A Study of Pressure Gradient Effects on the Interaction of a Rotating Magnetic Field with the Plasma in the Kinetic Approximation

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Outline

- Conductivity operator
  - ⋆ Hamiltonian kinetic approach
  - ⋆ Canonical perturbation theory
  - ⋆ Finite Larmor radius expansion
- Results
  - ⋆ Benchmarking with ideal MHD
  - ⋆ Power absorption and torque with and without pressure gradients
- Conclusions

Kinetic equation

$$\frac{\partial f}{\partial t} + \{f, H\} = \nu\left[f_0\left(\mathbf{x}, \mathbf{P} - \frac{e}{c}\tilde{\mathbf{A}}\right) - f\right],$$

$$f = \bar{f} + \tilde{f}, \qquad H = \bar{H} + \tilde{H},$$

$$\frac{\partial \bar{f}}{\partial t} = \{\bar{H}, \bar{f}\} + \overline{\left\{\tilde{H}, \tilde{f}\right\}},$$

$$\frac{\partial \tilde{f}}{\partial t} = \left\{H_0, \tilde{f}\right\} + \{\tilde{H}, \bar{f}\} + \widetilde{\left\{\tilde{H}, \tilde{f}\right\}},$$

Small amplitudes, Fourier analysis in time

$$\bar{f} = f_0(\mathbf{x}, \mathbf{P}) = f_0(\mathbf{J}),$$

$$\mathbf{E}(\mathbf{x}, t) = \Re\left\{\mathbf{E}(\mathbf{x})\mathrm{e}^{-i\omega t}\right\},$$

Transformation of coordinates and velocities

The canonical frequencies are

$$\Omega^i = \frac{\partial H_0}{\partial J_i},$$

$$v_0^i = \dot{x}^i = \frac{\partial H_0}{\partial P_i} = \frac{\partial H_0}{\partial J_l}\frac{\partial J_l}{\partial P_i} = \Omega_0^l \frac{\partial x^i}{\partial x^k}\frac{\partial J_l}{\partial P_k}$$

$$= \Omega_0^l \left\{x^i, J_l(\mathbf{x}, \mathbf{P})\right\}_{(\mathbf{x},\mathbf{P})} = \Omega_0^l \left\{x^i(\boldsymbol{\theta}, \mathbf{J}), J_l\right\}_{(\boldsymbol{\theta},\mathbf{J})} = \frac{\partial x^i}{\partial \theta^l}\Omega^l$$

the form of the transformation is

$$x^1 = r = r_0(P_\vartheta, P_z) + \rho^r(\phi_c, \mathbf{J}), \qquad v_0^r = \frac{\partial \rho^r}{\partial \phi_c}\Omega^\phi,$$

$$x^2 = \vartheta = \vartheta_c + \rho^\vartheta(\phi_c, \mathbf{J}), \qquad v_0^\vartheta = \Omega^\vartheta + \frac{\partial \rho^\vartheta}{\partial \phi_c}\Omega^\phi,$$

$$x^3 = z = z_c + \rho^z(\phi_c, \mathbf{J}), \qquad v_0^z = \Omega^z + \frac{\partial \rho^z}{\partial \phi_c}\Omega^\phi.$$

Current density in linear theory

$$\mathbf{j}(\mathbf{x}) \;=\; \int \mathrm{d}^3 v\, e\, \mathbf{v}\, f(\mathbf{x},\mathbf{v}) \;=\; \mathbf{j}_U(\mathbf{x}) + \mathbf{j}_M(\mathbf{x}).$$

Applying Gauss theorem the current density $\mathbf{j}_U$ can be re-written in the form

$$\mathbf{j}_U(\mathbf{x}) \;=\; -\frac{e^2 n_0(r)}{m}\frac{1}{i\omega}\tilde{\mathbf{E}}(\mathbf{x}) \;=\; \int \mathrm{d}^3 P\, e\, \mathbf{v}_0(r) \left(\frac{e}{c}\tilde{\mathbf{A}}(\mathbf{x}) \cdot \frac{\partial f_0(r,\mathbf{P})}{\partial \mathbf{P}}\right),$$

$$\mathbf{j}_M(\mathbf{x}) \;=\; \int \mathrm{d}^3 P\, e\, \mathbf{v}_0(r) \tilde{f}(\mathbf{x},\mathbf{P}).$$

$\mathbf{j}_U$ ... purely reactive unmagnetized plasma response,

$\mathbf{j}_M$ ... (main) magnetized plasma response,

$\mathbf{v}_0(r) = \frac{1}{m}\left(\mathbf{P} - \frac{e}{c}\mathbf{A}_0(r)\right)$ ... velocity in canonical coordinates.

Current density perturbation

$$\tilde{\mathbf{j}}(\mathbf{x}) \;=\; \mathbf{j}_U(\mathbf{x}) + \mathbf{j}_M(\mathbf{x}) \;=\; \int \mathrm{d}^3 P \, e \, \mathbf{v}_0(r,\mathbf{P}) \tilde{f}^t(\mathbf{x},\mathbf{P})$$

$$\tilde{f}^t(\mathbf{x},\mathbf{P}) \;=\; \tilde{f}(\mathbf{x},\mathbf{P}) + \frac{e}{c}\tilde{\mathbf{A}}(\mathbf{x}) \cdot \frac{\partial f_0(r,\mathbf{P})}{\partial \mathbf{P}}.$$

Contravariant current density

$$\begin{aligned} j^k(\mathbf{x}) \;&=\; \int \mathrm{d}^3 P \int \mathrm{d}^3 r' \, \delta\left(\mathbf{r} - \mathbf{r}'\right) \, e \, v_0^k(\mathbf{r}',\mathbf{P}) \tilde{f}^t(\mathbf{r}',\mathbf{P}) \\ &=\; \int \mathrm{d}^3 J \int \mathrm{d}^3 \theta \, \frac{1}{\sqrt{g}} \, \delta\left(\mathbf{x} - \mathbf{x}(\boldsymbol{\theta},\mathbf{J})\right) \, e \, v_0^k(\boldsymbol{\theta},\mathbf{J}) \tilde{f}^t(\boldsymbol{\theta},\mathbf{J}). \end{aligned}$$

