Assessment of the trapped particle confinement in optimized stellarators

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Introduction

A number of optimized stellarator configurations are analyzed with respect to trapped particle confinement using new target functions [1] which are related to collisionless α-particle confinement. These functions are based on specific averages of the bounce averaged \( \nabla B \) drift velocity of trapped particles across magnetic surfaces. Computations of such simplified criteria require smaller computing times than direct computations of particle losses. An essential part of calculations relates to some new versions of the NCSX configuration. For comparison, computations for different types of stellarator configurations are performed. In addition, the poloidal drift of trapped particles is calculated with the aim to identify particles with vanishing poloidal drift velocity. This is of interest since such particles can be quickly lost from the plasma.

Basic formulas

The main optimization target introduced in [1] is the mean-square average over the pitch angle and over the magnetic surface of the bounce averaged \( \nabla B \) drift velocity of trapped particles across magnetic surfaces,

\[
\hat{v}_{an} = \frac{1}{2\sqrt{2}} \Gamma_v \frac{\rho_L}{R_0}, \tag{1}
\]

with \( \rho_L = v/\omega_{ci} \) being the characteristic Larmor radius and \( R_0 \) being the major radius of the torus. The dimensionless factor \( \Gamma_v \) manifests the role of a specific magnetic configuration,

\[
\begin{align*}
\Gamma_v &= \sqrt{\frac{\Gamma_w}{\Gamma_s}}, \\
\Gamma_s &= \frac{\pi}{2\sqrt{2}} \frac{1}{\langle |\nabla \psi| \rangle} \left( \frac{1}{\langle |\nabla \psi| \rangle} \right)^{1/2} \frac{1}{\langle B \rangle_{B_{\text{abs}}^\text{max}}} \\
\Gamma_w &= \frac{\pi R_0^2}{\sqrt{2}} \lim_{L_s \to \infty} \left( \int_0^{L_s} \frac{ds}{B} \right)^{-1} \frac{1}{\langle |\nabla \psi| \rangle^2} \frac{B_{\text{abs}}^\text{max}/B_0}{B_{\text{abs}}^\text{min}/B_0} \int_{b'} \sum_{j=1}^{\text{max}} \left( \frac{d\gamma_j}{db'} \right)^2 \left( \frac{d\gamma_j}{db'} \right)^{-1} \tag{2}
\end{align*}
\]

Here, \( \psi \) is the magnetic surface label, \( B_0 \) is the reference magnetic field and \( B_{\text{abs}}^\text{min} \) and \( B_{\text{abs}}^\text{max} \) are the absolute minimum and maximum of \( B \) within the sufficiently large interval of \( s, [0,L_s] \),

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respectively \((s)\) is the length along the magnetic field line). The index \(j\) is numbering the field line intervals \([s_{j}^{\text{min}}, s_{j}^{\text{max}}]\) where \(b' - B/B_0 \geq 0, b' = v^2/(J B_0)\). \(\langle A \rangle\) denotes the flux surface average for \(A\). The quantities \(g_j\) and \(\hat{I}_j\) are determined as

\[
\begin{align*}
  g_j & = \frac{1}{3} \int_{s_{j}^{\text{min}}}^{s_{j}^{\text{max}}} \frac{ds}{B} \sqrt{1 - \frac{B}{B_0 b'}} \left( 4 \frac{B_0}{B} - \frac{1}{b'} \right) |\nabla \psi| k_G, \\
  \hat{I}_j & = \int_{s_{j}^{\text{min}}}^{s_{j}^{\text{max}}} \frac{ds}{B} \sqrt{1 - \frac{B}{B_0 b'}},
\end{align*}
\]

with \(k_G\) being the geodesic curvature of the magnetic field line. The representation in (3) has the advantage that it can be computed efficiently in a field line tracing code. For a specific configuration, \(\Gamma_v\) is only a function of the minor radius. Minimization of \(\Gamma_v\) corresponds to the optimization of the trapped particle collisionless confinement.

**Computations of \(\Gamma_v\)**

Using Boozer spectra for \(B\) and for cylindrical coordinates of magnetic surfaces, computations of \(\Gamma_v\) are carried out for two sets of magnetic configurations of NCSX (e04, e04003, e04T8G, e04T8H and N3A0, N3AKB, N3ARAB, respectively). These configurations are results of further optimizations of configurations presented in Ref. [2]. For comparison, computations are also done for the standard and the high mirror configurations of W7-X (w7x-sc1 and w7x-hm1, respectively), two LHD inward shifted configurations [3] (for \(R_{a\ell}=3.6\) and 3.53 m, \(R_{a\ell}\) is a major radius of the magnetic axis), and an optimized quasi-isodynamical stellarator configuration (QI-opt) with poloidally closed contours of \(B\) on magnetic surfaces [4].

Computational results for NCSX are presented in Figs. 1 and 2, whereas results for the other configurations together with the best NCSX configuration are presented in Fig. 3. From Figs. 1 and 2 follows that for NCSX the best results are realized for configurations e04003 from the 1\(st\) set and N3ARAB from the 2\(nd\) set. In a wide interval of \(r/a\) N3ARAB is the best configuration. For a classical stellarator \(\Gamma_v\) is close to unity [1]. One can clearly see, that \(\Gamma_v\) for N3ARAB in the whole volume is improved by a factor of more than 5 compared to a classical stellarator. In the range \(0.35 \leq r/a \leq 0.8\) the improvement is even more than a factor of 10.

