Prospects of the Stochastic Mapping Technique

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Abstract: The Stochastic Mapping Technique (SMT) is an efficient method to solve the Drift Kinetic Equation (DKE) in the long mean free path (LMFP) regime. Similar to conventional Monte Carlo (MC) approaches it has no limitations in the problem geometry. Computations can be done for an arbitrary complex magnetic field topology including islands and ergodic zones. To overcome the problem of low computational speed, particles in SMT are followed only on Poincaré cuts instead of following stochastic orbits as done in conventional MC methods. For this purpose, various pre-computed maps, e.g., of particle motion between minimum-B cuts, are used. Some aspects of a possible usage are discussed here, namely, the evaluation of transport properties in complex equilibria, confinement studies for collisionless orbits, kinetic effects of ECRH/ECCD and pertinent computation of the non-Maxwellian part of the electron distribution function and related fluxes for global flux balance.

1. Introduction

The evaluation of neoclassical transport coefficients is one of the essential elements of stellarator studies. It is needed for the optimization of magnetic configurations, and for the analysis and planning of experiments. It is also of relevance for the stellarator specific issues of a fusion reactor. In arbitrary 3-dimensional magnetic field configurations of a stellarator this problem has to be solved numerically. Methods which provide the most general solution at the present time are the conventional MC method, which has been realized in numerous codes, and DKES (Drift Kinetic Equation Solver), a “regular” code which employs a variational principle where the solution is expressed using a series of Fourier-Legendre test functions [1, 2]. A summary of these methods can be found in a joined benchmarking paper, Ref. [3].

Another important issue relevant to reactor scale stellarator optimization is the evaluation of confinement characteristics of fast particles. The most reliable measure here is the fraction of particles lost from the confinement volume due to their drift motion. It is computed by solving equations of drift motion for a large number of test particles with starting positions distributed within the whole volume and velocity space [4].

All methods mentioned above have no principal limitation from the geometry of the device or from the confinement regime, however, as an adverse consequence of problem generality, these methods have low computational efficiency in certain collisionality regimes. This low efficiency becomes a sufficient obstacle for optimization procedures where new, more effective methods are necessary. High computational speed is desirable also for creation of neoclassical databases for a fixed magnetic field configuration. Such
databases are used for the analysis and planning of experiments and are planned at the IPP Greifswald [3]. For creation of such a database, one needs to resolve the nonlinear dependence of transport coefficients on 3 parameters - radius, collision frequency, and radial electric field. Generation of such 3D arrays of data causes a significant numerical effort. It is also especially important for finding a self-consistent solution for the radial electric field through the ambipolarity condition [5] including fluxes of supra-thermal particles (e.g. generated by ECRH).

2. DKE solver in the LMFP-regime

On the way to fast DKE solvers which meet the needs mentioned above, our code development is focused mainly on methods which can provide the DKE solution in a fixed collisionality regime, in particular, in the LMFP-regime which is of main interest for reactor applications.

2.1 Field Line Integration Technique

The ability to handle arbitrarily complex magnetic fields is a strength of the field-line-integration technique [6]. Here, the drift kinetic equation is considerably simplified by assuming that the fast motion along field lines constitutes the only particle drift within a flux surface, ultimately allowing one to express the transport through the flux surface as a weighted integral of the geodesic curvature along a field line of infinite length (i.e. sufficiently long to cover the entire magnetic surface). Evaluating the integral numerically, as done in the NEO code [6], provides an efficient means of determining the radial transport for any non-symmetric magnetic configuration in the 1/ν-regime. The advantages of this method are the very high computational speed and the possibility to use real space coordinates as well as magnetic coordinates. The disadvantage of the method comes from the fact that it is limited to regimes without a radial electric field.

2.2 Stochastic Mapping Technique

The most general method to solve the DKE is the MC method. It has no limitations regarding to the problem geometry or to the collisionality regime. At the same time, a conventional MC method is the slowest numerical method applied to the DKE. This is caused by the fact that modern stellarators have a quite complex magnetic field geometry where numerous toroidal and poloidal modes contribute to the magnetic field spectrum. This makes a direct computation of the field and of the drift orbits “expensive” in terms of computing time.

Therefore, it was of high interest to develop a MC method which allows for a proper treatment of general magnetic configurations. The Stochastic Mapping Technique (SMT) is such a method to solve the DKE in the long mean free path regime [7, 8, 9]. Within this method, instead of following stochastic orbits, particles are traced only on Poincaré cuts using the pre-computed information about particle motion in between the cuts. The SMT method is capable of dealing with a general magnetic field geometry in real space coordinates and shows a strong gain in computational speed.

In Ref. [9] the SMT for stellarators is described in detail, various applications of this technique are discussed and the results of benchmarking with other methods are presented. Basically, the applications of SMT are the same as of conventional MC methods [10]. In addition, various weighting schemes developed for conventional MC methods are fully applicable to SMT. On the other hand, the high computational speed gained with SMT
(2 orders of magnitude or more) allows for the solution of problems where the usage of conventional MC methods is not reasonable because of large computation times.