Perturbed distribution function

$$\tilde{H} = -\frac{e}{c}\mathbf{v}_0 \cdot \tilde{\mathbf{A}} = \frac{ie}{\omega} v_0^k E_k = \frac{ie}{\omega}\Omega^\alpha \mathcal{E}_\alpha, \qquad \mathcal{E}_\alpha(\boldsymbol{\theta},\mathbf{J}) = \frac{\partial x^i}{\partial \theta^\alpha}\tilde{E}_i\left(\mathbf{x}(\boldsymbol{\theta},\mathbf{J})\right).$$

$$\tilde{H}(\boldsymbol{\theta},\mathbf{J}) = \sum_{\mathbf{m}} \tilde{H}_{\mathbf{m}}(\mathbf{J})\, \mathrm{e}^{i\mathbf{m}\cdot\boldsymbol{\theta}}, \qquad \tilde{f}(\boldsymbol{\theta},\mathbf{J}) = \sum_{\mathbf{m}} \tilde{f}_{\mathbf{m}}(\mathbf{J})\, \mathrm{e}^{i\mathbf{m}\cdot\boldsymbol{\theta}},$$

$$\tilde{f}_{\mathbf{m}} = \frac{1}{\mathbf{m}\cdot\boldsymbol{\Omega} - \omega - i\nu}\left(m_\alpha \tilde{H}_{\mathbf{m}} + \nu\frac{e}{\omega}\left(\mathcal{E}_\alpha\right)_{\mathbf{m}}\right)\frac{\partial f_0}{\partial J_\alpha},$$

$$\tilde{f}^t_{\mathbf{m}}(\mathbf{J}) = \frac{ie}{\omega}\frac{\left(\mathcal{E}_\alpha(\boldsymbol{\theta},\mathbf{J})\right)_{\mathbf{m}}}{\mathbf{m}\cdot\boldsymbol{\Omega}(\mathbf{J}) - \omega - i\nu}\left\{\Omega^\alpha(\mathbf{J})\left(\mathbf{m}\cdot\frac{\partial f_0(\mathbf{J})}{\partial \mathbf{J}}\right) - \frac{\partial f_0(\mathbf{J})}{\partial J_\alpha}\left(\mathbf{m}\cdot\boldsymbol{\Omega}(\mathbf{J}) - \omega\right)\right\},$$

The low frequency limit

Only the potential part of the electric is considered:

$$
\begin{aligned}
\tilde{\mathbf{A}}(\mathbf{x}) &= \mathbf{0}, \qquad \tilde{\mathbf{E}}(\mathbf{x}) = -\nabla\tilde{\Phi}(\mathbf{x}), \\
(\mathcal{E}_\alpha)_{\mathbf{m}} &= -\left(\nabla\tilde{\Phi}(\mathbf{x}(\boldsymbol{\theta},\mathbf{J}))\cdot\frac{\partial\mathbf{x}(\boldsymbol{\theta},\mathbf{J})}{\partial\theta^\alpha}\right)_{\mathbf{m}} = -\left(\frac{\partial\tilde{\Phi}(\mathbf{x}(\boldsymbol{\theta},\mathbf{J}))}{\partial\theta^\alpha}\right)_{\mathbf{m}} = -i\,m_\alpha\tilde{\Phi}_{\mathbf{m}}(\mathbf{J}), \\
\tilde{f}^t_{\mathbf{m}}(\mathbf{J}) &= \frac{e\tilde{\Phi}_{\mathbf{m}}(\mathbf{J})}{\mathbf{m}\cdot\boldsymbol{\Omega}(\mathbf{J})}\left(\mathbf{m}\cdot\frac{\partial f_0(\mathbf{J})}{\partial\mathbf{J}}\right).
\end{aligned}
$$

Here it is important to have the current $\mathbf{j}_U$ under the integral so that the (Larmor radius expanded) contributions from $\mathbf{j}_U$ and $\mathbf{j}_M$ exactly add up in such a way that ω cancels and the result is finite in the limit $\omega \to 0$.

Finite Larmor radius expansion

$$
\begin{aligned}
\tilde{\mathbf{E}}(\mathbf{x}) &= \tilde{\mathbf{E}}(r)\mathrm{e}^{i(k_\vartheta \vartheta + k_z z)}, \\
(\mathcal{E}_\alpha)_{\mathbf{m}} &= \frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{d}\phi_c \, \mathrm{e}^{-i m_\phi \phi_c} \, \mathrm{e}^{i\left(k_\vartheta \rho^\vartheta + k_z \rho^z\right)} \, \frac{\partial x^k}{\partial \theta^\alpha} \, \tilde{E}_k(r_0 + \rho^r) \\
&= \left[\left(a_\alpha^k\right)_{m_\phi} + \left(b_\alpha^k\right)_{m_\phi} \frac{\partial}{\partial r_0} + \cdots \right] \tilde{E}_k(r_0),
\end{aligned}
$$

$$
\begin{aligned}
\frac{1}{2\pi}\int_{-\pi}^{\pi} \mathrm{d}\phi_c \, \mathrm{e}^{i m_\phi \phi_c} \delta\left(r - r_0 - \rho^r\right) \frac{\partial x^k}{\partial \theta^\alpha} \mathrm{e}^{-i\left(k_\vartheta \rho^\vartheta + k_z \rho^z\right)} &= \\
= \left[\left(a_\alpha^k\right)^*_{m_\phi} - \frac{\partial}{\partial r_0} \left(b_\alpha^k\right)^*_{m_\phi} + \cdots \right] \delta\left(r - r_0\right),
\end{aligned}
$$

The Fourier coefficients of $\mathbf{a}_\alpha, \mathbf{b}_\alpha$ are computed analytically. The expansion for $\mathbf{a}_\alpha$ and $\mathbf{b}_\alpha$ has to be done carefully in such a way that the correct low frequency limit is obtained.

Current density in 2-mode FLR expansion

$$\tilde{j}^k(r) = \frac{2\pi i e^2}{\omega r} \sum_{m_\phi} \int_0^\infty dJ_\perp \int_{-\infty}^\infty \mathrm{d}u_\parallel$$

$$\times \left[\left(a_\alpha^k\right)^*_{m_\phi} - \frac{\partial}{\partial r} \left(b_\alpha^k\right)^*_{m_\phi} \right] \frac{\Omega^\alpha \, J \, S^\beta}{\mathbf{m} \cdot \mathbf{\Omega} - \omega - i\nu} \left[\left(a_\beta^j\right)_{m_\phi} + \left(b_\beta^j\right)_{m_\phi} \frac{\partial}{\partial r} \right] E_j(r),$$

$$S^\beta = \Omega^\beta \, \mathbf{m} \cdot \frac{\partial f_0}{\partial \mathbf{J}} - \frac{\partial f_0}{\partial J_\beta} \left(\mathbf{m} \cdot \mathbf{\Omega} - \omega \right).$$

For a Maxwellian plasma

$$\frac{\partial f_0}{\partial J_\beta} = \frac{1}{m} \frac{\partial f_0}{\partial v_j} \left\{ \theta^\beta, P_j \right\} = -\frac{f_0}{T_0} v^j \frac{\partial \theta^\beta}{\partial x^j} = -\frac{f_0}{T_0} \Omega^\beta \quad \Rightarrow \quad S^\beta = -\omega \frac{f_0}{T_0} \Omega^\beta.$$

Therefore, in the case of a homogeneous Maxwellian plasma only absorption of power is possible.

