perpendicular components of the magnetic field and plasma velocity inside are determined from the first order linearized equations. The linearized equations for the perpendicular components turn out to be formally identical to the Kelvin–Helmholtz equation for the surface waves (Heyn and Semenov, 1996). Thus, in addition to generating non-linear MHD waves reconnection also acts as a source for surface waves, which transfer the perpendicular components of the magnetic field and plasma velocity outwards from the X-line. Outside the outflow region the surface waves produce disturbances of the current sheet. For the symmetric case the propagation speed is just U_A, and we obtain the following results

$$h^\alpha = \left(0;0;0;\left(\frac{4\pi Q - B_0^2}{B_0^2}\right)^{1/2} E^*\left(x^0 - \frac{c}{U_A}x^1\right)\right) \quad (7)$$

$$f(x^0, x^1) = \frac{x^1\rho_0}{u^1\rho B_0}E^*\left(x^0 - \frac{c}{U_A}x^1\right), \quad (8)$$

where $h^i = {}^*F^{ik}u_k$ is the space-like 4-vector of the magnetic field inside the outflow region, $h^ih_i < 0$, ${}^*F^{ik}$ is the dual tensor of the electromagnetic field, and the function $f(x^0,x^1)$ describes the shape of the slow shock in the first quadrant. These results evidently depend on the behaviour of the reconnection rate in contrast to the results of the Riemann problem.

2.3. External disturbances

The results of the previous two parts can provide the boundary conditions for disturbances in the inflow region. Under the restriction of weak reconnection these disturbances are small $\sim\varepsilon$ and can be found in a similar way to the problem of flow around a thin aerofoil. To calculate the perturbations of the magnetic field and plasma as a function of time it is convenient to use the Cagniard–deHoop method which originally had been applied in seismology to the problem of elastic waves propagation (Heyn and Semenov, 1996).

The reconnection electric field $E^*(x^0)$ which is dissipative in the origin, is implemented in the ideal MHD as a boundary condition at the reconnection X-line. The consequence of this procedure is that the first-order disturbances in x-component of the magnetic field has a logarithmic singularity at the X-line like in the original Petschek solution. It is possible, so far only in the non-relativistic case, to construct a solution in the diffusion region and to match it to the solution in the convective zone. This gives a relation connecting the reconnection rate with plasma resistivity in the diffusion region. The singular behavior of the first-order perturbation in the x-component of the magnetic field plays an important role in the matching procedure (Erkaev et al., 2000).

3. Analysis of the solution

The solution (2)–(8) is obtained for arbitrary behaviour of the reconnection rate $E^*(x^0)$. To model a pulse of reconnection, we choose the following function $E^*(x^0)$

$$E^*(x^0) = \begin{cases} 0.2E_A \sin(\frac{\pi x^0}{ct_0}) & 0 \leqslant x^0 < ct_0 \\ 0 & x^0 \geqslant ct_0. \end{cases} \quad (9)$$

The switch-on or active phase of reconnection corresponds to the period $0 \leqslant x^0 < ct_0$ when the dissipative electric field is generated in the diffusion region (Fig. 2). Simultaneously, $E^*(x^0)$ is transferred to other parts of the current sheet through the MHD waves and acts just like a convective electric field there, $\mathbf{E}^* \cdot \mathbf{j} > 0$, which results in the conversion of magnetic field energy into plasma energy. The current sheet itself resolves into two pairs of slow shocks of switch-off type, and the latter propagate rapidly with the relativistic Alfvén speed along the sheet and collectively form the outflow for the plasma streaming towards the current sheet from both sides. Plasma entering the outflow region is accelerated to the relativistic Alfvén speed and heated, and this energy conversion through the slow shocks is one of the characteristic features of reconnection. The volume of the outflow region with accelerated and heated plasma rapidly grows in time during switch-on or active phase of reconnection (Fig. 2), and the whole process has an explosive-like character.

