



B A C H E L O R A R B E I T

# Spintronics

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# 1 Introduction

The term *spintronics*<sup>1</sup> arised from the relatively new approach of using the electrons spin magnetic moment to create microelectronic devices. Behaviors the electron exhibits in particular magnetic junctions often derive from quantum effects associated with the spin magnetic moment as an additional degree of freedom. These quantum effects boosted the number of effects available for creating microelectronic devices.

In fact, the discovery of the GMR (Giant Magneto Resistance) effect in 1988 can be considered the birth of spintronics. Because of the importance of the new field of research and of the already developed applications, the discoverers of this effect have been awarded the nobel price of physics in 2007 [3].

This thesis aims at describing the fundamental theories of spintronics and at giving a review of the basic spintronic devices that have already been developed. The goal is *not* to give detailed information on the newest developments, because things become obsolete quickly in this fast-paced field of research.

The fist part (section 2) deals with fundamentals of spin injection, spin-dependent tunneling and relaxation. A basic model of the behaviour of particles and spin in solid state systems is described, aiming at giving an idea what happens in a metal/semiconductor when an electrical field is applied. Calculations are made F/N Junctions are described as a basic model of spin injection which leads to an understanding of the spin valve effect.

The second chapter (section 3) deals with the two most important effects in today's realized applications.

## 2 Fundamentals

### 2.1 spin injection

This approach follows in principle the treatment of [2], Sec. II A-B. The difference lies in the parallel examination of spin and particles. Moreover, some calculations have been reproduced and steps have been given more exactly.

The term *spin injection* refers to injection of nonequilibrium spin from a ferromagnetic region (non-zero polarization) into a non-ferromagnetic region (mostly a conductor). This can be seen as the transfer of spin polarization by electrical current. It has to be distinguished from the reverse effect called *spin extraction*.

The first examinations aim at getting an explicit representation of the **particle density**  $n(x, t)$  and **spin density**  $s(x, t)$  functions of electrons, which give us the simplest model of what happens at border regions (e.g. electrical contact between iron and aluminum).

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<sup>1</sup>composed of *spin* and *electronics*, also (formerly) called *magnetoelectronics*

### 2.1.1 Drift-Diffusion Model

Particle density and spin density quantities can be expressed in terms of spin up ( $\uparrow$ ) and spin down ( $\downarrow$ ) electron densities:

$$n = n_{\uparrow} + n_{\downarrow} \quad (1)$$

$$s = n_{\uparrow} - n_{\downarrow} \quad (2)$$

If one can find a representation of  $n(x, t)$ , it's easy to get one for  $s(x, t)$  as well.

#### Derivation of the drift-diffusion equations for $n$ and $s$ - random walk

Now consider electrons moving through a metal or a semiconductor under the influence of an electric field  $\mathbf{E}$ . They can scatter off impurities, phonons or boundaries. A good first approximation of how electrons behave in such a system is the one-dimensional random walk.

The basic quantities that define the random walk are

$v$  ... velocity of an electron moving through the crystal

$l$  ... distance the electron moves before changing its direction (mean free path)

In order to get a constant step time  $\tau = \frac{l}{v}$ , it is important that the electric field is not strong enough to change the velocity significantly:

$$\Delta v = \left| \frac{eE}{m} \tau \right| \ll v \quad (3)$$

$\Delta v$  ... change of the velocity during one step

$m$  ... mass of the electron

$e$  ... charge of the electron

The average velocity can be changed by influence of the field and due to friction of the electron, which is proportional to the average velocity itself:

$$\dot{v}_{av} = -\frac{eE}{m} - \frac{v_{av}}{\tau}$$

Supposing that  $v_{av}$  is constant, another important quantity can be obtained:

$$\text{drift velocity ... } v_d = -\frac{eE\tau}{m} \left( \stackrel{(3)}{\ll} v \right) \quad (4)$$

The electron density  $n$  at time  $t$  can be expressed by the densities at time  $t - \tau$  and positions  $x + l$  respectively  $x - l$  and the probabilities  $p_+$  and  $p_-$  that a particle moves right or left ( $p_+ + p_- = 1$ ):

$$n(x, t) = n(x - l, t - \tau)p_+ + n(x + l, t - \tau)p_- \quad (5)$$

Considering here also the spin magnetic moment, this balance equation can also be written for spin-up and spin-down densities:

$$n_{\uparrow}(x, t) = n_{\uparrow}(x - l, t - \tau)(1 - w)p_{+} + n_{\uparrow}(x + l, t - \tau)(1 - w)p_{-} \\ + n_{\downarrow}(x - l, t - \tau)wp_{+} + n_{\downarrow}(x + l, t - \tau)wp_{-} \quad (6)$$

where  $w$  is the propability that spin flips in the time of  $\tau$ . So the first two terms describe the ratio of the densities in the case of that the spin didn't flip during the last step, the second two if it did flip.

Expanding this into a taylor series around  $x$  and  $t$ , one gets:

$$n_{\uparrow}(x, t) \approx n_{\uparrow}(1 - w) - l\Delta p \frac{\partial n_{\uparrow}}{\partial x} - \tau \frac{\partial n_{\uparrow}}{\partial t} + \frac{1}{2}l^2 \frac{\partial^2 n_{\uparrow}}{\partial x^2} + n_{\downarrow}w$$

From this it follows that

$$\frac{\partial n_{\uparrow}}{\partial t} = \underbrace{\frac{1}{2} \frac{l^2}{\tau}}_D \frac{\partial^2 n_{\uparrow}}{\partial x^2} - \underbrace{\frac{l}{\tau} \Delta p}_{v_d} \frac{\partial n_{\uparrow}}{\partial x} - \frac{w}{\tau} (n_{\uparrow} - n_{\downarrow})$$

$D$  ... diffusion coefficient<sup>2</sup>

Because of (Eq. 4),  $\Delta p$  must be  $\ll 1$ . So one finds the

DRIFT-DIFFUSION EQUATIONS FOR  $n_{\uparrow}(x, t)$  AND  $n_{\downarrow}(x, t)$

---

$$\frac{\partial n_{\uparrow}}{\partial t} = D \frac{\partial^2 n_{\uparrow}}{\partial x^2} - v_d \frac{\partial n_{\uparrow}}{\partial x} - \frac{w}{\tau} (n_{\uparrow} - n_{\downarrow}) \quad (7c)$$

$$\frac{\partial n_{\downarrow}}{\partial t} = D \frac{\partial^2 n_{\downarrow}}{\partial x^2} - v_d \frac{\partial n_{\downarrow}}{\partial x} - \frac{w}{\tau} (n_{\downarrow} - n_{\uparrow}) \quad (7d)$$

<sup>2</sup>According to S. Pramanik, S. Bandyopadhyay, M. Cahay (see [7]), especially in quantum wires, the spin diffusion coefficient and the charge diffusion coefficient are *not* the same and can differ vastly.