Power flux and power absorption

Averaged over ϑ and z, the work of the wave electric field on the current is separated into a divergence of some material flux density, a locally absorbed power and a fake work

$$\frac{1}{2}\Re\left\langle \mathbf{j}\cdot\mathbf{E}^*\right\rangle \; = \; \frac{1}{2}\Re\left\{j^k(r)E_k^*(r)\right\} \; = \; \frac{1}{r}\frac{\partial}{\partial r}rF_M^r(r)+p_L(r)+p_f,$$

$$F_M^r(r) \; = \; -\frac{\pi e^2}{\omega r}\Re\left\{i\sum_{m_\phi}\int_0^\infty \mathrm{d}J_\perp\int_{-\infty}^\infty \mathrm{d}u_\parallel \left(b_\alpha^k\right)^*_{m_\phi}\frac{\partial E_k^*(r)}{\partial r}\frac{J\Omega^\alpha\Omega^\beta}{\mathbf{m}\cdot\boldsymbol{\Omega}-\omega}\mathbf{m}\cdot\frac{\partial f_0}{\partial\mathbf{J}}\left[\left(a_\beta^j\right)_{m_\phi}+\left(b_\beta^j\right)_{m_\phi}\frac{\partial}{\partial r}\right]E_j(r)\right\},$$

$$p_L(r) \; = \; -\frac{\pi^2 e^2}{\omega r}\sum_{m_\phi}\int_0^\infty \mathrm{d}J_\perp\int_{-\infty}^\infty \mathrm{d}u_\parallel\, J\,\delta\left(\mathbf{m}\cdot\boldsymbol{\Omega}-\omega\right)\mathbf{m}\cdot\frac{\partial f_0}{\partial\mathbf{J}}\left|\Omega^\beta\left[\left(a_\beta^j\right)_{m_\phi}+\left(b_\beta^j\right)_{m_\phi}\frac{\partial}{\partial r}\right]E_j(r)\right|^2,$$

$$p_f \; = \; -\frac{\pi e^2}{\omega r}\Re\left\{i\sum_{m_\phi}\int_0^\infty \mathrm{d}J_\perp\int_{-\infty}^\infty \mathrm{d}u_\parallel\, E_k^*(r)\left[\left(a_\alpha^k\right)^*-\frac{\partial}{\partial r}\left(b_\alpha^k\right)^*\right]J\Omega^\alpha\frac{\partial f_0}{\partial J_\beta}\left[a_\beta^j E_j(r)+b_\beta^j\frac{\partial E_j(r)}{\partial r}\right]\right\}.$$

Due to the indefinite sign of $\mathbf{m}\cdot\partial f_0/\partial\mathbf{J}$, power absorption is guaranteed for a Maxwellian plasma only. The fake work p_f stems from the finite Larmor radius expansion of the subintegrand in current $\mathbf{j}_U$. For a Maxwellian plasma p_f can be shown to be a flux divergence.

Number of modes

If $\dfrac{\partial}{\partial r}$ is interpreted as $k_\perp$ with $\mathbf{k} = (k_\perp, 0, k_\parallel)$, the Maxwell tensor

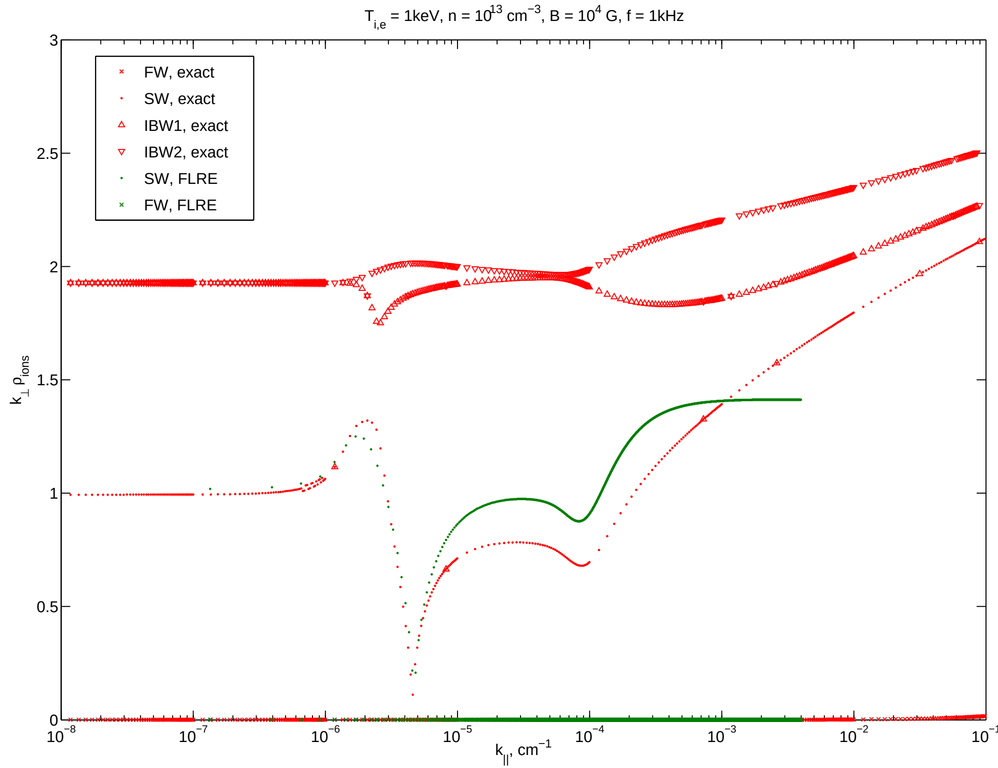
$$\Lambda_{ij} \;=\; N_i N_j - \delta_{ij} N^2 + \varepsilon_{ij},$$

has the following structure

$$\mathbf{\Lambda} \;\widehat{=}\; \begin{pmatrix} k_\perp^0 + k_\perp^{2(M-1)} & k_\perp^0 + k_\perp^{(2M-1)} & k_\perp^1 + k_\perp^{(2M-1)} \\ k_\perp^0 + k_\perp^{(2M-1)} & k_\perp^2 + k_\perp^{2M} & k_\perp^0 + k_\perp^{2M} \\ k_\perp^1 + k_\perp^{(2M-1)} & k_\perp^0 + k_\perp^{2M} & k_\perp^2 + k_\perp^{2M} \end{pmatrix}.$$

For this particular FLR expansion of order M which fulfills positive energy definiteness and has a finite low frequency limit, there exist $6M-2$ roots of the dispersion equation, i.e. $3M-1$ modes in the system. Therefore, the number of modes is $2, 5, 8, \ldots$.

Dispersion: comparison with the exact solution



Red: roots of the exact dispersion equation; a fast mode, a slow mode and 2 Bernstein type modes are shown.

Green: roots of the 2-mode FLR expanded dispersion equation.

Only the fast and slow mode are coupled in the Alfvén resonance zone.

