

# WAVE STRUCTURES EXCITED IN COMPRESSIBLE PETSCHERK-TYPE MAGNETIC RECONNECTION

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**Abstract.** We present a method to analyze the wave and shock structures arising from Petschek-type magnetic reconnection. Based on a time-dependent analytical approach developed by Heyn and Semenov (1996) and Semenov et al. (2004), we calculate the perturbations caused by a delta function-shaped reconnection pulse, which permits a representation of the plasma variables in the form of Green's functions. Different configurations for the initial conditions are considered (Penz et al., 2006). In the case of symmetric, antiparallel magnetic fields and symmetric plasma density, the well-known structure of an Alfvén discontinuity, a fast volume wave, a slow shock, a slow wave, and a tube wave occurs. In the case of asymmetric, antiparallel magnetic fields, additionally surface waves are found. We also discuss the case of symmetric, antiparallel magnetic fields and asymmetric densities, which leads to faster propagation in one half plane, causing side waves forming a Mach cone in the other half plane. Complex effects like anisotropic propagation characteristics, intrinsic wave coupling, and the generation of different non-linear and linear wave modes in a finite  $\beta$  plasma are retained. The temporal evolution of these wave and shock structures is shown.

## 1. Introduction

The mechanism of wave generation in the Petschek-type model is the following (Heyn et al., 1988). Due to the local enhancement of the plasma resistivity, an electric field tangential to the current sheet is generated, giving rise to a normal component of the magnetic field. During this process, a mass flux perpendicular to the disturbed surface will occur. Since the inflow of plasma takes place from both sides of the surface, it is necessary that the surface broadens, forming a thin boundary layer. To satisfy all conservation laws, several shocks and discontinuities will arise, known as the general Riemann problem (Akhiezer et al., 1975; Heyn et al.,

1988). Heyn and Semenov (1996) and Semenov et al. (2004) developed an analytical model of Petschek-type reconnection for a compressible plasma and skewed magnetic field, which allows to present the disturbances in the outer regions as well as the internal structure of the boundary layer caused by a given reconnection electric field.

The aim of this paper is to study the propagation of the different waves caused by reconnection in the outflow region for a compressible plasma. We give a detailed description of all waves occurring in the context of Petschek-type magnetic reconnection. According to the general Riemann problem, slow shocks and Alfvén waves will arise due to magnetic reconnection, propagating along the current sheet and causing disturbances in the ambient magnetic field. Because of the conservation of total pressure, an interaction between the upper and the lower half plane appears, leading to the development of characteristic features like fast or slow waves, tube waves, surface waves, and also side waves. In the analytical model developed by Heyn and Semenov (1996) and Semenov et al. (2004), the disturbances in the outflow region are given as convolution integrals of a kernel with the reconnection electric field. In order to unambiguously identify the contribution of each structure, we use a delta-shaped reconnection electric field, allowing a representation of the problem in the form of Green's functions (Penz et al., 2006). All wave structures arising in this description can be found as poles or branch points of the corresponding Green's function. This allows a detailed analysis of the different waves present in Petschek-type magnetic reconnection.

## 2. Wave generation in the ambient plasma environment

In this section, we perform an analysis of the case of asymmetric reconnection in a compressible plasma, using the equations for ideal MHD, which are

$$\begin{aligned}
 \frac{\partial \rho}{\partial t} + \nabla \cdot (\rho \mathbf{v}) &= 0, \\
 \frac{\partial (\rho \mathbf{v})}{\partial t} \\
 + \nabla \cdot \left[ \rho \mathbf{v} \mathbf{v} + p \mathbf{I} - \frac{1}{4\pi} \left( \mathbf{B} \mathbf{B} + \frac{B^2}{2} \mathbf{I} \right) \right] &= 0, \\
 \frac{\partial}{\partial t} \left( \rho \frac{v^2}{2} + \rho e + \frac{B^2}{8\pi} \right) \\
 + \nabla \cdot \left[ \rho \mathbf{v} \left( \frac{v^2}{2} + e + \frac{p}{\rho} \right) + \frac{1}{4\pi} \mathbf{B} \times (\mathbf{v} \times \mathbf{B}) \right] &= 0 \\
 \frac{\partial \mathbf{B}}{\partial t} + \nabla \cdot (\mathbf{B} \mathbf{v} - \mathbf{v} \mathbf{B}) &= 0,
 \end{aligned}$$