Finally, adding (subtracting) these equations (see Eq. 1 and 2) leads to

DRIFT-DIFFUSION EQUATION FOR  $n(x, t)$  AND  $s(x, t)$

---

$$\frac{\partial n}{\partial t} = D \frac{\partial^2 n}{\partial x^2} - v_d \frac{\partial n}{\partial x} \quad (8)$$

$$\frac{\partial s}{\partial t} = D \frac{\partial^2 s}{\partial x^2} - v_d \frac{\partial s}{\partial x} - s \underbrace{\frac{1}{\tau_s}}_{= \frac{2w}{\tau}} \quad (9)$$

$\tau_s$  ... spin relaxation time,

### Solutions to the diffusion equations

In the case of  $E = 0 \stackrel{(4)}{\Rightarrow} v_d = 0$ , one can look only at the diffusion, without the disturbance of drift. Let there be an initial number of particles  $N_0$ , with spin  $S_0$  for the spin case. This gives the initial conditions

$$n(x, 0) = N_0 \delta(x) \quad (10)$$

$$s(x, 0) = S_0 \delta(x) \quad (11)$$

It can be easily shown that the standard deviation evolves with time as

$$\sigma(t) = \sqrt{2Dt}$$

in  $d = 1$  dimension. A solution can be guessed, since diffusion takes place at the length-scale  $\sim \sqrt{Dt}$ :

$$n(x, t) = N_0 \frac{1}{\sqrt{Dt}} f\left(\frac{x}{\sqrt{Dt}}\right)$$

For spin, another function of time can be expected to play a role in the solution, in which the spin relaxation time  $\tau_s$  has to be included as divisor. This leads to the separation ansatz

$$s(x, t) = N_0 \frac{1}{\sqrt{Dt}} f\left(\frac{x}{\sqrt{Dt}}\right) g\left(\frac{t}{\tau_s}\right)$$

#### charge (particle)

Using the ansatz for eq. (8) (remember  $v_d = 0$ ) and substituting  $\xi := \frac{x}{\sqrt{Dt}}$  gives

#### spin

For this case, additionally using  $\eta := \frac{t}{\tau_s}$  for the argument for  $g$  leads to

the diff. equation. This result is slightly

different from that obtained in [2], Eq. II.22. The factor 1/2 is missing there.

$$f'' + \frac{\xi}{2}f' + \frac{1}{2}f = 0$$

$$(-2*\eta+1)-2*\eta\frac{g'}{g} = -\frac{1}{f}(\xi f' + 2 * f'') = \lambda \quad (12)$$

with  $\lambda \dots$  separation parameter

Both sides can be solved independently:

The right hand side is almost the same as for the charge case, except that the equation now is inhomogeneous because of the separation constant. The left hand side is just a linear differential equation and can be integrated easily, resulting into an exponential function.

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SOLUTIONS TO THE DIFFUSION EQUATIONS FOR  $n(x, t)$  AND  $s(x, t)$

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$$n(x, t) = \frac{N_0}{\sqrt{4\pi Dt}} e^{-\frac{x^2}{4Dt}} \quad (13)$$

$$s(x, t) = \frac{S_0}{\sqrt{4\pi Dt}} e^{-\frac{x^2}{4Dt}} e^{-\frac{t}{\tau_s}} \quad (14)$$

The initial condition can be considered as particles released at a point in space. The result of this model is that the Dirac Distribution dissolves into a Gaussian, finally leading to a homogeneous distribution of the particles when going to infinite time. In the spin case, the relaxation term leads to decay of spin, meaning that it is not conserved.

In fig. 1, the solutions are presented for the case of silicon. The initially released particles simply diffuse, leading to a homogeneous distribution, whereas spin additionally relaxes. After some time, relaxation dominates the diffusion, leading to spin  $s \approx 0$  at  $t = 200$  ns.

### charge (particle)

For particles, the overall number is a constant:

$$N_0 = \int_{-\infty}^{\infty} xn(x, t)dx$$

### spin

Spin is not a conserved quantity, it decays exponentially:

$$S = \int_{-\infty}^{\infty} xs(x, t)dx = S_0 \exp\left(\frac{-t}{\tau_s}\right)$$

The standard deviation at time  $t = \tau_s$  gives an idea of how far spin diffuses until

it relaxes:

$$\sigma = \sqrt{2D\tau_s} = \sqrt{2}L_s$$

with  $L_s \dots$  spin diffusion length.

### solutions to the spin drift-diffusion-equation

If the electric field does not vanish ( $E \neq 0$ ), solving eq. 9 analytically is difficult. For this approach a numeric solution is suitable enough to get an idea of what difference a non-zero drift makes for diffusing spin.

The following examinations are based on numeric calculations using Wolfram Mathematica<sup>TM3</sup> for the case of electrons in silicon. Mathematica's function `NDSolve` is able to solve numerically many different types of ordinary differential equations as well as partial differential equations. The spin drift-diffusion equation (Eq. 9) in principle is of the type of the heat equation, which can be solved by `NDSolve`.

The sizes of the physical quantities in silicon at  $T = 300\text{K}$  and  $E = 10^3\text{Vcm}^{-1}$  are as follows:

Quantity <sup>4</sup>	Value
$D$	$10^{-4} \text{ m}^2/\text{s}$ ([10])
$\tau_s$	$50 \text{ ns}$ ([2])
$v_d$	$10^4 \text{ m/s}$ ([1])
$n_i$	$10^{16}\text{m}^{-3}$ ([5])

These are used to simulate the case of a thin wire with an area cross-section of  $A \approx 10^{-12}\text{m}^2$ , having a initial spin density of<sup>5</sup>  $s_0 = An_i = 10^4/\text{m}$ :

#### a) no electric field applied, no given spin current

This simple case simulates the behaviour of spin when there is no influence on the spin density, except of relaxation and diffusion.

$$\frac{\partial s}{\partial t} = D \frac{\partial^2 s}{\partial x^2} - \frac{s}{\tau_s}; t \in [0, 500\text{ns}], x \in [0, 100\mu\text{m}]$$

The boundary conditions must be specified for both edges, and have been chosen to:

$$s(0, t) = s(d, t) = s_0$$

with  $d = 100\mu\text{m}$  being the distance between the right and the left boundary, which has been chosen to give particles enough space to diffuse.

