Optimization Studies of TJ-II Stellarator¹

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Abstract
A new numerical code SORSSA [1] has been developed for computing the energy content of different stellarator configurations based on a simple transport model depending on the neoclassical effective helical ripple, $\epsilon_{\text{eff}}$. The procedure consists in estimating $\epsilon_{\text{eff}}$ by following magnetic field lines, for arbitrary coil systems, and thus for $\beta = 0$ using the NEO code [2]. This method is particularly suited for existing stellarators since it allows neoclassical transport estimations just depending on the coil currents. SORSSA has been applied to analyze the available parameter space of TJ-II [3] configurations as well as for studying the possible benefits of splitting the actual single toroidal coil system into five different groups. Preliminary results indicate only moderate improvement for coil current variations of $\pm 20\%$.

Introduction
The flexibility of TJ-II, a four field period medium size stellarator ($R = 1.5m$, $a < 0.2m$), in obtaining different configurations arises from its four different sets of independently fed coils: 32 helically displaced toroidal coils (responsible for the main magnetic field), one central circular coil, one central helical coil (following the winding law of the toroidal coils), and two vertical field coils. Because full neoclassical transport estimations in stellarators are very expensive, in computer CPU terms, no systematic calculations scanning TJ-II configurations have been done so far. SORSSA offers the possibility to give a figure of merit about the goodness of neoclassical properties skipping the part of the usual neoclassical calculations by working in real space coordinates. The price one has to pay is to make only vacuum calculations $\beta = 0$ and only obtaining the effective helical ripple $\epsilon_{\text{eff}}$, thus only valid in the $1/\nu (\ln f p)$ collisionality regime and for zero radial electric field ($E_r = 0$). On the other hand it’s estimation can be used as an upper bound for the neoclassical confinement properties. Moreover, when attached to a minimization procedure it can be used to find neoclassically optimized configurations using directly as free parameters the currents flowing through the device coils. The goal of this work is to obtain the best possible TJ-II configuration using the SORSSA transport estimation. To this end the single TJ-II toroidal coil system has been split in five different coil groups (maintaining the periodicity and the stellarator symmetry) to allow for more freedom in the optimization, and study the possible benefit of such configurations.

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Optimization of energy confinement

In [1] an optimization procedure for the stored energy in the plasma has been developed to analyze the confinement properties of the TJ-II device for the $1/\nu$ regime. For the calculations of the heat conductivity, $\kappa_\perp$, the NEO code [2] has been used.

In the optimization procedure the total stored energy in the plasma volume is used as fitness parameter with an energy source, $Q(r) = \frac{Q_0}{r}\delta(r)$, which is localized at the magnetic axis. It is assumed that the temperature profile is defined by the heat conductivity equation

$$\frac{1}{r} \frac{\partial}{\partial r} r \kappa_\perp \frac{\partial T}{\partial r} + Q(r) = 0 \tag{1}$$

with the boundary conditions $T(a) = 0$ and $\lim_{r \to 0} (r \frac{dT}{dr}) = 0$ (here $a$ is the boundary of the plasma). So, the heat conductivity, $\kappa_\perp$, is proportional to $\epsilon_{\text{eff}}^{3/2} T^{7/2}$, and computation of $\epsilon_{\text{eff}}^{3/2}$ for sets of computed magnetic surfaces is an essential part of the optimization procedure. The normalized stored energy

$$\hat{W} = \int_0^a dr \, r \hat{n}(r) \left( \int_r^a \frac{dr'}{r' \epsilon_{\text{eff}}^{3/2}(r')} \right)^{2/9} \tag{2}$$

can be obtained by integrating the temperature profile resulting from (1) ($\hat{n}$ is a normalized plasma density).

Only “good” flux surfaces can be used for computation of the total stored energy. Field lines forming magnetic islands (the integration in (2) goes over $r_{\text{eff}}$) and stochastic zones have to be excluded from the computation.