Even more interesting is the fact that as soon as the active phase terminates for $x^0 > ct_0$ the shocks detach from the former reconnection line and propagate along the cur-

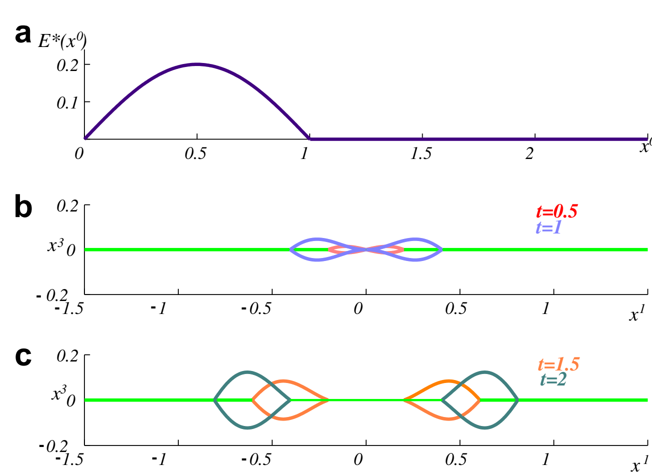

Fig. 2. Development of reconnection. (a) The reconnection rate $E^*(x^0)/E_A$ as a function of x^0 normalized to ct_0, which has been used to calculate the shape of the outflow region during switch-on (b) and switch-off (c) phase.

rent sheet as some kind of solitary objects but certainly not as solitons. It is a fact that during this switched-off phase, energy conversion still continues at the shocks. Gradually, all the plasma inside the reconnected flux tube is accelerated and heated and then collected inside the outflow region. As a consequence, the width of the outflow region linearly grows with distance x^1 to the X-line (see Eq. (8)), hence, the outflow region continues to change its form in the course of motion along the current sheet. A new current sheet is restored behind the separating outflow regions where the magnetic field strength becomes less than that before the reconnection pulse. Magnetic energy and reconnected flux are transported away by the moving outflow regions.

In the vicinity of the reconnection line $x^1 \ll v_x t$, $x^3 \ll v_x t$, and during the switch-on phase of reconnection when $E^*(x^0) \neq 0$, the solution (2)–(8) is reduced to the steady-state Petschek solution considered in Blackman and Field (1994), and Lyubarsky (2005).

The process of reconnection is particularly efficient for the case of a strong magnetic field $B_0^2/8\pi \gg \rho_0 c^2$ and a cold plasma $p_0 \ll B_0^2/8\pi$. In this case, the Lorentz-factor turns out to be:

$$u^0 = \left(\frac{4\pi Q}{4\pi Q - B_0^2}\right)^{1/2} \approx \left(\frac{v_a^2}{c^2 + c_s^2/(\gamma - 1)}\right)^{1/2}. \tag{10}$$

The plasma is accelerated up to relativistic velocities,

$$v_x = \frac{cB_0}{\sqrt{4\pi\rho_0(c^2 + c_s^2/(\gamma-1) + v_a^2)}} \approx c - \frac{c}{2v_a^2}\left(c^2 + \frac{c_s^2}{\gamma - 1}\right), \tag{11}$$

and simultaneously significantly compressed (Fig. 3) and heated (Fig. 4),

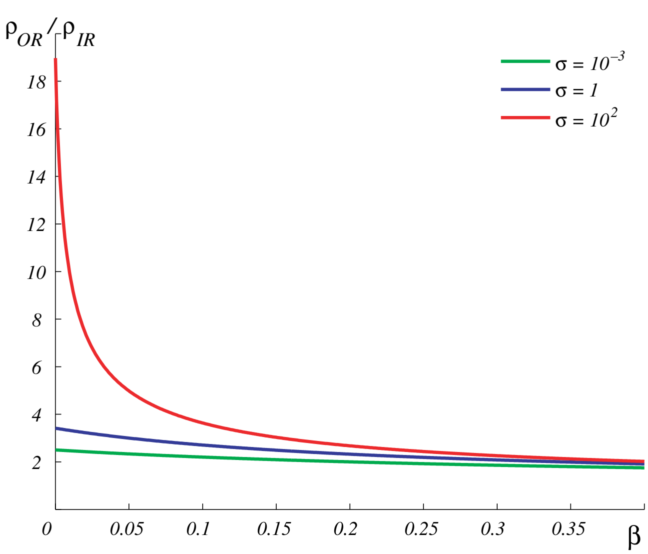

Fig. 3. Plasma density jump across the slow shock as a function of the upstream plasma beta $\beta = 8\pi p_0/B_0^2$ for different magnetization parameters $\sigma = B_0^2/8\pi\rho_0 c^2$.