The initial condition also has to be specified:

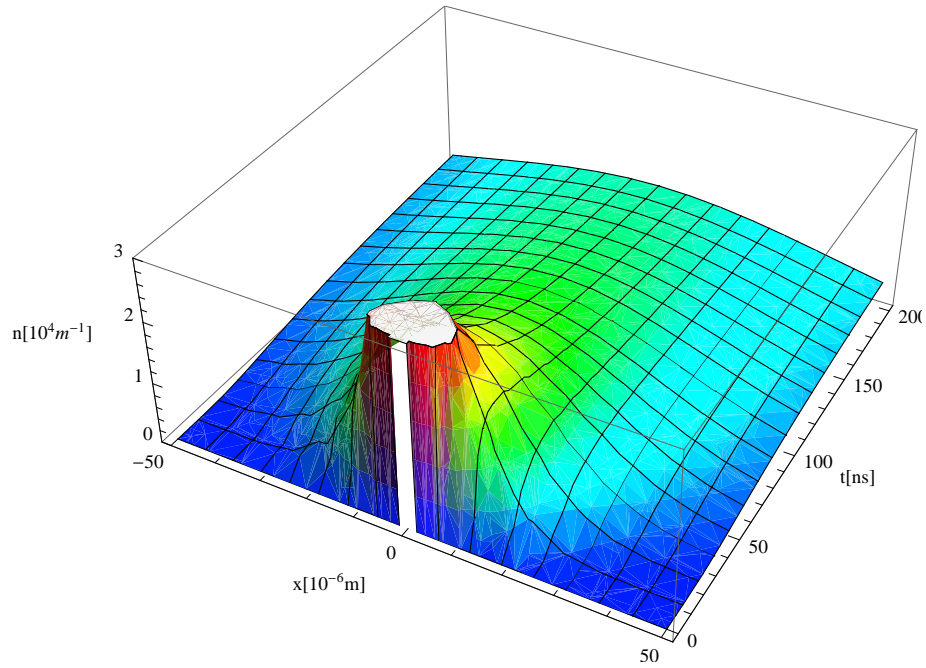
$$s(x, 0) = s_0$$

<sup>3</sup>Version 7.0.0

<sup>5</sup>Under the assumption of that the spin density is of the same order of magnitude as the charge carrier density



a)



b)

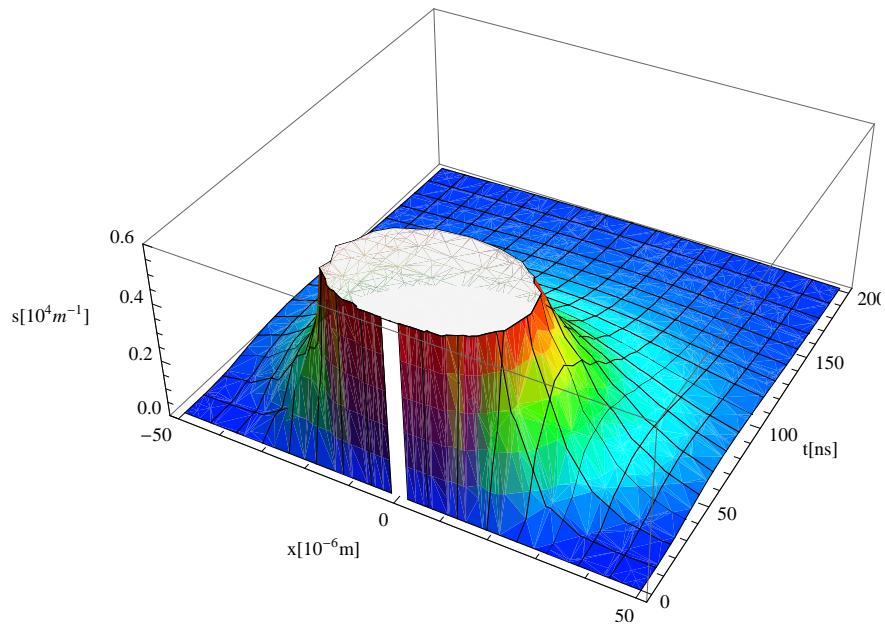


Figure 1: Solutions to the diffusion equations: a)  $n(x,t)$  and b)  $s(x,t)$ . The constants have been chosen to approximately match the case for silicon:  $D \approx 4 \cdot 10^{-3} m^2/s$  (at  $T = 300K$ ,  $E = 10^3 Vcm^{-1}$ , see [10]) and  $\tau \approx 50ns$  (as discussed in [2], sec IV.E2),  $N_0 = 10^4 m^{-3}$

meaning that at  $t = 0$ , the area between the two points has a constant density.

The resulting solution is plotted in Fig. 2. At a sufficient spatial distance from the boundary  $s(0, t)$ , the value of the solution evaluates exactly to  $1/e$  at  $t = \tau_s = 50$  ns, as it is expected from an exponential decay of spin over time. The spatial, as well as the time dependent decay of spin can be seen in the figure. After a certain time, the initial condition does not play a role anymore, resulting into the expected pure spatial exponential decay. This is called the **steady state**.

b) **non-zero electric field, no given spin current**

A generated spin drift is linearly dependent on the applied electric field (Eq. 4). For this case, the field has been chosen to be oscillating:

$$\frac{\partial s}{\partial t} = D \frac{\partial^2 s}{\partial x^2} - v_d \sin(\omega t) \frac{\partial s}{\partial x} - \frac{s}{\tau_s}; t \in [0, 500 \text{ ns}], x \in [0, 700 \mu\text{m}]$$

The boundary- and initial conditions again have been chosen to:

$$s(0, t) = s(d, t) = s(x, 0) = s_0$$

and  $\omega = 5 \cdot 10^7$ /sec, which leads to a good visualization on what is going on in the system when the field raises and decreases. The resulting solution is plotted in Fig. 3. The electric field injects the spin into the region between the boundaries. Especially interesting is the evolution of the waves. When looking at a peak, one can see that, when following the way further into space, it moves also a little bit in time. Raising the relaxation time would result into broader peaks, while raising the drift velocity gives a deeper propagation into space.

c) **oscillating spin current, spin pumping**

Another possibility to inject spin is to control the gradient of  $s$  at the boundary, which is the condition for **spin pumping**.

$$\frac{\partial s}{\partial t} = D \frac{\partial^2 s}{\partial x^2} - \frac{s}{\tau_s}; t \in [0, 500 \text{ ns}], x \in [0, 100 \mu\text{m}]$$

For this case only one boundary has to be chosen:

$$s(0, t) = s_0$$

The initial condition stays the same:

$$s(x, 0) = s_0$$

The difference now is the following condition:

$$\frac{\partial s}{\partial x} \Big|_{x=0} = j_0 \sin(\omega t), \omega = 5 \cdot 10^7 / \text{sec}$$