Quasilinear theory, calculation of the torque

$$\frac{\partial n_0}{\partial t} \equiv \left\langle \int \mathrm{d}^3 P \, \frac{\partial \bar{f}}{\partial t} \right\rangle_{\vartheta,z} = \left\langle \int \mathrm{d}^3 P \, \overline{\left\{ \tilde{H}, \tilde{f} \right\}} \right\rangle_{\vartheta,z}$$

$$= -\frac{1}{r}\frac{\partial}{\partial r} r \left\langle \int \mathrm{d}^3 P \, \overline{\left\{ r, \tilde{H} \right\} \tilde{f}} \right\rangle_{\vartheta,z} \equiv -\frac{1}{r}\frac{\partial}{\partial r} r V_r,$$

radial fluid velocity:

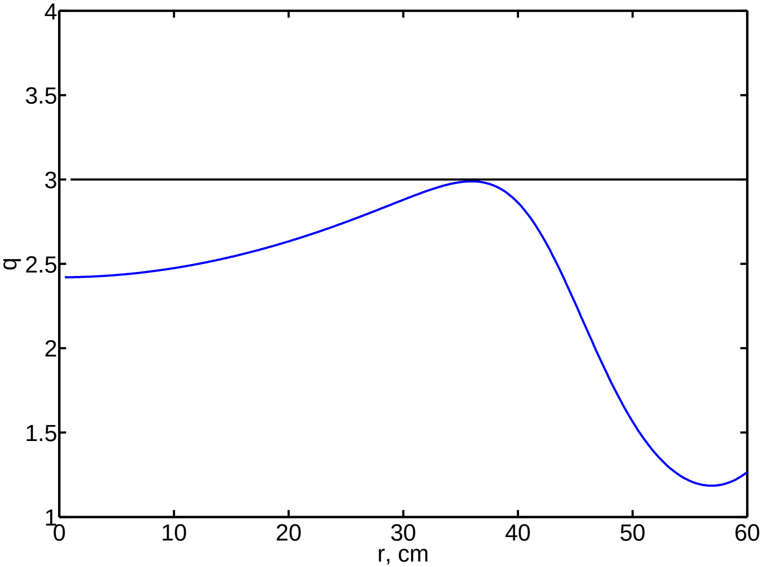
$$V_r = -\frac{\pi}{2 r n_0} \sum_{m_\phi} \int \mathrm{d}\phi_c \int \mathrm{d}^3 J \, \delta\left(r - r\left(\phi_c, \mathbf{J}\right)\right) \mathbf{m} \cdot \frac{\partial r\left(\phi_c, \mathbf{J}\right)}{\partial \mathbf{J}}$$

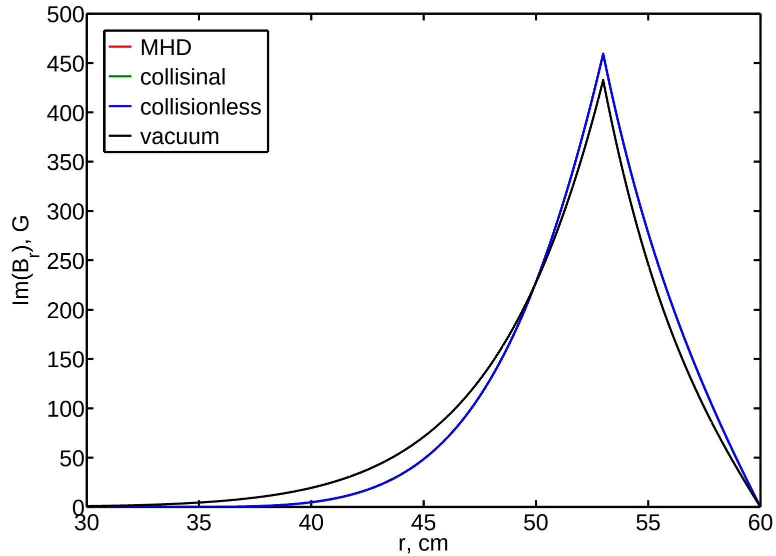
$$\times \left| H_{\mathbf{m}} \right|^2 \delta\left(\mathbf{m} \cdot \Omega - \omega\right) \mathbf{m} \cdot \frac{\partial f_0}{\partial \mathbf{J}} = \frac{c k_s}{\omega n_0 e B_0} p_L = \frac{c}{B_0} \frac{F_s}{n_0 e}$$

Force density in $\mathbf{e}_s = \left(\mathbf{B}_0 / B_0\right) \times \mathbf{e}_r$ direction:

$$F_s = \frac{k_s}{\omega} p_L .$$

Comparison of B_r profile with ideal MHD



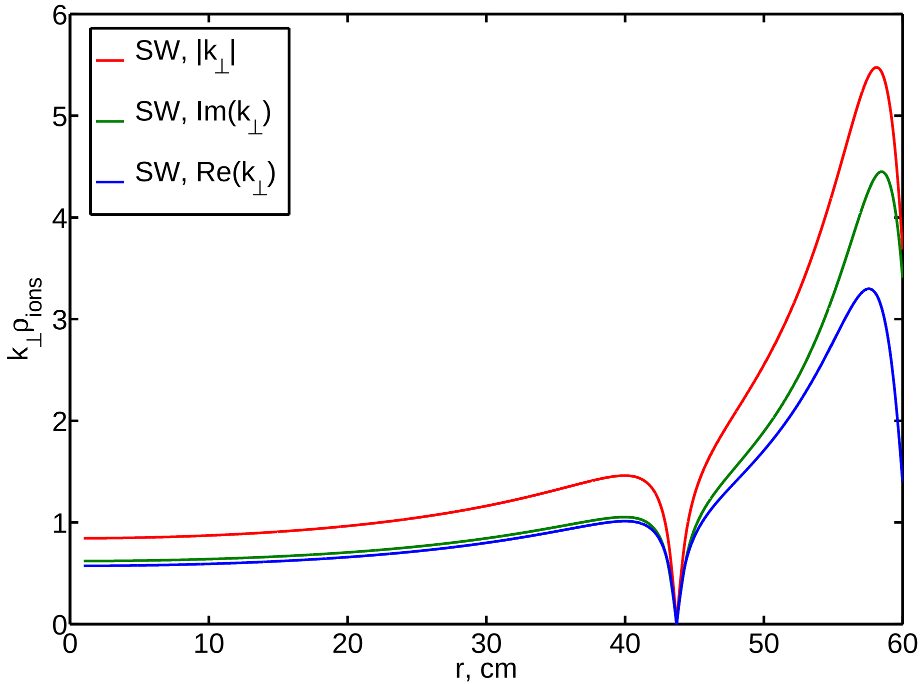


q – profile with extremum close to the resonance

B_r – radial magnetic field

Equilibrium plasma currents modify the profile when compared to the vacuum case. Ideal MHD and 2-mode FLR expansion results are in excellent agreement.