For LHD with \(R_{a\ell}=3.6\) m (\(\sigma\) optimized [5] for \(r/a \approx 0.5\)) \(\Gamma_v\) is only 2÷2.5 times smaller than for a classical stellarator. For LHD with \(R_{a\ell}=3.53\) m (neoclassical-transport-optimized configuration of LHD [5]) for \(r/a \geq 0.5\) \(\Gamma_v\) is \(\approx 2.5\) times smaller than unity (except for the near-boundary layer). The W7-X configurations are slightly better in this respect. The minimum \(\Gamma_v\) values are found to be about 3 and 4 times smaller than unity (high-mirror and the standard configurations, respectively) for \(r/a \approx 0.7\). Beside the improved NCSX version, the QI-opt configuration is the best one in the range \(r/a < 0.95\).

**Computations of the poloidal drift of trapped particles**

The poloidal motion of trapped particles \(d\theta/dt\) is computed using Eq. (55) of Ref. [1] with the radial electric field being zero. The results are analyzed in a normalized form \(d\theta/dt_{\text{norm}}\). For normalization, \(d\theta/dt\) for a deeply trapped particle near the magnetic axis of an \(l=2\) stellarator
is used. Characteristic results are presented which are chosen from computations for the first 90 minima of $B$ along a magnetic field line. The results cover roughly one toroidal period. Figures 4 to 6 show results for $d\theta/dt_{\text{norm}}$ as functions of the pitch $\gamma=v_{//0}/v_{\perp0}$ for ncsx-N3ARAB, w7x-hm1 and QI-opt, respectively. Here, $v_{//0}$ is $v_{//}$ at a local minimum of $B$ and $v_{\perp0}=\sqrt{\gamma_c J}$. The curves for $d\theta/dt_{\text{norm}}$ are marked according to the numbering of the minima of $B/B_0$ along the magnetic field lines which are also presented in the figures. Here, $n$ is the number of integration steps with 1280 steps per magnetic field period. In addition, plots for $\gamma_c$ as functions of $\gamma$ are shown. The $\gamma_c$ quantity represents the angle (in $\pi/2$ units) between a magnetic surface cross-section and a contour of $J_{//}=\text{const}$ in the point where these two curves cross each other. The $\gamma_c$ value is calculated as $\gamma_c=(2/\pi)\arctan(v_{\text{an}}/rd\theta/dt)$ where $v_{\text{an}}$ is the bounce averaged radial drift velocity of a trapped particle calculated with formula (16) of Ref. [1]. When $d\theta/dt$ approaches zero, $\gamma_c\approx\pm1$ which corresponds to a condition where the $J_{//}=\text{const}$ contour is perpendicular to the magnetic surface. With high probability this corresponds to $J_{//}=\text{const}$ contours which are not closed and this, in turn, leads to quick losses of the corresponding trapped particles.

It follows from Fig. 4 that for ncsx-N3ARAB ($r/a\approx0.8$), $d\theta/dt_{\text{norm}}$ is close to zero for deeply trapped particles (small $\gamma$) in those local minima of $B$ which are located in the lower regions of the $B/B_0$ distribution. For smaller values of $r/a$ (the results are not shown in the figures here) the general property is that trapped particles corresponding to higher regions of the $B/B_0$ distribution have $d\theta/dt>0$. At the same time, particles trapped near the local minima of $B$ in the lower regions of $B/B_0$ have $d\theta/dt<0$. So, with increasing $\gamma$ for all trapped particles of such a kind $d\theta/dt$ turns into zero for a certain value of $\gamma$. Figure 5 shows characteristic results for the high-mirror configuration of W7-X ($r/a\approx0.56$). For a rather big fraction of minima of $B$, $|\gamma_c|$ is close to unity but the $\gamma$ intervals corresponding to such values of $|\gamma_c|$ are small. Figure 6 shows results for poloidal motion in QI-opt for $r/a\approx0.84$. For this surface $d\theta/dt\neq0$ for all minima of $B$ and for a majority of these minima $\gamma_c$ is smaller than 0.2. Such and even better results are realized for surfaces corresponding to smaller values of $r/a$. For surfaces with $r/a>0.84$ trapped particles with $d\theta/dt=0$ appear. For example, at $r/a=0.91$ six minima from 90 have values of $d\theta/dt=0$.

From computations for LHD, $R_{\text{ax}}=3.53$ m, (the results are not shown here) it follows that for $r/a\approx0.49$ trapped particles with $d\theta/dt=0$ exist only for the deepest ripple wells for $\gamma$ close to the trapping boundary in such a ripple. With increasing $r/a$ to $r/a\approx0.7$ the trapped particles corresponding to $d\theta/dt=0$ disappear.

**Conclusion**

The employed technique has the advantage that the computations can be efficiently performed using a field line tracing code. From the computations it follows that for the N3ARAB configuration of NCSX the averaged radial $VB$ drift velocity of trapped particles is smaller than for an equivalent classical stellarator by more than a factor of five in total. This is the best
result in the comparison with other optimized stellarator configurations. A numerical analysis of velocities of the radial drift as well as the poloidal drift allows for a quick assessment of $J_{||}$ contours in the vicinity of a given magnetic surface. Contours of $J_{||}$=const perpendicular to magnetic surfaces are found for the majority of the configurations. An exception is only the inner part of the QI-opt configuration. However, the intervals of pitches for the corresponding trapped particles are small and the fractions of such particles are expected to be small.

References