1. Introduction

External or intrinsic magnetic perturbations can provide a stochastic field topology in fusion edge plasmas, both in tokamaks and in stellarators. In these edge plasma regions ergodic, island and "laminar" zones, can coexist and, certainly, mutually influence each other in a complicated manner. Fig. 1 shows the footprints of the magnetic field lines (Poincaré map) in TEXTOR-94 (project DED, [1]).

The simplest possible model for the plasma flow under such conditions, therefore, must at least have the following ingredients:

- The complete field line pattern, including all three zones.
- A model for transport in these fields, with homogeneous boundary conditions only at the separatrix between perturbed (SOL) and unperturbed region, consistently linking the various regions of the SOL.
- A model for the 3D recycling process. Distinct from regular (2D) boundary plasmas here we have to deal with a breaking of the symmetry between the magnetic topology (hence: parallel plasma flow) and the vacuum chamber (hence: recycling). This can lead, due to overloading of particular field lines with recycling, to complicated local flow patterns with flow reversal even under globally low recycling conditions.

Previous 3D Monte Carlo codes for the stellarator periphery (EMC3) [2] are based on "global" magnetic coordinate system, which is aligned to the magnetic surfaces. Hence such procedure is less suitable in the presence of ergodic layers.
Instead, we propose a Multi Coordinate System Approach (MCSA), combined with a mapping technique.

2. Basic equation in magnetic coordinates

Plasma fluid equations in some general curvilinear coordinates $x^i$ can be written in the form

$$\frac{\partial u}{\partial t} = \frac{1}{\sqrt{g}} \frac{\partial}{\partial x^i} \sqrt{g} \left( D^{ij} \frac{\partial u}{\partial x^j} - V^i u \right) + Q_u. \tag{1}$$

In the case of heat conductivity equation for electrons and ions $u_{e,i} = 3n T_{e,i}/2$ is the internal energy density. $n$ and $T_{e,i}$ are plasma density and electron and ion temperatures, respectively. The diffusion tensor in (1) (omitting the indices "e" and "i" here and in the following) is taken to be of the form

$$D^{ij} = D^g g^{ij} + D^h h^i h^j,$$

where $g^{ij} = (\nabla x^i)(\nabla x^j)$ and $h^i = h \nabla x^i$ are contravariant components of the metric tensor and the unit vector along the magnetic field $h = B/B$. $D^g$ and $D^h$ are classical parallel and anomalous perpendicular heat conductivity coefficients, respectively. The velocity components arising in the heat convection term are given by

$$V^i = \frac{5}{3} V^i_\rho + D^{ij} \frac{\partial}{\partial x^j} \log n, \tag{2}$$

where $V^i_\rho$ are fluid velocity components. The heat source term $Q_u$ comprises all non-divergent terms in energy balance equation.

3. Multi Coordinate System Approach (MCSA)

For the numerical solution of Eqs. (1) a set of many (typically $M \sim 20$) local magnetic coordinate systems $S_m, \ m = 1..M$ is used in combination with a mapping procedure derived from field line tracing [3]. Every local coordinate system is chosen such that for two coordinates the condition

$$h \nabla x^i_m = 0, \quad i = 1, 2. \tag{3}$$

holds. I.e. two coordinates are chosen normal to $B$ for each coordinate system. The sets of local magnetic coordinate systems are constructed starting from quasitoroidal coordinates $r, \theta, \varphi$. In our particular case, Poincaré sections are introduced as the surfaces $\theta = \theta_m \equiv (m-1)\Delta \theta, \ m = 1, \ldots, M, \quad \Delta \theta = 2\pi/M$. Consider an arbitrary point $P_0 = (r_0, \theta_0, \varphi_0)$. The magnetic coordinates $x^1_m(P_0), x^2_m(P_0)$ in the system $S_m$ are defined as the small radius and the toroidal angle of the projection along the magnetic field line to the Poincaré section $m$. Hence $x^1_m, x^2_m$ are linked with the quasitoroidal coordinates by the characteristics of Eq. (3),

$$x^1_m = \rho(r, \theta, \varphi; \theta_m), \quad x^2_m = \chi(r, \theta, \varphi; \theta_m). \tag{4}$$

$\rho$ and $\chi$ satisfy the magnetic field line equations with the initial conditions $\rho(r, \theta, \varphi; \theta) = r, \chi(r, \theta, \varphi; \theta) = \varphi$. The remaining variable $x^3$ has been chosen to coincide with the poloidal angle, $x^3 = \theta$. The lengthy derivations of the expression for the metrical coefficients will be given in a technical report [4].
4. Monte-Carlo procedure

The internal energy contained in the system is distributed between an ensemble of $N_p$ "test particles". In order to derive the elementary particle jump we rewrite (1) in conservative Fokker-Planck form for the pseudoscalar density $N$ of test particles $N = u\sqrt{\Omega}/w$. $w$ is the weight of test particles and $\Omega$ is the plasma volume,

$$\frac{\partial N}{\partial t} = \frac{\partial}{\partial x^i} \left[ \frac{\partial}{\partial x^j} D^{ij} N - V_c^i N \right] + Q_N \quad \text{with} \quad V_c^i = V^i + \frac{1}{\sqrt{\Omega}} \frac{\partial}{\partial x^j} \sqrt{\Omega} D^{ij}. \quad (5)$$