where  $\rho$  is the mass density,  $\mathbf{B}$  is the magnetic field,  $\mathbf{v}$  is the plasma velocity,  $p$  is the isotropic pressure, and  $e$  is the internal energy. We consider two antiparallel magnetic fields of different field strength, namely  $B_a$  in the upper half plane, and  $B_b$  in the lower half plane, which are separated by a tangential discontinuity. The background magnetic fields and the total pressure are assumed to be constant. Additionally, we consider a fixed plasma in the inflow region in zeroth order. For weak reconnection, which implies that quantities perpendicular to the tangential discontinuity are small compared with tangential components, the problem can be tackled in two different steps. First, we can evaluate the components tangential to the discontinuity from the non-linear system of MHD equations in zeroth order. Then, we can determine the perpendicular components from the linearized system of MHD equations in the first order approximation.

The linearized MHD equations can be transformed into Fourier-Laplace space and a displacement vector can be introduced (Penz et al., 2006). After that, a solution for the displacement vector can be found in Fourier-Laplace space. The backward transformation into coordinate-time space is done by using the Cagniard-deHoop method (Heyn and Semenov, 1996; Penz et al., 2006). Thus, one achieves a representation of the MHD parameters  $[U]$  in form of convolution integrals of the reconnection electric field  $E$  and an integration kernel  $K$ ,

$$U(x, z, t) = \int K(x, z, \tau) E(t - \tau) d\tau.$$

For the task of studying the different waves excited by reconnection, it is convenient to use a delta-function shaped reconnection electric field,

$$E(t) = \delta(t - t_0),$$

as an input function, because the convolution with a spatial extended electric field may blur different waves. By using a delta-shaped pulse, the integration over the Cagniard contour is eliminated, and therefore the Green's functions of the first order MHD variables are given as analytical functions, e.g., the plasma density is given as (Penz et al., 2006)

$$\begin{aligned}
 \rho^{(1)} &= \frac{\sqrt{(1 + v_{Aa}^2 s^2)(1 + v_{Ab}^2 s^2)(u_b^2 + c_{sb}^2 v_{Ab}^2 s^2)}}{\sqrt{(1 + c_{sa}^2 s^2)(1 + c_{sb}^2 s^2)(u_a^2 + c_{sa}^2 v_{Aa}^2 s^2)}} \\
 &\quad \times \frac{-B_a \rho_a^{(0)} \rho_b^{(0)}}{\pi} \frac{1}{L_a + L_b} \frac{Q(s)}{\tau_s},
 \end{aligned}$$

where  $v_A$  and  $c_s$  are the Alfvén and the sound speed, respectively, and  $u^2 = v_A^2 + c_s^2$ .

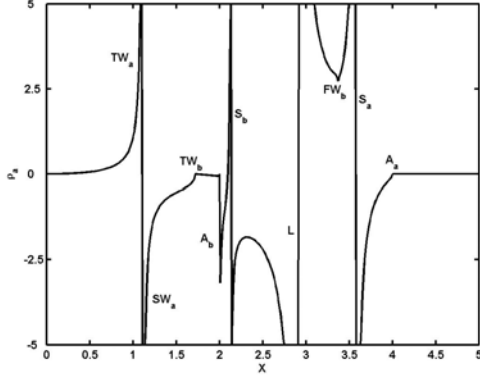
Using the exact compressible wave motions driven by sources created by magnetic reconnection, the system response function is derived in a way that it can be used for linear plane wave problems. The poles and branch points found in the Green's function arise from four different terms in the previous equation:

- from the source function  $Q(s)$ , describing Alfvén waves and slow shocks,
- from  $L_a + L_b$  leading to the appearance of surface waves,
- from  $\tau_s$  giving a fast volume wave, and
- from the branch points of the first term in leading to waves travelling with fast, slow, and tube speed.