The concept of local magnetic coordinate systems (LMCS) [11] used in SMT permits the general case of the magnetic field topology, which may include also islands and ergodic magnetic field regions. This is not possible in DKE solvers, where the existence of embedded flux surfaces is assumed. The concept of LMCS allows for a clear separation of slow cross-field transport from the fast parallel motion, and thus strongly reduces the problem of numerical diffusion and pertinent memory requirements. In addition, a small Larmor radius approximation for displacements of the footprints on the Poincaré cuts due to magnetic and electric drifts is introduced. These displacements can be presented in the form of a Taylor series up to second order in the Larmor radius expansion. The expansion coefficients are 3D functions of the position on the cut and of the particle pitch. This keeps the memory requirement in a reasonable range. Moreover, for some range of radial electric fields the dependence on the radial electric field is of parametric type only. Therefore, iterations with varying electrostatic potential can be done without re-computation of maps, an essential advantage for iteration schemes with changing radial electric field to fulfill the ambipolarity condition in the establishment of either the electron root or the ion root. For higher values of the radial electric field, maps can be generated for fixed profiles of the electrostatic potential.

Well established tools for evaluation of transport coefficients and fast-particle confinement are working in flux coordinates only. This reduces the applicability of those methods only to configurations with embedded magnetic surfaces. However, this simple topology is only an idealized model of the real stellarator magnetic configuration which also may contain islands and ergodic layers. Such realistic MHD equilibria can, e.g., now be computed with the PIES code [12, 13, 14]. This code is capable to compute equilibria with complex topology including islands and ergodic zones. Conventional MC codes as well as SMT can handle transport problems in such types of equilibria. In addition, it was already shown that the field line tracing method can also provide transport coefficients for such equilibria in the regions where embedded flux surfaces exist [15, 16].

Because of various possibilities to separate numerical tasks, both the pre-computation of maps as well as actual MC runs can be parallelized. This either can be done on parallel computers or on grids of independent computers because data exchange between tasks can be kept very small.

3. Status and outlook

3.1 Equilibria

Since originally SMT was realized for magnetic fields given in real space coordinates, a version of the code working directly with magnetic fields represented in flux (Boozer) coordinates is being developed at the moment. This kind of magnetic field representation is the most common in various applications, and, very often, the magnetic field is available only in flux coordinates. This step is necessary to establish the link also from this code to the international database on stellarator configurations and to the benchmarking and neoclassical database effort at IPP Greifswald.

A version of the SMT code applicable to stellarator equilibria of general topology provided by the PIES code will be developed on the basis of the SMT version in flux coordinates. In regions with embedded flux surfaces, it would allow for the evaluation
of transport coefficients in regimes where the radial electric field is important. It would also allow the computation of the fast particle loss fraction in the whole configuration.

3.2 Fast particle confinement

The application of the SMT code to fast evaluation of fast particle losses will be carried out. For this purpose it is necessary to parallelize both the preloading part of the code and the orbit solver. As a result, the SMT solver will be able to trace the orbits in the approximation where the parallel adiabatic invariant is conserved. This is relevant to optimization methods where the condition of closure of the contours of the parallel adiabatic invariant for the majority of trapped particles is one of the figures of merit [17, 18, 19].

3.3 Kinetic effects of ECRH/ECCD

High power ECRH/ECCD leads to a significant distortion of the distribution function of electrons from a Maxwellian. This distortion causes changes in power absorption coefficients and subsequently in power deposition profiles as well as current density profiles. Therefore, it is necessary to compute the electron distribution function selfconsistently with the wave electric field. Such computations in stellarators require the solution of the DKE in the whole configuration space. A solution to this seems only to be possible with MC methods. In particular, SMT is a favorable candidate for such a solver because of its large gain in computational speed.

In such a selfconsistent solution, one has to take into account the nonlinear nature of wave-particle interaction which is important in many cases. Such a MC method of evaluation of the electron distribution function taking into account realistic orbits of electrons during their nonlinear cyclotron interaction with the wave beam has been realized in Ref. [20]. There, the focus is on a proper description of the particle interaction with a wave beam while the geometry of the main magnetic field outside the beam was a simplified slab model. The problem is solved by introducing Poincaré cuts at each side of the beam. With this, the confinement volume is split into two regions: a narrow toroidal region where the wave particle interaction takes place, and the rest of the volume where this interaction is negligibly small. As described in Ref. [20], the governing kinetic equation is reduced to a set of integral relations which map the pseudo-scalar particle flux densities through neighboring cuts. Combined together using the periodicity of the problem, the relations which map the flux through the wave beam and through the outer region form an integral equation which is then solved with MC techniques. Here, it turned out to be very efficient to use discretized transition probabilities for the energy of perpendicular motion when crossing the wave beam.

Recently (see this workshop) such a selfconsistent solution has been realized for tokamak geometry neglecting the influence of trapped particles. In addition, the code ECNL has been coupled with the beam tracing code TORBEAM. In stellarators, the solution to the problem becomes more complicated because of the presence of additional effects connected with convective transport of electrons. Such a convective transport of non-Maxwellian electrons can play a significant role in the energy balance of stellarators in case of high power electron cyclotron heating. Consequently, together with the neoclassical thermal particle fluxes also the non-Maxwellian electron flux should be taken into account in the flux ambipolarity condition which in turn defines the self-consistent radial electric field. Since neoclassical particle fluxes are non-linear functions of the radial elec-
tric field, one needs an iterative procedure to solve the ambipolarity condition where the 
on-Maxwellian electron flux should be calculated for each iteration because those fluxes 
are very sensitive to the detailed structure of the non-Maxwellian particle source [9, 21]. 
Conventional Monte-Carlo methods used earlier for evaluation of supra-thermal electron 
fluxes [5] are rather slow in performing the iterations in reasonable computer time. Here, 
SMT which is more effective than a conventional MC method, can be used instead [21].

References