Slow mode dispersion: collisional electron model

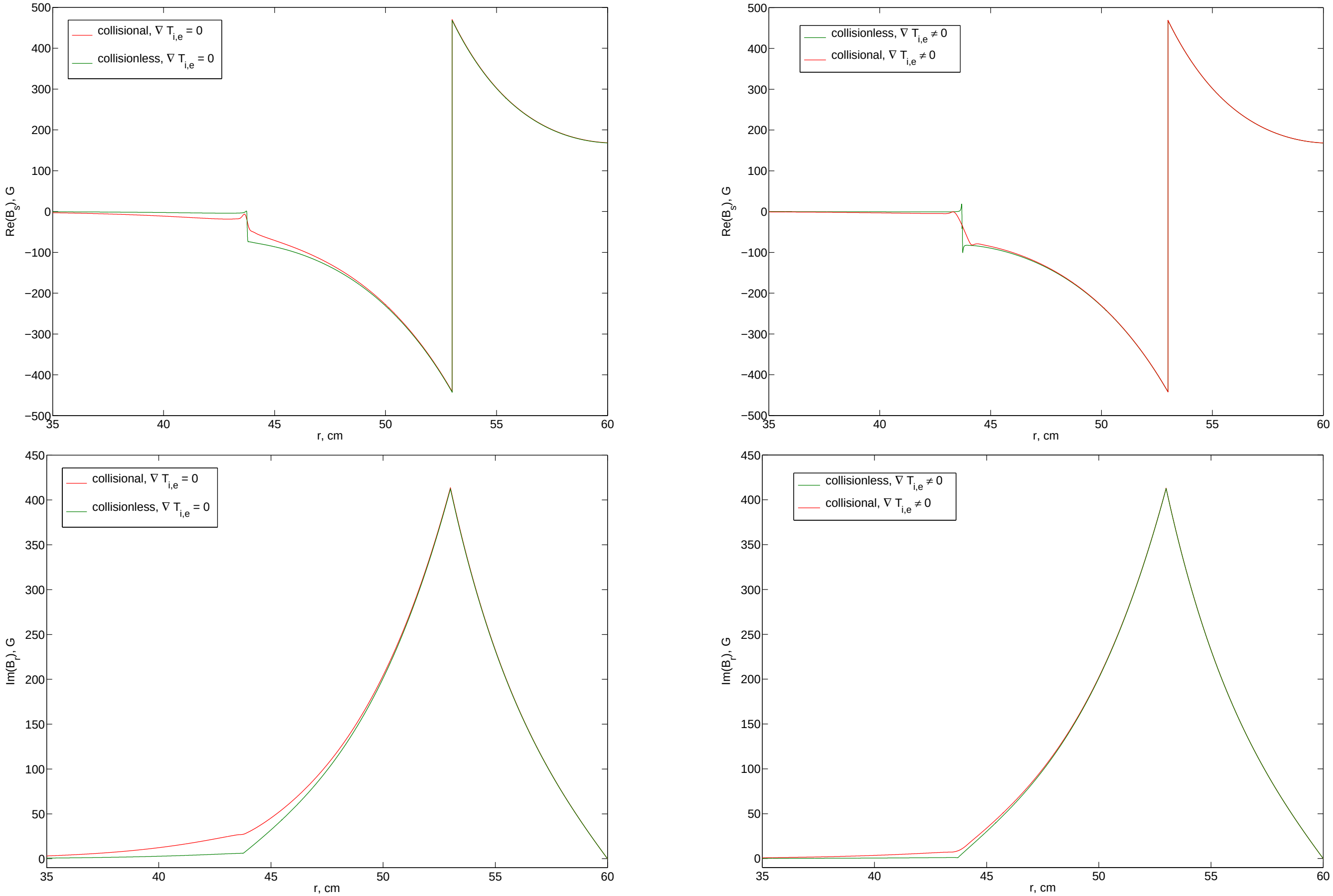


Blue: real part. Green: imaginary part. Red: modulus.

Periodic cylinder model of the plasma torus with constant densities and temperatures and a q profile such that the resonant radius $r_{\rm res} \approx 43$.

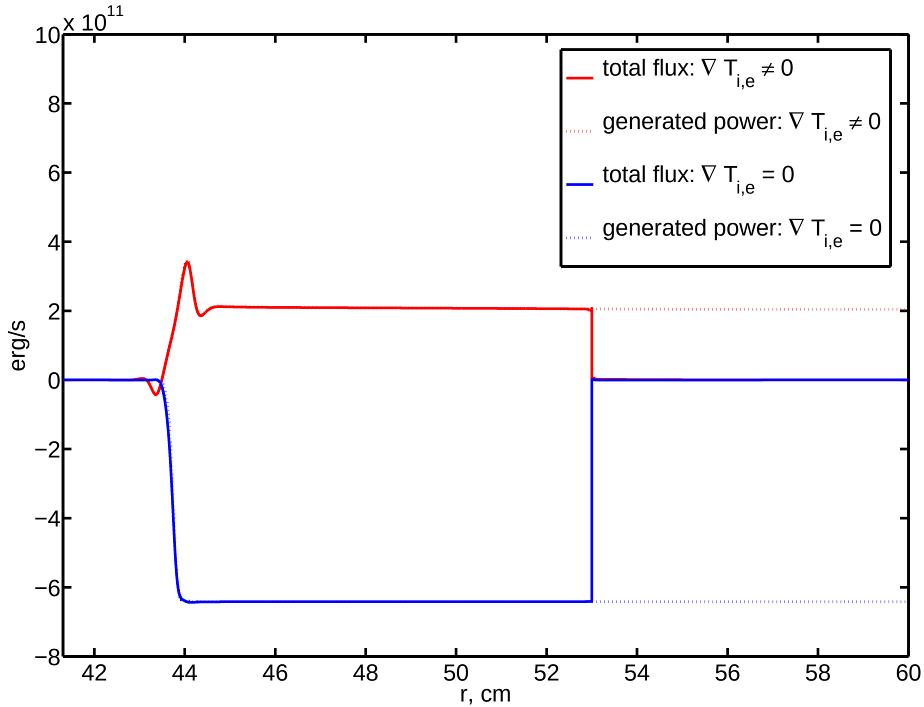
central values: n_0 = 10^{13} cm,$^{-3}$, B_0 = 2 T, T_0 = 1 keV.