### 3. Case study 1: Asymmetric magnetic fields and low plasma beta

In this section, we consider antiparallel magnetic fields with different field strength in the upper and the lower half plane for a low plasma beta. We use  $B_a=1$ ,  $B_b=-0.5$ ,  $\beta_a=0.1$ , and  $\beta_b=3.4$ . Due to the slightly asymmetric conditions, slow shocks will appear and no rarefaction waves. The source function  $Q(s)$  gives poles corresponding to the propagation of the Alfvén discontinuity and the slow shock in the upper and the lower half space. The structure with the fastest propagation velocity is the Alfvén discontinuity followed by the slow shock in the upper half space. These contributions are seen in Fig. 1 at  $x=4$  and  $x=3.576$ , respectively. The contributions of the Alfvén wave and the slow shock generated in the lower half space, which are also visible in the upper half space, can be found at  $x=2$  and  $x=2.132$  in Fig. 1. A surface wave comes from  $L_a + L_b$ , since there is a shear velocity between the upper and the lower half plane. It can be seen at  $x=2.911$  in Fig. 1. Additionally, the contributions from the three branch points can be found at  $x=1.15$ ,  $x=3.35$ , and  $x=1.1$  in Fig. 1. These are a slow wave for the upper half plane, since for  $\beta_a < 1$ , the slow wave velocity corresponds to the sound speed. In contrast, the other branch point gives a fast wave in the lower half plane, since  $\beta_b > 1$ , so that the sound speed corresponds to the fast wave speed in this case. The tube wave is the slowest wave in both half planes. Now all seven wave structures appearing in this configuration are specified. For the lower half plane, the wave structure is similar, only the contribution from the tube wave differs, giving a location at  $x=1.7$ .

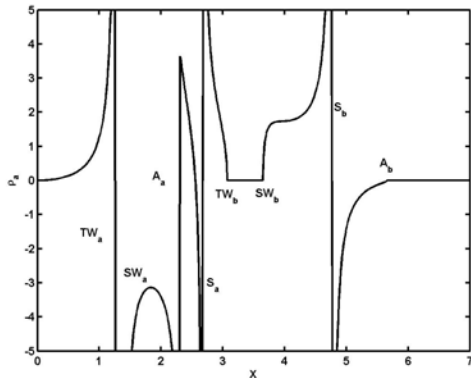
For increasing  $z$ , most of the fronts disappear. Only the surface wave from  $L_a + L_b$  and the front of the fast body wave give significant contributions for  $z > 0.2$ .



**Fig. 1:** Sequence of wave and shock structures excited for antisymmetric magnetic fields and a low plasma beta. A, S, FW, SW, TW, and L means Alfvén wave, slow shock, fast wave, slow wave, tube wave, and surface wave, respectively, while subscripts a and b denote upper and lower half plane.

#### 4. Case Study 2: The appearance of side waves

If some waves in the lower half plane are propagating with a velocity faster than the fastest one in the upper half plane, so-called side waves arise in the upper half plane. They are caused by the disturbances of the propagating waves in the lower half plane and show the characteristic form of a Mach cone in the upper half plane. In order to achieve a fast propagation velocity in the lower half plane, we increase the plasma density in the upper half plane to 3.0, while keeping it at 0.5 in the lower half plane. The magnetic fields are chosen to be symmetric as  $B_a = 1.0$



**Fig. 2:** Sequence of wave and shock structures for the case of side waves. It is obvious that all disturbances in the lower half plane are moving with velocities faster than the fastest disturbance in the upper half plane. Additionally, between SW and TW in the lower half plane, a lagoon solution can be found.

