Calculation of Self-consistent Radial Electric Field in Presence of Convective Electron Transport in a Stellarator

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Abstract. Convective transport of supra-thermal electrons can play a significant role in the energy balance of stellarators in case of high power electron cyclotron heating. Here, together with neoclassical thermal particle fluxes also the supra-thermal electron flux should be taken into account in the flux ambipolarity condition, which defines the self-consistent radial electric field. Since neoclassical particle fluxes are non-linear functions of the radial electric field, one needs an iterative procedure to solve the ambipolarity condition, where the supra-thermal electron flux has to be calculated for each iteration. A conventional Monte-Carlo method used earlier for evaluation of supra-thermal electron fluxes [1] is rather slow for performing the iterations in reasonable computer time. In the present report, the Stochastic Mapping Technique [2, 3] (SMT), which is more effective than the conventional Monte Carlo method, is used instead. Here, the problem with a local monoenergetic supra-thermal particle source is considered and the effect of supra-thermal electron fluxes on both, the self-consistent radial electric field and the formation of different roots of the ambipolarity condition are studied.

FLUX BALANCE AND NEOCLASSICAL PARTICLE FLUXES

In a stellarator the constraint that the ion and electron fluxes be equal determines the radial electric field. Thus, the equation for the flux balance, \( \Gamma_e^{nc} + \Gamma_e^{s} = Z_i \Gamma_i^{nc} \), has do be fulfilled on each flux surface. Here, \( \Gamma_{\alpha}^{\text{nc}} \) with \( \alpha = e, i \) are the neoclassical particle fluxes [1],

\[
\Gamma_{\alpha}^{\text{nc}} = -n_{\alpha} \left\{ D_{\alpha 11} \left( \frac{n_{\alpha}}{n_{\alpha}} - \frac{q_{\alpha} E_r}{T_{\alpha}} \right) + D_{\alpha 12} T_{\alpha} \right\},
\]

with \( q_{\alpha}, n_{\alpha}, T_{\alpha}, E_r \) being the particle charge, density, temperature and the radial electric field, respectively, and prime denotes a derivative with respect to a formal radius.

The neoclassical diffusion coefficients \( D_{11}^{\alpha} \) and \( D_{12}^{\alpha} \) are computed according to the Shaing-Houlberg-model [4], where instead of \( \epsilon_h \) the effective ripple \( \epsilon_{\text{eff}} \) [5] is used [6]. The balance equation is a non-linear equation in the radial electric field which might have multiple roots.

SUPRA-THERMAL PARTICLE FLUXES

The supra-thermal particle flux, \( \Gamma_e^{s} \), is of particular importance for the confinement since it can influence the radial electric field through the ambipolarity condition [1]. Following the SMT approach [2, 3], the usual expression for particle flux through the magnetic surface \( \hat{\psi} = \hat{\psi}_0 \) defined in guiding center variables and flux coordinates (\( \hat{\psi}, \theta, \phi \)) can be written as an average over Poincaré cuts of the phase space flux density,

\[
\Gamma_e^{s} = 2\pi \sum \int d^5 u \Gamma_m(u) \delta(t - u^5) \Theta(\hat{\psi}(Z(z_m, \tau_m)) - \hat{\psi}_0) - \Theta(\hat{\psi}(Z(z_m, 0)) - \hat{\psi}_0).
\]

Here, \( \Gamma_m(u) \) is the pseudo-scalar particle flux density through those Poincaré cuts, \( u \) denotes the five variables \( x^1, x^2, p, \lambda, t \), where \( x^1, x^2 \) are contravariant coordinates in a local magnetic coordinate system, and \( p, \lambda, t \) are the
momentum modulus, the particle pitch and the time, respectively. The summation over \( m \) is a summation over contributions from different Poincaré cuts and \( \Theta \) is the Heaviside step function. In SMT, \( Z(z_m, \tau) \) is the solution to equations of particle drift motion with \( z_m \) being the initial value of phase space variables on the Poincaré cut with index \( m \), and with \( \tau_m \) being a transition time between cuts. All details of SMT can be found in Ref. [2, 3]. The energy flux is obtained in the same way and it differs from \( \Gamma'_{\psi} \), by the factor \( w_k(u) \) in the sub-integrand, where \( w_k \) is the value of the kinetic energy of the particle at the phase space point \( u \) on the cut.