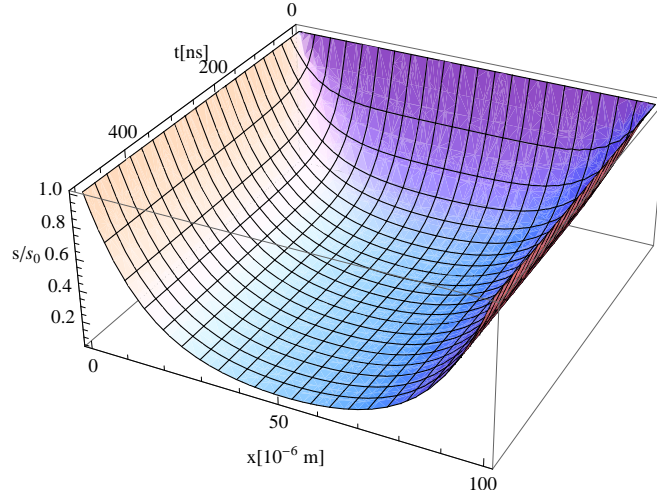


Figure 2: Numerical solution delivered by Mathematica of the spin drift-diffusion equation for the case of no drift and no spin current.

where<sup>6</sup>  $j_0 = 2 \cdot 10^6 m^{-2}$ . Fig. 4 shows the results. The difference between the case of the electric field and this one is, that an electric field can only inject spin which is already existing at the boundary, whereas the spin pumping boundary condition allows to bring new spin into the system due to the derivative.

d) **both oscillating spin drift and current**

As a last example, both the drift term and the derivative at the boundary can be used to inject spin.

$$\frac{\partial s}{\partial t} = D \frac{\partial^2 s}{\partial x^2} - v_d \sin(\omega t) \frac{\partial s}{\partial x} - \frac{s}{\tau_s}; t \in [0, 300 ns], x \in [0, 700 \mu m]$$

Boundary- and initial conditions are as follows:

$$s(0, t) = s(x, 0) = s_0, \frac{\partial s}{\partial x} \Big|_{x=0} = j_0 \sin(16 \cdot \omega t)$$

where again  $\omega = 5 \cdot 10^7 / \text{sec}$ . With the factor 16 in front of  $\omega$ , the result is a beating wave, where the spin pumping part lets raise the spin over 1 with the higher frequency. The drift part then pulls the gained spin further away as long as it is positive. When the sign changes, the spin flows back to the edge, which produces the curve for all the peaks of the pumped spin. Fig. 5 shows the results.

<sup>6</sup>The value of  $j_0$  has been chosen in order to provide an insight into the results mathematically, rather than being physically realistic

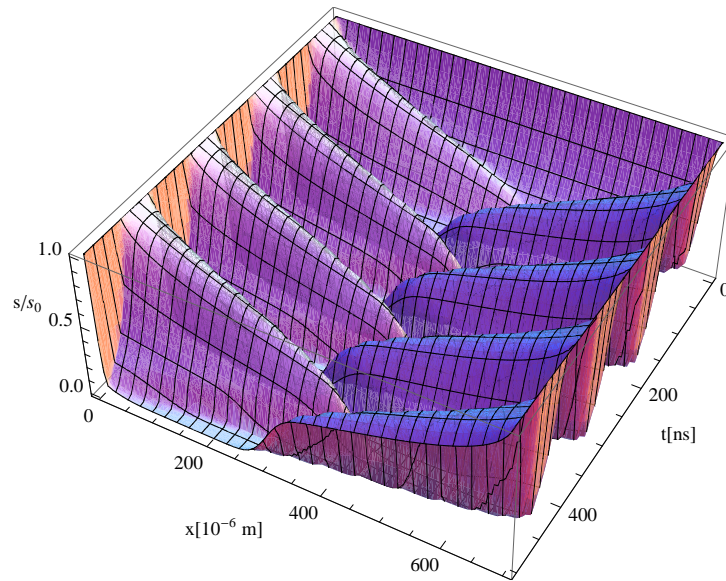


Figure 3: Numerical solution delivered by Mathematica of the spin drift-diffusion equation for the case of an oscillating electric field derived by Mathematica.

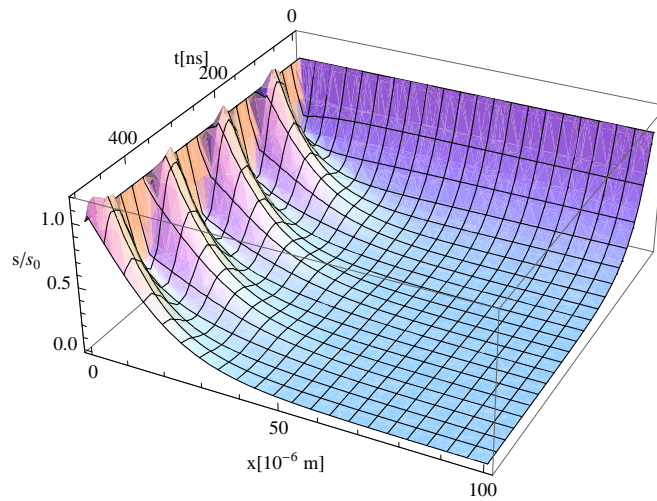


Figure 4: Numerical solution delivered by Mathematica of the spin drift-diffusion equation for the case of an oscillating spin current derived by Mathematica.

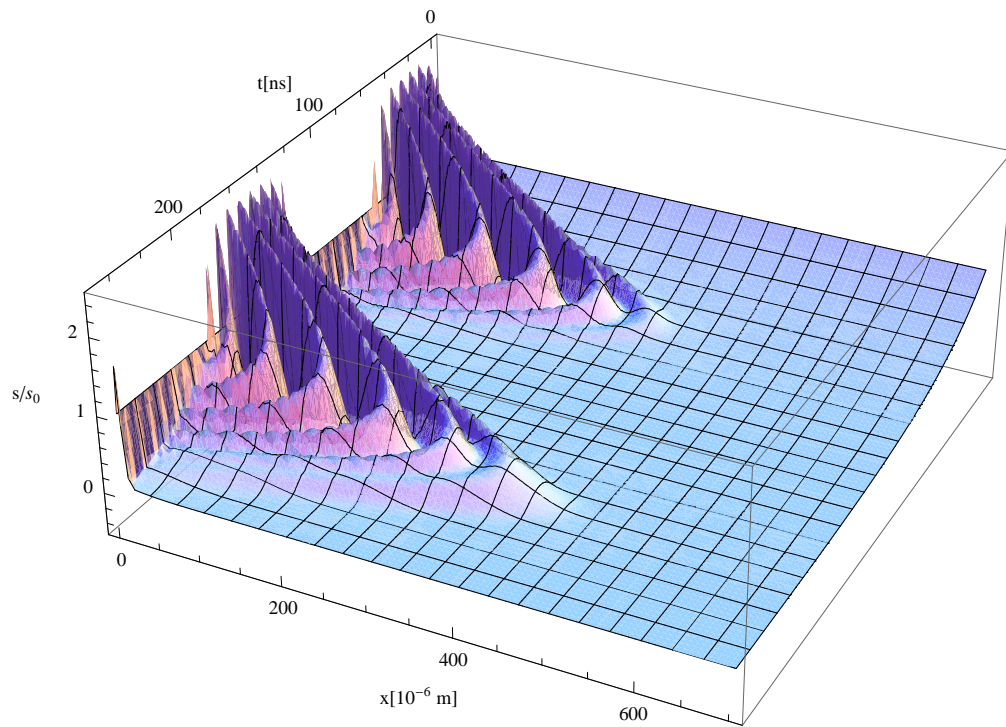


Figure 5: Numerical solution delivered by Mathematica of the spin drift-diffusion equation for the case of both an oscillating electric field and oscillating spin current with different frequencies.