The source terms $Q_N = Q_u \sqrt{\Omega}/w$ are accounted for by weight adjustment, and here we describe the dynamics of test particles only. The random process governing the test particle motion is

$$x^i(t + \Delta t) = x^i(t) + \Delta x^i, \quad (6)$$

The Fokker-Planck equation for the density of test particles $N$ subjected to random process (6) coincides with (5) if $\langle \Delta x^i \Delta x^j \rangle = 2D^{ij}(x(t))\Delta t$ and $\langle \Delta x^i \rangle = V^i(x(t))\Delta t$. Thus, the random step $\Delta x^i = \sqrt{2\Delta t}\alpha^{ik} \xi_k + V^i \Delta t$. Here $\xi_k$ are standardized random numbers, $\langle \xi_k \rangle = 0, \langle \xi_k \xi_k \rangle = \delta_{ik}$, $\alpha^{ik}$ is the square root matrix $\alpha^{ik}\alpha^{ij}\delta_{jl} = D^{ij}$ (we use Cholesky decomposition to extract it). We introduce time intervals $\Delta t$ small compared with characteristic relaxation time of the relevant plasma parameters. The transport coefficients are computed from the estimates of plasma parameters from the previous time step and are kept constant during $\Delta t$. This "explicit" procedure is repeated until a stationary solution is reached. Due to our definition (4) the magnetic coordinates $x^{1,2}_m$ coincide with quasitoroidal coordinates as long as particle remains on one section $\theta = \theta_m$. This permits to calculate all parameters for random steps $\Delta x^{1,2}_m$ from plasma and magnetic field data in the quasitoroidal coordinates. The step component $\Delta x^3$ is the displacement along the magnetic field line. Since this displacement must be a multiple of $\Delta \theta$ in our approach, the final position of test particle is on one of the neighbouring Poincaré sections. Therefore, the new particle position in quasitoroidal coordinates can be obtained by mapping

$$\begin{align*}
\theta(t + \Delta t) &= \theta_{m'}, \quad m' = m \pm 1 \\
r(t + \Delta t) &= \rho \left( r(t) + \Delta x^1, \varphi(t) + \Delta x^2, \theta_m, \theta_{m'} \right), \\
\varphi(t + \Delta t) &= \chi \left( r(t) + \Delta x^1, \varphi(t) + \Delta x^2, \theta_m, \theta_{m'} \right). \quad (7-9)
\end{align*}$$

Up to the precision of the map $\{\rho, \chi\}$ this random process introduced above induces no artificial cross field transport arising from the fast parallel transport, i.e., our procedure is perfectly aligned despite of the possibly ergodic character of the magnetic field. The maps $\{\rho(r, \theta_m, \varphi, \theta_{m+1}), \chi(r, \theta_m, \varphi, \theta_{m+1})\}$, $\{\rho(r, \theta_m, \varphi, \theta_{m-1}), \chi(r, \theta_m, \varphi, \theta_{m-1})\}$, $m = 1, \ldots, M$, are precomputed once for each configuration by field line tracing and constructed during the Monte Carlo computation from bicubic splines.

5. Results and conclusions

The new code E3D is employed here to study the heat transfer in the edge plasma, expected during the DED operation in TEXTOR.
The results illustrate the influence of the perturbation field on the temperature profile (Fig. 2). The heat load pattern on the limiter is shown to be strongly influenced by the effect of perturbation field (Fig.3). In addition to the typical TEXTOR data [1], we have chosen the following input parameters: radius of the perturbation coils $r_c = 53 \text{cm}$; number of Poincaré sections $M = 19$; heat flux from core into SOL ($r = 42 \text{cm}$) $Q_{e,i} = 0.45 \text{MW}$ in electrons and ions each; parallel heat conduction: classical; cross field heat conduction coefficient $\chi_{\perp} = nD_{\perp}$ with $D_{\perp} = 5 \text{m}^2/\text{s}$; plasma fluid velocity $V_p = 0$; plasma density $n = 7 \times 10^{12} \text{cm}^{-3}$; perturbation field $\vec{B}$ varied from 0 to 0.1 T. The results confirm the qualitative picture of the heat flux patterns on the bumper limiter ("divertor target"), [5], essentially as a consequence of the field structure in the "laminar region" alone (compare Fig. 1 and Fig. 2).

Quantitatively, we find a redistribution of global heat load from the ALT limiter to the wall and bumper. However, due to the localization of the heat flux into narrow stripes, the peak power load on the bumper is increasing about 2 times faster with increasing perturbation field. We conclude from this first applications to a strongly simplified (and hence: still physically obvious) case that the procedure seems to work both correctly and economically (turn around time: 2 hours on CRAY-T3E with the fully parallelized version).

References