**COMPUTATIONAL RESULTS**

For numerical computations, the magnetic field from the W7-AS stellarator [7] was used in its real space representation. In Figure 1, particle and energy fluxes of supra-thermal particles are shown, respectively. The particle source is on the magnetic axis in the magnetic field minimum located at the elliptic cross section of W7-AS. Trapped particles with a pitch value \( \lambda_0 = 0.1 \) and fixed energies \( w_0 \) ranging from \( w_0 = 2T_0 \) to \( w_0 = 9T_0 \) are generated there. The source rate in these computations was \( v_{\text{stat}} = P_{\text{source}}/w_0 \) where \( P_{\text{source}} = 400 \) kW is the source power. The profiles of the equilibrium parameters were the following, \( T_0(\Psi) = T_0(1.2 - \Psi), n_0(\Psi) = n_0(1.2 - \Psi)^2, \Phi(\Psi) = T_0\Psi/e \), where \( T_0 = 3 \) keV, \( n_0 = 3 \cdot 10^{13} \) cm\(^{-3} \), and \( a = 17.4 \) cm, respectively. The quantity \( \Psi_b = \Psi = (r/a)^2 \) was chosen as a formal flux label, where the radius \( r = R - R_0 \) is computed in the mid plane of the symmetric section and \( R_0 \) is the radius of the magnetic axis. It can be seen that the energy of the source particles has a significant influence on the profiles of supra-thermal fluxes.

The results of self-consistent modeling are presented in Figures 2, 3, 4 and 5, where a modified density profile \( n_0(1.2 - \Psi^2) \) is used. Figure 2 shows the self-consistent \( E_r \)-profile with neoclassical fluxes only. In Figure 3 supra-thermal electron fluxes are given for \( \lambda_0 = 0.1 \) and two energies, in each case with its respective \( E_r \)-profile. One can clearly see that particles with lower energy (4\( T_0 \)) have time to slow down, whereas particles with higher energy (9\( T_0 \)) quickly drift out of the plasma.

Figure 4 shows the formation of the “electron root” in a rather narrow region near the magnetic axis. In addition, Figure 5 shows the dependence of fluxes on \( E_r \) in two radial positions. One can see that at \( r = 5.5 \) cm two stable solutions exist, which finally result in the formation of the “electron root”. The decision which root has to be chosen is based on the minimization of a generalized heat production rate [1]. Following that approach, the position of the poloidal shear layer can be determined from

\[
P(r) = \int_{E_r^l}^{E_r^s} (Z_1\Gamma_1^{nc} - \Gamma_e^{nc} - \Gamma_e^c) dE_r = 0,
\]

where \( E_r^l \) and \( E_r^s \) are the stable solutions for \( E_r \) in the “ion” or “electron root”, respectively. The “ion root” is then realized for \( P > 0 \) and the “electron root” for \( P < 0 \) [1]. Basically, the ion root is realized almost everywhere. When
FIGURE 2. Radial electric field $E_r$ resulting from neoclassical fluxes only versus radius $r$. In addition, the profiles of $n_e$ and $T_e$ in dimensionless units are given.

FIGURE 3. Supra-thermal particle flux $\Gamma^s_e$ versus radius $r$. The supra-thermal fluxes are computed for $w_0 = 4T_e$ (full) and $w_0 = 9T_e$ (dashed) in each case with a selfconsistent $E_r$.

approaching the axis, the neoclassical fluxes are decreasing together with the magnetic surface area, but at the same time the supra-thermal flux is increasing. Finally, the neoclassical bifurcation occurs and the root is changed from “ion” to “electron”. Further inward, the “electron root” disappears which can be seen in Figure 4 where the pertinent root vanishes. This event is an artifact of the neoclassical transport model used in the present computation where the ion flux is decreasing with increasing $E_r$ and cannot balance the supra-thermal flux anymore. As discussed in Ref. [1], the validity of the neoclassical theory may be violated in such a case of a very strong radial electric field.

FIGURE 4. Radial electric field $E_r$ versus radius $r$ for two energies of source particles and for the neoclassical equilibrium.
FIGURE 5. Total electron flux at $r = 5.5$ cm (1) and at $r = 5.0$ cm (2), supra-thermal electron flux (3) and ion flux (4) versus radial electric field $E_r$, respectively, for the $w_0 = 9T_0$-case. Circles mark stable roots, whereas crosses mark unstable ones.

SUMMARY

The application of SMT to a “global” computation of supra-thermal particle fluxes in a stellarator shows that this method is fast enough to allow for iterations of the radial electric field using the ambipolarity condition taking into account fluxes from supra-thermal particles. For this purpose, SMT is the ideal tool, because the computation of one self-consistent profile requires only tens of minutes on a DEC Alphastation 500 depending on accuracy. Therefore, SMT combined with a neoclassical balance code permits the self-consistent modeling of particle and energy balance in a stellarator with strong electron or ion cyclotron heating where the convective transport of supra-thermal particles plays a significant role. It is also shown that convective fluxes are very sensitive to the detailed structure of the supra-thermal particle source. In the case of ECRH, non-linear effects of wave-particle interaction are dominant in the formation of such a source [8]. The method for modeling this effects has been recently developed and will be included in future models based on SMT.

ACKNOWLEDGMENTS

This work has been carried out within the Association EURATOM-ÖAW and with funding from the Austrian Academy of Sciences.

REFERENCES