### 2.1.2 electrical spin and charge currents

Naturally, electronics as well as spintronics rely on having a current flowing through any kind of material. Again looking at the drift-diffusion equation (Eq. 8, 9), one can see that factoring out  $\frac{\partial}{\partial x}$  leads to the continuity equations

$$\frac{\partial n}{\partial t} + \frac{\partial}{\partial x} \left[ -\frac{e\tau}{m} En - D \frac{\partial n}{\partial x} \right] = 0 \quad (15)$$

$$\frac{\partial s}{\partial t} + \frac{\partial}{\partial x} \left[ -\frac{e\tau}{m} Es - D \frac{\partial s}{\partial x} \right] = -\frac{s}{\tau_s} \quad (16)$$

$\mu = \frac{e\tau}{m}$  ... electron mobility

The expressions in the brackets are

$$\text{electron (particle) current ...} \quad J = -\mu En - D \frac{\partial n}{\partial x} \quad (17)$$

$$\text{spin (particle) current ...} \quad J_s = -\mu Es - D \frac{\partial s}{\partial x} \quad (18)$$

Furthermore, when multiplying with charge one obtains

$$\text{electrical (charge) current ...} \quad j = -eJ = -\sigma E - eD \frac{\partial n}{\partial x} \quad (19)$$

$$\text{spin (charge) current ...} \quad j_s = -eJ_s = -\sigma_s E - eD \frac{\partial s}{\partial x} \quad (20)$$

$\sigma = e\mu n$  ... conductivity

$\sigma_s = e\mu s$  ... spin conductivity

Now polarization can be expressed easily:

$$\text{density spin polarization ...} \quad P_n = \frac{n_{\uparrow} - n_{\downarrow}}{n} = \frac{s}{n} \quad (21)$$

$$\text{current spin polarization ...} \quad P_j = \frac{j_{\uparrow} - j_{\downarrow}}{j} = \frac{j_s}{j} \quad (22)$$

#### Influence of an electric field

If a static electric field  $E = -\nabla\phi$  is applied, (Eq. 19) can be written as

$$j = -\sigma \nabla\phi + eD \nabla n$$

In a system where the particle density changes across the sample, a term for  $\nabla n$  is needed.

Considering a Fermi gas in equilibrium, the particle density is given by

$$n(\mathbf{r}) = \int_0^{\infty} d\varepsilon \mathcal{N}(\varepsilon) f(\varepsilon - e\phi - \eta)$$

where<sup>7</sup>

<sup>7</sup>For a more detailed description, see [2] or the courses on statistical- and solid state physics

$\varepsilon$  ... Energy of the state

$\mathcal{N}(\varepsilon)$  ... density of states per unit volume

$f(\varepsilon - \eta - e\phi) = \frac{1}{e^{(\varepsilon - \eta - e\phi)/k_B T} + 1}$  ... Fermi-Dirac Distribution

The total electron energy is  $\varepsilon - e\phi$ . So the density of the gas can be rewritten as a function of the *chemical potential*  $\eta$  minus the energy  $e\phi$  of the electric field.

$$n(\mathbf{r}) = n_0(\eta + e\phi)$$

That means, in contrast to the case of having no field applied, only the energy of the electrons changes.

For this dependency, an expression for  $\nabla n$  can be obtained. Consequently for the electrical current one obtains

$$j = -\sigma \nabla \phi + eD \nabla n = -\sigma \nabla \phi + eD \frac{\partial n_0}{\partial \eta} e \nabla \phi$$

$$j = \nabla \phi \left( -\sigma + e^2 D \frac{\partial n_0}{\partial \eta} \right)$$

When speaking of equilibrium,  $j$  must be zero, leading to

EINSTEIN'S RELATION

$$\sigma = e^2 D \frac{\partial n_0}{\partial \eta} \quad (23)$$

recognizing that  $\frac{\partial n_0}{\partial \eta} = \mathcal{N}(\mathcal{E}_f)$ , where  $\mathcal{E}_f$  is the *fermi energy*.

The aim is to get an expression for  $j \neq 0$ . In order to go on with the same approach there must be some assumptions made:

- The electron distribution depends *only* on the energy of the electron state
- The chemical potential  $\eta$  still makes sense, meaning that relating the change of the internal energy to the change of particle density as a physical quantity is reasonable.
- Adding another energy term  $\eta_1 = e\mu(x)$ , where  $\mu$  is the *electrochemical potential*<sup>8</sup>, to the chemical potential describes the current flow. This quantity represents the chemical potential caused by the charge of the electron in a system of a non-zero electric field.

<sup>8</sup>Here, the term from [12] is used, instead of "quasichemical potential" which is used in [2]. It sounds more convenient to me because the quantity is related to the electric potential.

- Einstein's relation (Eq. 23) still holds <sup>9</sup>

Then (Eq. 19) helps in getting a useful expression now:

$$j = \nabla\phi \left( -\sigma + e^2 D \frac{\partial n_0}{\partial \eta} \right) + e^2 D \frac{\partial n_0}{\partial \eta} \nabla\mu$$

The terms within brackets vanish, so the last term makes the difference to the equilibrium case. Using Einstein's relation (Eq. 23) leads to

CHARGE CURRENT IN A DIFFUSIVE REGIME

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$$j = \sigma \nabla\mu \tag{24}$$

This is an important result of the drift-diffusion model for spin injection. The next steps aim at getting an equation for  $\mu$  since this quantity seems to be important for spin injection.