Magnetic field perturbations B_s and B_r



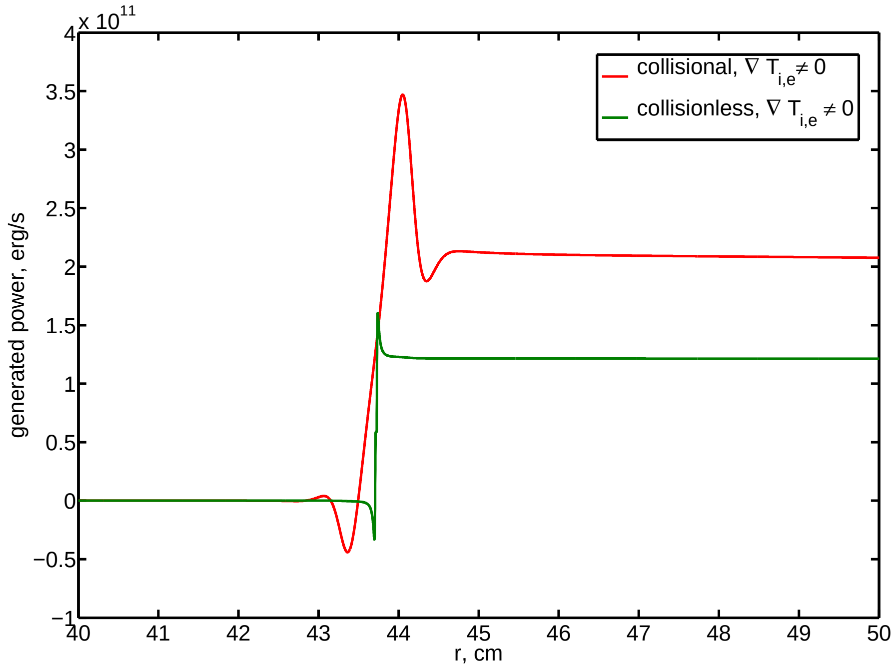
n=const, $T_{i,e}$=const. n=const, $T_i = T_e = T_0\left(1 - r^2/a^2\right)$.

Total Power flux and integrated locally absorbed (generated) power



Blue: no pressure gradients. Red: with pressure gradients. a) the energy equation is satisfied b) the value obtained from the expression p_L for the absorbed (generated) power within the plasma volume is in perfect agreement with the work of the local electric field on the antenna currents $\frac{1}{2}\Re\left\{\tilde{\mathbf{E}} \cdot \tilde{\mathbf{j}}_a^*\right\}$.

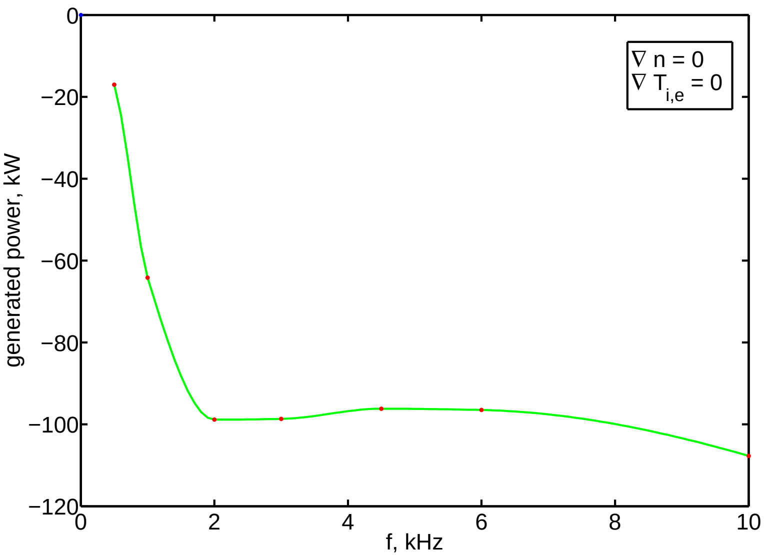
Integrated locally generated power and electron collisions

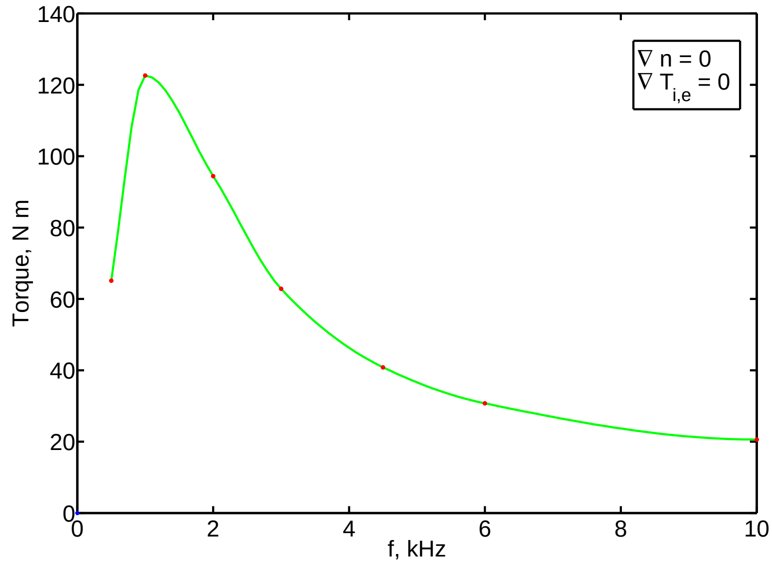


Red: with electron collisions. Green: collisionless model.

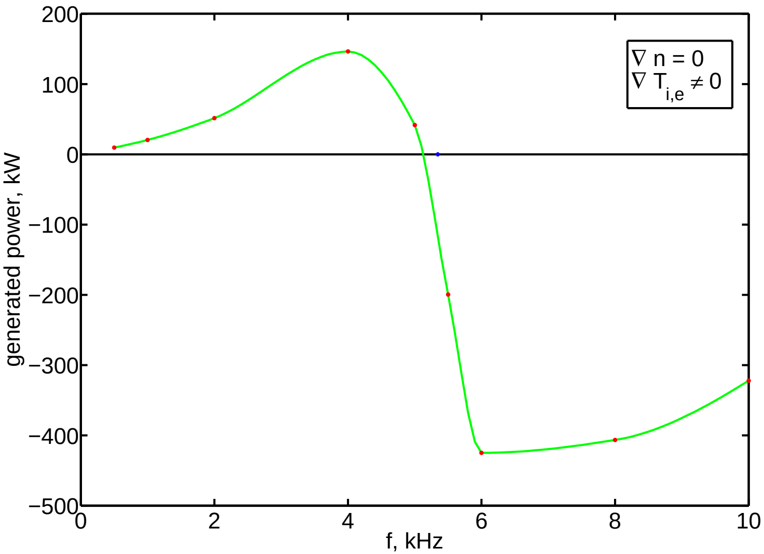
Electron collisions broaden the resonance region and enhance the amount of power generated within this region.