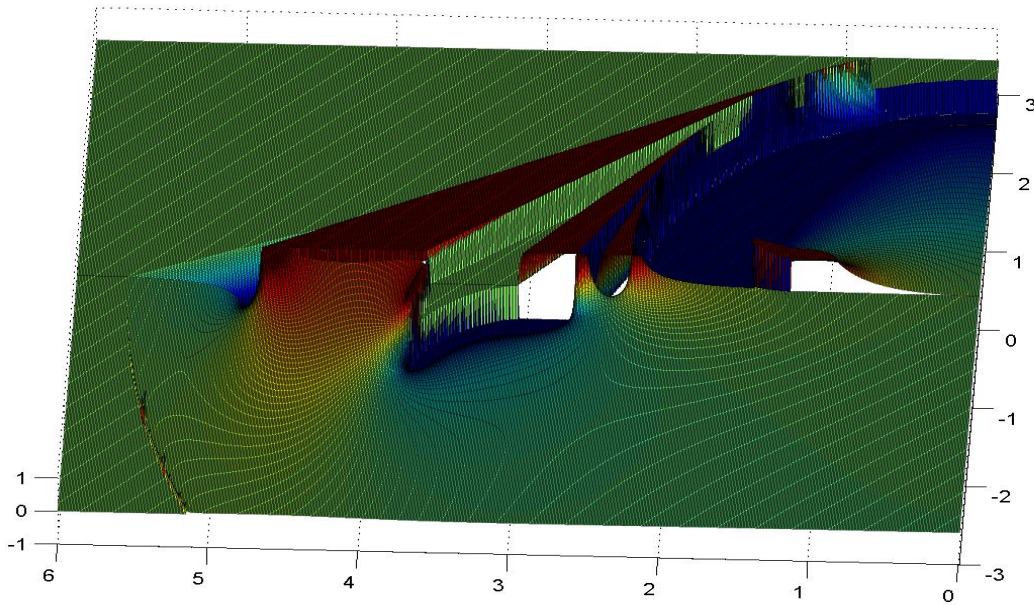
and  $B_b = -1.0$ , while  $\beta_a = \beta_b = 0.5$ . For these parameters, it is obvious that the disturbances in the lower half plane are propagating faster. The fastest wave in the lower half plane is the Alfvén wave, followed by the slow shock, the slow wave, and the tube wave (Fig. 2). Compared with the fastest disturbance in the upper half plane, the slow shock, all disturbances in the lower half plane are moving faster. This leads to the occurrence of four side waves in the upper half plane, caused by the four structures from the lower half plane. These side waves form a Mach cone with an apex angle proportional to the fast wave speed in the upper half plane, as can be seen in Fig. 3. Additionally, between the side wave caused by the slow wave and the side wave from the tube wave, a lagoon can be observed, where the density disturbances vanish. Since we consider symmetric magnetic fields, no contribution from  $L_a + L_b$  can be observed.

#### 5. Conclusions

We presented a method to analyze the behavior of waves caused by magnetic reconnection in a compressible plasma. It is shown that there appear different wave and shock structures, namely Alfvén waves and slow shocks in the upper and the lower half plane caused by the source term, surface waves caused by  $L_a + L_b$ , the fast body wave, as well as perturbations due to branch points giving structures moving with sound and tube speed.

For the case of asymmetric antiparallel magnetic fields, a complex behavior is found. In this case, the Alfvén and slow shocks do not merge, so that each of them causes a perturbation in the ambient plasma. In this case a surface wave appears because of the contribution from  $L_a + L_b$ . Depending on the value of beta, the contribution from the branch points gives a slow wave ( $\beta < 1$ ) or a fast wave ( $\beta > 1$ ), while the third branch point gives a tube wave, which propagates always slower than the Alfvén and the slow wave. Also a fast body wave is observed. If the wave and shock structures in the lower half plane propagate faster compared to those in the upper half plane, side waves appear. Under certain circumstances, a lagoon-type solution can also appear.

The method presented here allows to gather information about the internal structure of the boundary layer and even about the conditions in the other half plane from analyzing the separation and the sequence of wave structures observed outside of the boundary layer in one half space. This can be used for the analysis of satellite measurements of reconnection-associated signatures in the Earth's magnetosphere. It also may help to interpret wave structures observed at the Sun's surface. However, in



**Fig. 3:** The density distribution in the  $x$ - $z$  plane. The first side wave arises from the Alfvén discontinuity, the second from the slow shock, the third one from the slow wave. The fourth side wave, which comes from the tube wave is separated from the other side waves by a lagoon. After the four side waves, the wave structure is similar as in the previous examples.

the MHD approach only low-frequency modes can be described, and therefore, in reality the wave structures excited by magnetic reconnection might be even more complicated due to the appearance of additional modes in kinetic theory.

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