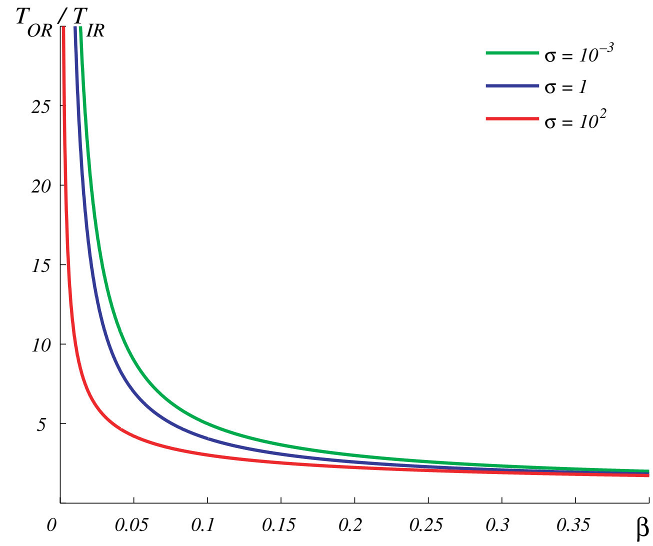

Fig. 4. Plasma temperature jump across the slow shock as a function of the upstream plasma beta.

$$\rho = \frac{\rho_0^2}{\gamma - 1}\frac{c_s^2 + \gamma v_a^2/2}{\sqrt{Q(Q - B_0^2/4\pi) - \rho_0 c^2}} \approx \frac{\rho_0}{\gamma - 1}\frac{\gamma v_a/2}{\sqrt{c^2 + c_s^2/(\gamma - 1)}} \tag{12}$$

$$T^{00} = \left(c^2\rho + \frac{\gamma}{\gamma - 1}p\right)(u^0)^2 - \bar{p} \approx \frac{\rho_0}{\gamma - 1}\frac{\gamma v_a^4/2}{c^2 + c_s^2/(\gamma - 1)}. \tag{13}$$

It can be seen from (12) and (13) that the plasma density ρ and the energy density $T^{00} \to \infty$ when $B_0^2/8\pi \gg \rho_0 c^2$ and $B_0^2/8\pi \gg p_0$. At the same time, the cross size of the outflow region $z_{OR} \sim (c^2 + c_s^2/(\gamma - 1))/v_a^2 \to 0$. Hence, z_{OR} becomes much less than the z-size of the reconnected magnetic flux tube. This means that the relativistic plasma is concentrated to a small volume.

It is interesting to note that the plasma compression at a relativistic slow shock is more strong than that at a non-relativistic shock (see Fig. 3), while the plasma is less heated on a relativistic shock as compared to the case $v^2 \ll c^2$, which clearly can be seen in Fig. 4, where the relativistic curves are well below the non-relativistic limit. It is a fact that the plasma pressure in the outflow region is fixed by the total pressure outside, and the plasma density increases significantly. Hence, the pressure downstream of the shock increases mainly due to a dramatic density compression rather than a temperature growth, so, the plasma inside the outflow region is relatively cold and very dense.

4. Conclusions

The localized initiation of reconnection and subsequent evolution of the system has the following features:

i. Disruption of the current sheet and the appearance of the reconnection electric field $E^*(t)$ inside the diffusion region, the magnitude of which acts as a quantitative measure of the reconnection rate.
ii. Rearrangement of the magnetic field distribution to form a topologically new region of reconnected flux.
iii. The generation of MHD waves inside the diffusion region.
iv. Conversion of magnetic energy to plasma energy mainly as a result of propagation of MHD waves and the corresponding spreading of $E^*(t)$ along the current sheet.
v. Streaming of plasma towards the current sheet from both sides and a rapid outflow along it. The outflow region serves as a transporter of magnetic flux, plasma, energy, momentum and disturbances in the inflow region.

The process of relativistic reconnection is especially efficient for the case of a strong magnetic field and cold plasma, i.e., when the magnetic energy must be bigger than the rest mass energy and the thermal energy of the background plasma near the initial current sheet. In this case, relativistic reconnection concentrates plasma and energy into a very tiny volume so that the width of the outflow region tends to zero. Relativistic reconnection can accelerate plasma to nearly the speed of light and effectively converts magnetic energy into the plasma energy with very small characteristic time.

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