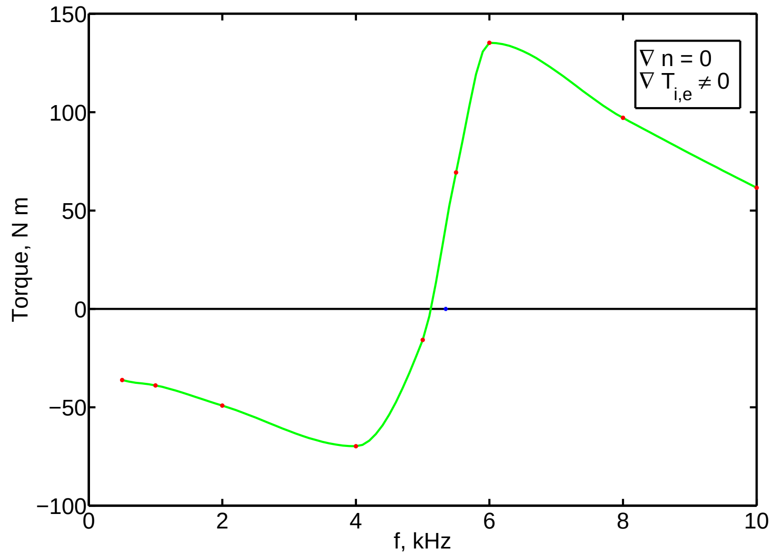
Power and torque, T=const and n=const





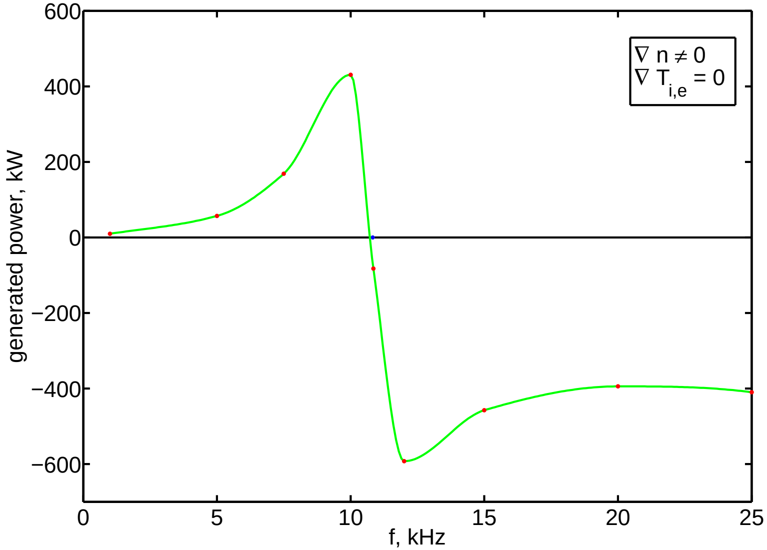
Power and torque, T varying and n=const

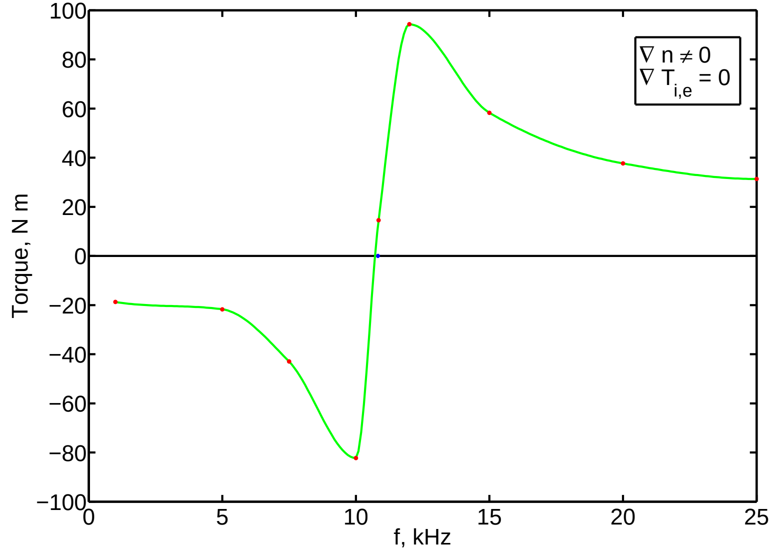




Blue asterix: diamagnetic drift frequency

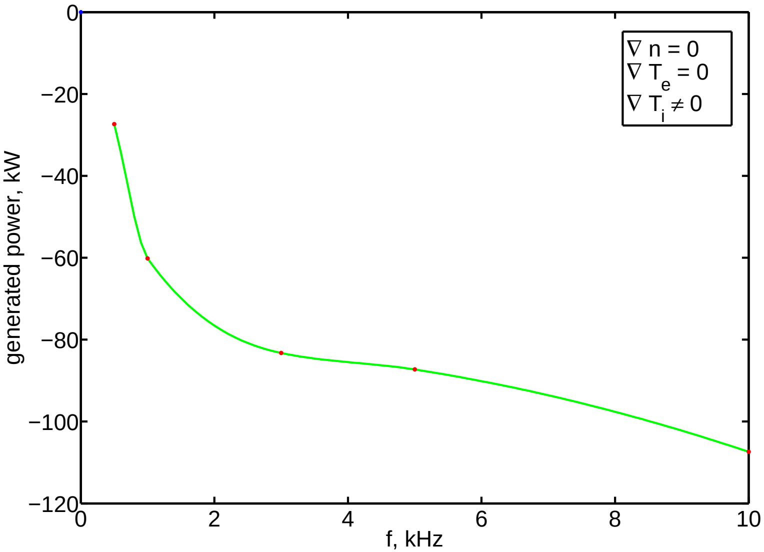
Power and torque, T=const and n=varying

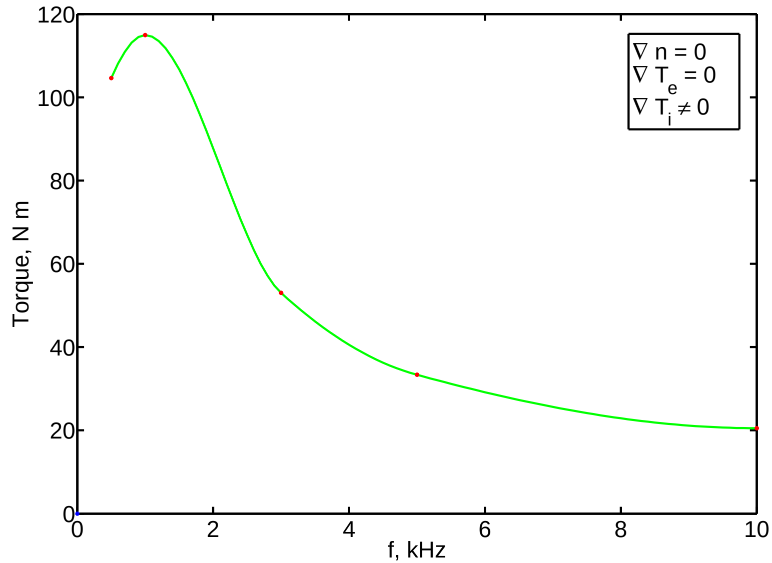




Blue asterix: diamagnetic drift frequency

Power and torque, T_e, n=const and T_i=varying





Qualitatively the same as the n, T= const case.

Why power can be generated?