The magnetic axis, which is needed for island detection, is found by fitting the effective radius $r_{\text{eff}}$ vs. the starting value $R_{\text{beg}}$ for the field line integration. For the axis $r_{\text{eff}}$ is zero.
The detection of magnetic islands is done in the $\varphi = 0$ plane because, there, $R_{\text{beg}}$ is known (the field line integration is started at $z = 0$ and $\varphi = 0$, because there the direction of the normal vector to the flux surface is known [4]). For this purpose the angle $\alpha$ between the vectors $P$ and $\nabla \Psi$ is used (see Fig. [2]). The vector $P$ points from the magnetic axis $MA$ to a point $P$ on the magnetic surface. The magnetic axis is the innermost (degenerated) magnetic surface with $r_{\text{eff}} = 0$. For a “good” magnetic surface $\cos \alpha$ is negative for all points on the surface. If the magnetic axis is not located inside the flux tube, as for magnetic islands, the numbers of positive and negative values for $\cos \alpha$ are close to each other.

Field lines which belong to a stochastic zone do not form a magnetic surface. For such field lines the module of $\nabla \Psi$ increases continuously. This behavior can be explained by the fact that, in this case, the function $\Psi$ (as well as $\nabla \Psi$) is not a single valued function of the position.

![Fig.2. Island detection: Here a “good” flux surface and an island corresponding to $\epsilon = 1/4$ is shown. $MA$ . . . magnetic axis, $S$ . . . magnetic surface.](image)

For the optimization procedure the well known Simulated Annealing algorithm [5] has been used. For these studies the optimizer has been working in an eight dimensional parameter space, defined by the helical coil current $I_{\text{hel}}$, the current for the central circular coil $I_{\text{centr}}$, the current for vertical field coil $I_{\text{vert}}$ and the currents of the five toroidal coil groups. The range of variation for the parameters has been limited to $\pm 20\%$ to stay close to the experimental setup.

**Results**

During the optimization procedure around 5000 configurations have been obtained. The final result of the procedure can be seen in Fig. 3, where the stored energy, normalized to the standard configuration, is plotted versus a configuration index.
(notice that this index is arranged in decreasing stored energy).

Fig.3. Stored energy normalized to the standard configuration. The standard configuration is marked with a blue star.

There are better (up to 40% better) as well as worse (much worse) configurations than the standard. Since Eq. 2 measures not only the improvement in the effective helical ripple but also in the plasma volume, those configurations below the standard usually display regions with islands (that were excluded from the calculation). On the other hand, configurations better than the standard don’t have large islands and have a smaller $\epsilon_{\text{eff}}$. The effect of the islands can be seen in the Poincaré cuts of the field lines for the standard (Fig. 4 for $\varphi = 0, 45$) as well as for the best (Fig. 5 $\varphi = 0$) and worst (Fig. 6 $\varphi = 0$) configurations considered so far.

Fig.4. Cross sections for the “standard” configuration ($B_\perp/B_0 \approx 2.5\%$).
The radial profiles of the effective helical ripple show that the main reason for higher values of the stored energy is the reduction of $\epsilon_{eff}$ (Fig. 7), whereas a comparison with Fig. 6 shows that the reduction cannot only be attributed to the reduction of the plasma volume because of islands. An increase of $\epsilon_{eff}$ is also responsible for lower values of the total stored energy. These effects can be seen in the rotational transform profiles for the three configurations (Fig. 8).

Fig.7. Effective ripple of the best, the standard and the de-optimized configuration. Only “good” field lines (marked with crosses) are used for computing the total stored energy.
Fig. 8. Rotational transform of the best, the standard and the de-optimized configuration. “Good” field lines are marked with crosses.

Conclusions
SORSSA code has been successfully applied to study neoclassical transport properties of TJ-II stellarator configurations. The configuration parameter space of TJ-II has been extended by splitting TJ-II’s toroidal coil system into five independent groups. Preliminary results, constrained to current variations of ±20%, indicate only a moderate improvement. More work is under progress broadening the current variations to address the potential of the coil configuration of TJ-II.

References