### (Degenerate) Ferromagnetic Conductors

Ferromagnets are considered as a source of nonequilibrium spin. In fact, because of the differences of various quantities for spin-up and spin-down (namely minority- and majority-) electrons in ferromagnets, (Eq. 24) must be written to distinguish between the two possibilities:

$$j_{\uparrow(\downarrow)} = \sigma_{\uparrow(\downarrow)} \nabla\mu_{\uparrow(\downarrow)} \tag{25}$$

The conductivities and average quasichemical potentials can be expressed as

$$\sigma = \sigma_{\uparrow} + \sigma_{\downarrow} \tag{26}$$

$$\sigma_s = \sigma_{\uparrow} - \sigma_{\downarrow} \tag{27}$$

$$\mu = \frac{\mu_{\uparrow} + \mu_{\downarrow}}{2} \tag{28}$$

$$\mu_s = \frac{\mu_{\uparrow} - \mu_{\downarrow}}{2} \tag{29}$$

$$\mathcal{N} = \mathcal{N}_{\uparrow} + \mathcal{N}_{\downarrow} \tag{30}$$

$$\mathcal{N}_s = \mathcal{N}_{\uparrow} - \mathcal{N}_{\downarrow} \tag{31}$$

---

<sup>9</sup>In eq. 19, one can see that in this model the current  $j$  depends only linearly on  $E$  and  $\nabla n$ . It is essential that the electron density  $n$  does *not* depend on the Field.



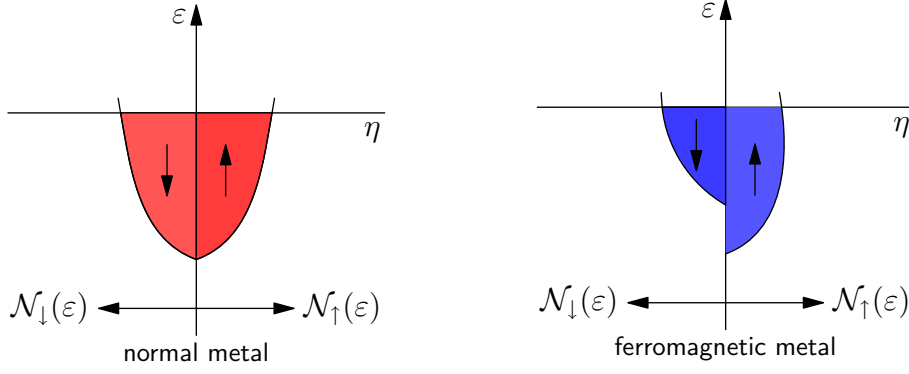


Figure 6: Schematic drawing of the density of states of electrons depending on spin (degenerate conductors). The differences between a normal metal and a ferromagnetic metal are shown.

Going on with (Eq. 25) and using it for expressing the particle- and spin charge currents (Eq. 20 leads to,19) again:

$$j = j_{\uparrow} + j_{\downarrow} = \sigma \nabla \mu + \sigma_s \nabla \mu_s \quad (32)$$

$$j_s = j_{\uparrow} - j_{\downarrow} = \sigma_s \nabla \mu + \sigma \nabla \mu_s \quad (33)$$

Extracting  $\nabla \mu$  from the first equation

$$\nabla \mu = \frac{1}{\sigma} (j - \sigma_s \nabla \mu_s) \quad (34)$$

and putting this in the second one gives

$$\begin{aligned} j_s &= \overbrace{\frac{\sigma_s}{\sigma}}^{P_\sigma} (j - \sigma_s \nabla \mu_s) + \sigma \nabla \mu_s \\ &= P_\sigma j - \left( \frac{\sigma_s^2}{\sigma} - \frac{\sigma^2}{\sigma} \right) \nabla \mu_s \\ &= P_\sigma j - \left( \frac{(\sigma_{\uparrow} - \sigma_{\downarrow})^2}{\sigma} - \frac{(\sigma_{\uparrow} + \sigma_{\downarrow})^2}{\sigma} \right) \nabla \mu_s \\ &= P_\sigma j + 4 \frac{\sigma_{\uparrow} \sigma_{\downarrow}}{\sigma} \nabla \mu_s \end{aligned} \quad (35)$$

with  $P_\sigma \dots$  conductivity spin polarization. Taking the gradient of the equation under the assumption that the charge current does not change in space (*steady state*):

$$\nabla j_s = 4 \frac{\sigma_{\uparrow} \sigma_{\downarrow}}{\sigma} \nabla^2 \mu_s \quad (36)$$

Another different expression for  $\nabla j_s$  containing information about  $\mu_s$  is needed. When speaking of degenerate conductors, only electrons at the Fermi level contribute to currents. Consequently, the deviations from the chemical potential can be considered small,

whereby the particle density  $n_{\uparrow(\downarrow)}$  can be expanded into

$$n_{\uparrow(\downarrow)} = n_{\uparrow(\downarrow)0}(\eta + e\mu_{\uparrow(\downarrow)} + e\phi) \approx n_{\uparrow(\downarrow)0} + \frac{\partial n_{\uparrow(\downarrow)0}}{\partial \eta}(e\mu_{\uparrow(\downarrow)} + e\phi)$$

which, when adding up the two possibilities, leads to

$$n = n_0 + e\mathcal{N}(\mu + \phi) + e\mathcal{N}_s\mu_s$$

In fact, in degenerate conductors *local charge neutrality* can be imposed, which means that  $n = n_0$ :

$$\mathcal{N}(\mu + \phi) + \mathcal{N}_s\mu_s = 0 \Rightarrow \mu + \phi = -\frac{\mathcal{N}_s\mu_s}{\mathcal{N}} \quad (37)$$

Doing the same for the spin density gives

$$\begin{aligned} s &= s_0 + e\mathcal{N}_s(\mu + \phi) + e\mathcal{N}\mu_s \\ &\stackrel{(\text{Eq. 37})}{=} s_0 + e\mu_s \left( -\frac{\mathcal{N}_s^2}{\mathcal{N}} + \mathcal{N} \right) \stackrel{(\text{Eq. 30,31})}{=} s_0 + e\mu_s \frac{-(\mathcal{N}_\uparrow - \mathcal{N}_\downarrow)^2 + (\mathcal{N}_\uparrow + \mathcal{N}_\downarrow)^2}{\mathcal{N}_\uparrow + \mathcal{N}_\downarrow} \\ &= s_0 + 4e\mu_s \frac{\mathcal{N}_\uparrow\mathcal{N}_\downarrow}{\mathcal{N}} \end{aligned} \quad (38)$$

This can be used now to get the second expression for  $\nabla j_s$ . According to (Eq. 20, 16), in the steady state

$$\nabla J_s = -\frac{s}{\tau_s}$$

For Ferromagnets, subtracting  $s_0$  from (Eq. 38) gives the nonequilibrium spin density

$$\delta s = s - s_0 \quad (39)$$

so that one finally obtains

$$\nabla j_s = e \frac{\delta s}{\tau_s} = 4e^2 \frac{\mu_s}{\tau_s} \frac{\mathcal{N}_\uparrow\mathcal{N}_\downarrow}{\mathcal{N}} \quad (40)$$