Constants of motions are used to build the distribution function:

$$H_0(J_\perp, P_\vartheta, P_z), \quad r_0(P_\vartheta, P_z), \quad u_\parallel(P_\vartheta, P_z),$$

$$f_0(H_0, u_\parallel, r_0) = \frac{n_0(r_0)}{(2\pi m T_0(r_0))^{(3/2)}} \times \exp\left\{-\frac{1}{T_0(r_0)}\left(H_0(\mathbf{J}) - e\Phi_0(r_0) - mV_0(r_0)u_\parallel + \frac{mV_0^2(r_0)}{2}\right)\right\}.$$

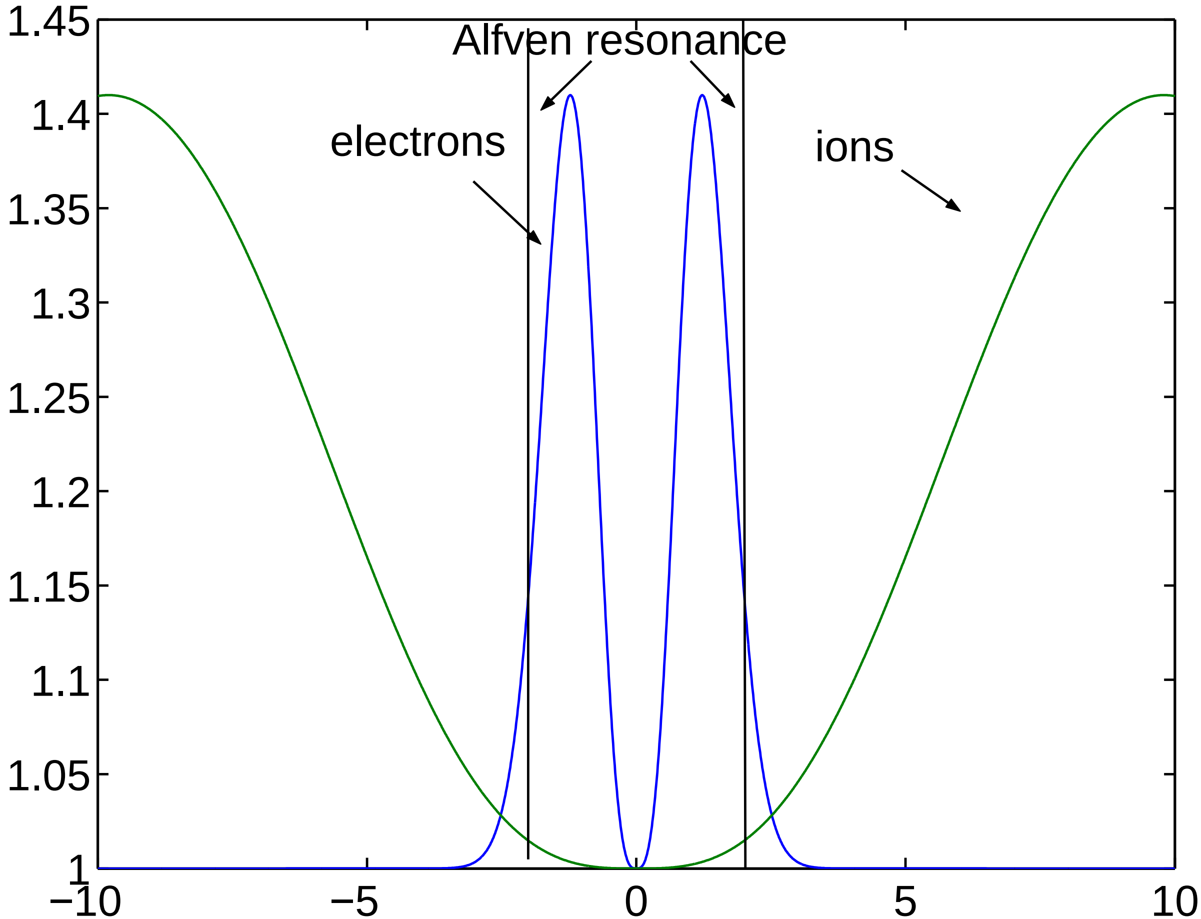
The velocity $V_0(r_0)$ is chosen such that electron current gives the desired q profile.

$$\mathbf{m}\cdot\frac{\partial r_0}{\partial \mathbf{J}} = \frac{ck_s}{eB_0}, \qquad \mathbf{m}\cdot\frac{\partial u_\parallel}{\partial \mathbf{J}} = \frac{k_\parallel}{m},$$

$$\begin{aligned}\delta\left(\mathbf{m}\cdot\mathbf{\Omega}-\omega\right)\mathbf{m}\cdot\frac{\partial f_0}{\partial \mathbf{J}} &= \delta\left(\mathbf{m}\cdot\mathbf{\Omega}-\omega\right)\left(\omega\frac{\partial f_0}{\partial H_0} + \mathbf{m}\cdot\frac{\partial r_0}{\partial \mathbf{J}}\frac{\partial f_0}{\partial r_0} + \mathbf{m}\cdot\frac{\partial u_\parallel}{\partial \mathbf{J}}\frac{\partial f_0}{\partial u_\parallel}\right)\\ &= \delta\left(\mathbf{m}\cdot\mathbf{\Omega}-\omega\right)\left(-\frac{\omega}{T_0}f_0 + \frac{ck_s}{eB_0}\frac{\partial f_0}{\partial r_0} + \frac{k_\parallel}{m}\frac{\partial f_0}{\partial u_\parallel}\right)\\ &\propto -\left(\omega-\omega_*\right) \quad \Longrightarrow \quad p_L \propto \omega\left(\omega-\omega_*\right),\end{aligned}$$

with ω_* the diamagnetic drift frequency. $\qquad F_s \propto k_s\left(\omega-\omega_*\right).$

Why the ions do not feel it?



Conclusions (model)

- A family of FLR expansions has been introduced with the property of a finite low frequency limit and positive definiteness of the absorbed power in a thermodynamic equilibrium plasma.
- The number of modes for this type of expansion is $3M - 1$, i.e., 2 – 5 – 8 –. . . , where M is an expansion order.
- Since the Bernstein type modes do not participate in the Alfvén resonance, the 2-mode FLR expansion has been used in the present study (5-mode expansion is planned in a future study).
- The 2-mode model is in good agreement with ideal MHD.
- The separation of the work of the electric field on the plasma currents allows 3 relatively independent methods to calculate the absorbed (generated) power: as a total power flux (Poynting plus material flux) towards the antenna, as a integral of the absorbed power density over the plasma volume and third, as the work of the antenna currents on the wave electric field.

Conclusions (modelling)

- For the case of a homogeneous plasma, the results for the power absorption and for the torque are in good agreement with other linear models.
- For DED frequencies ω below the electron diamagnetic drift frequency $\omega_* = k_s v_{de}$, the force acting on the plasma in always in the direction of the ion diamagnetic drift irrespective of the sign of ω.
- Measurements of the antenna impedance are of interest.