That is the desired second relation for  $\nabla j_s$ . Using it in combination with (Eq. 36, 23) leads to the important result

$$\begin{aligned} \nabla^2 \mu_s &= e^2 \frac{\mu_s}{\tau_s} \frac{\mathcal{N}_\uparrow\mathcal{N}_\downarrow}{\mathcal{N}} \frac{\sigma}{\sigma_\uparrow\sigma_\downarrow} \\ &= e^2 \frac{\mu_s}{\tau_s} \frac{\mathcal{N}_\uparrow\mathcal{N}_\downarrow}{\mathcal{N}} \frac{e^2 D_\uparrow \mathcal{N}_\uparrow + e^2 D_\downarrow \mathcal{N}_\downarrow}{e^2 D_\uparrow \mathcal{N}_\uparrow e^2 D_\downarrow \mathcal{N}_\downarrow} \\ &= \frac{\mu_s}{\tau_s} \frac{D_\uparrow \mathcal{N}_\uparrow + D_\downarrow \mathcal{N}_\downarrow}{\mathcal{N} D_\uparrow D_\downarrow} \\ &= \frac{\mu_s}{\tau_s \frac{\mathcal{N}}{\frac{1}{D_\downarrow} + \frac{1}{D_\uparrow}}} \end{aligned}$$

DIFFUSION EQUATION FOR THE SPIN ELECTROCHEMICAL  
POTENTIAL IN THE STEADY STATE

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$$\nabla^2 \mu_s = \frac{\mu_s}{L_s^2} \quad (41)$$

$$L_s = \sqrt{\bar{D}\tau_s} \quad \dots \quad \text{spin diffusion length}$$

$$\bar{D} = \frac{\mathcal{N}}{\mathcal{N}_\uparrow/D_\downarrow + \mathcal{N}_\downarrow/D_\uparrow} \quad \dots \quad \text{generalized diffusivity}$$

or *spin averaged diffusion coefficient* [12]

*Note:* From (Eq. 40) one can see that  $\mu_s$  is proportional to the nonequilibrium spin density  $\delta s$ :

$$\delta s = 4e\mu_s \frac{\mathcal{N}_\uparrow \mathcal{N}_\downarrow}{\mathcal{N}}$$

Therefore, the electrochemical potential is used for describing spin injection and is also called *spin accumulation*.

### 2.1.3 F/N junction

A widespread basic model for spin injection is the Ferromagnet/Nonferromagnet junction. The more compact approach of [12], which was originally introduced by Rashba ([9]) will be used for the following.

The next steps consist of continuing with the steady state, and looking at electrons moving in one direction through a contact between a ferromagnetic (index  $F$ ), and a paramagnetic metal (index  $N$ ) (or a degenerate semiconductor).

#### Contact Region

To use the diffusion equation (Eq. 41), the boundary conditions at the contact must be known.

In this model, the current spin polarization is set to be continuous

$$P_{j,F}(0^-) = P_{j,N}(0^+) \equiv P_j \quad (42)$$

This means that scatterings are neglected. Moreover, the electrochemical potential is discontinuous at  $x = 0$  (see Fig. 7), so a boundary condition can be found to get the value of the current at that point:

$$j_{\uparrow(\downarrow)} = \Sigma_{\uparrow(\downarrow)} [\mu_{\uparrow(\downarrow)N}(0) - \mu_{\uparrow(\downarrow)F}(0)] = \Sigma_{\uparrow(\downarrow)} \Delta\mu_{\uparrow(\downarrow)}(0)$$

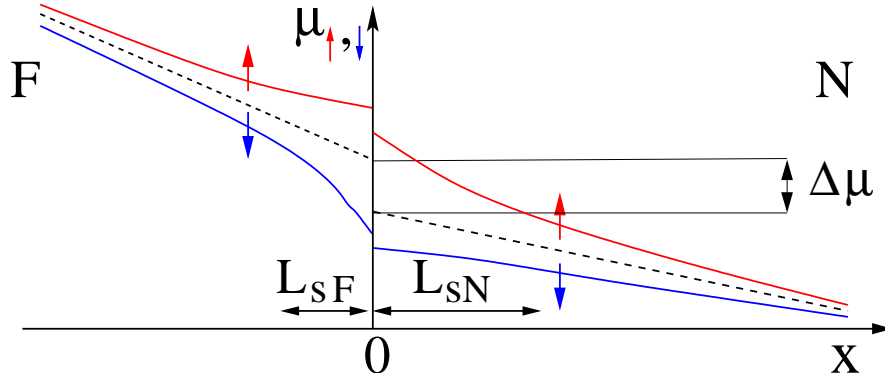


Figure 7: Principal drawing of the F/N junction, showing that the electrochemical potentials  $\mu$  are discontinuous. (From [12], Fig.7)

where  $\Sigma = \Sigma_{\uparrow} + \Sigma_{\downarrow}$  refers to to *conductivity of the contact*, which is of course spin-dependent. Again adding an subtracting, respectively, leads to

$$\begin{aligned} j &= \Sigma \Delta\mu(0) + \Sigma_s \Delta\mu_s(0) \\ j_s &= \Sigma_s \Delta\mu(0) + \Sigma \Delta\mu_s(0) \end{aligned}$$

Out of these two equations, the term for the current polarization at the contact can be obtained by eliminating  $\Delta\mu(0)$ :

$$P_{j,c} = \frac{j_s}{j} = \underbrace{\frac{\Sigma_{\uparrow} - \Sigma_{\downarrow}}{\Sigma}}_{P_{\Sigma}} + \frac{\Delta\mu_s(0)}{j} \underbrace{\frac{4\Sigma_{\downarrow}\Sigma_{\uparrow}}{\Sigma}}_{R_c} \quad (43)$$

Now, there is an expression for the drop of the spin electrochemical potential at  $x = 0$ :

$$\Delta\mu_s(0) = R_c j (P_j - P_{\Sigma}) \quad (44)$$

By dividing (Eq. 35) by  $j$ , the equation for the current polarization can be obtained:

$$P_j = P_{\sigma} + \frac{1}{j} \frac{4\sigma_{\downarrow}\sigma_{\uparrow}}{\sigma} \nabla\mu_s \quad (45)$$

The differences between the two regions are as follows:

Ferromagnetic Region

Nonmagnetic Region

For this case, the current polarization is

Here,  $\sigma_{\downarrow} = \sigma_{\uparrow}$ , and therefore  $P_{\sigma} = 0$ .

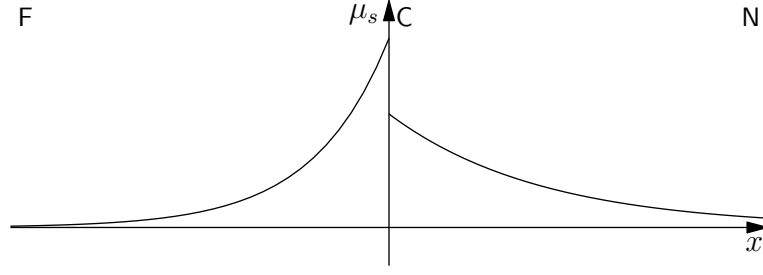


Figure 8: Behavior of the spin electrochemical potential at the contact region (plotted Eq. 48 and 49) .

Consequently,

$$P_{j,F}(0) = P_{\sigma,F} + \frac{1}{j} \frac{4\sigma_{F,\downarrow}\sigma_{F,\uparrow}}{\sigma_F} \nabla\mu_{s,F}(0) \quad (46) \quad P_{j,N}(0) = \frac{1}{j} \underbrace{\frac{4\sigma_{N,\downarrow}\sigma_{N,\uparrow}}{\sigma_N}}_{\sigma_N} \nabla\mu_{s,N}(0) \quad (47)$$

To get the value of  $\mu_{s,F}$ , the diffusion equation (Eq. 41) can be used. The differences between the two regions are as follows:

boundary conditions

$$\mu_{s,F}(-\infty) = 0$$

$$\mu_{s,N}(\infty) = 0$$

diffusion equations

$$\nabla^2 \mu_{s,F} = \frac{1}{L_{s,F}^2} \mu_{s,F}$$

$$\nabla^2 \mu_{s,N} = \frac{1}{L_{s,N}^2} \mu_{s,N}$$

solutions

$$\mu_{s,F}(x) = \mu_{s,F}(0) e^{x/L_{s,F}} \quad (48)$$

$$\mu_{s,N}(x) = \mu_{s,N}(0) e^{-x/L_{s,N}} \quad (49)$$

gradients

At  $x = 0$ , one gets simple terms for the gradients of the electrochemical potentials.

$$\nabla\mu_{s,F} = \frac{\mu_{s,F}}{L_{s,F}} \quad (50)$$

$$\nabla\mu_{s,N} = -\frac{\mu_{s,N}}{L_{s,N}} \quad (51)$$

Putting this back into (Eq. 46) and (Eq. 47) finally leads to

$$P_{j,F}(0) = P_{\sigma,F} + \frac{1}{j} \frac{\mu_{s,F}(0)}{R_F} \quad (52)$$

$$P_{j,N}(0) = -\frac{1}{j} \frac{\mu_{s,N}(0)}{R_N} \quad (53)$$

where the effective resistances are

$$R_F = \frac{\sigma_F}{4\sigma_{F,\downarrow}\sigma_{F,\uparrow}} L_{s,F} \quad (54) \quad R_N = \frac{1}{\sigma_N} L_{s,N} \quad (55)$$

Finally, as the expressions for the current polarizations for all the three cases are known, one can use the continuity condition (Eq. 42) to get the

SPIN INJECTION EFFICIENCY

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$$\langle P_\sigma \rangle_R = P_j = \frac{R_c P_\Sigma + R_F P_{\sigma,F}}{R_c + R_F + R_N} \quad (56)$$

$\langle P_\sigma \rangle_R$  means that  $P_\sigma$  is averaged over the three regions and weighted by the effective resistances. The term in the denominator is the *effective equilibrium resistance* of the whole junction.

### spin accumulation

Using the expression for the current polarization in the N region (Eq. 53) and again taking the continuity (Eq. 42) into account, one finds

$$\mu_{s,N}(0) = -jP_j R_N \quad (57)$$

The two cases are

$j < 0$ : spin injection electrons flow from F to N, spin accumulation is positive $\mu_{s,N}(0) > 0$	$j > 0$ : spin extraction electrons flow from N to F, spin accumulation is negative $\mu_{s,N}(0) < 0$
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#### 2.1.4 nonequilibrium resistance - spin bottleneck effect

The overall resistance of the contact is expected to be

$$R = \tilde{R}_F + \tilde{R}_c + \tilde{R}_N$$

where  $\tilde{R}_X$  stands for the effective resistances of the regions. When thinking of spin injection, spin accumulation is not zero except of  $\mu_{s,F}(-\infty) = 0$ , which leads to an additional resistance

$$R = \tilde{R}_F + \tilde{R}_c + \tilde{R}_N + \delta R \quad (58)$$

$\delta R$  ist the correction of the junction resistance for the non-equilibrium case of non-zero spin accumulation. This additional contribution can be calculated by the following:

Integrating (Eq. 34) and noting that  $P_{\sigma,N} = 0$ , the conditions for all 3 regions can be found:

$$\text{Ferromagnet:} \quad \mu_F(0) - \mu_F(-\infty) = j\tilde{R}_F - P_{\sigma,F}\mu_{s,F}(0) \quad (59)$$

$$\text{Contact:} \quad \mu_N(0) - \mu_F(0) = j\tilde{R}_c - P_{\Sigma}(\mu_{s,N}(0) - \mu_{s,F}(0)) \quad (60)$$

$$\text{Nonmagn. Region:} \quad \mu_N(\infty) - \mu_N(0) = j\tilde{R}_N \quad (61)$$

and for the entire resistance (Eq. 58)

$$\mu_N(\infty) - \mu_F(-\infty) = j(\tilde{R}_F + \tilde{R}_c + \tilde{R}_N + \delta R) \quad (62)$$

Putting the resistances of the 3 regions into the last equation leads to

$$\delta R = (P_{\Sigma} - P_{\sigma,F})\mu_{s,F}(0) - P_{\Sigma}\mu_{s,N}(0) = -P_{\sigma,F}\mu_{s,F}(0) + P_{\Sigma} \underbrace{(\mu_{s,F}(0) - \mu_{s,N}(0))}_{\Delta\mu_s(0) \dots \text{potential drop}}$$

The next step consists of replacing the remaining spin electrochemical potential by the expression obtained at  $x = 0$  for the current polarization (Eq. 52) and using the derived term for the potential drop at the contact (Eq. 44):

$$\delta R = -P_{\sigma,F}(P_j - P_{\sigma,F})R_F - P_{\Sigma}(P_j - P_{\Sigma})R_c$$

Finally, using (Eq. 56) for  $P_j$  leads to

NONEQUILIBRIUM RESISTANCE

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$$\delta R = \frac{R_N(P_{\Sigma}^2 R_c + P_{\sigma,F}^2 R_F) + R_F R_c (P_{\sigma,F} - P_{\Sigma})^2}{R_F + R_c + R_N} > 0 \quad (63)$$

### What causes this additional resistance?

When spin accumulation is not zero, there will be always nonequilibrium spin ranging from the ferromagnetic area to the contact region. Spin can be imagined to “pile up” at the contact, which leads to diffusion oriented opposite to the direction of the current flow. In other words, part of the polarized electrons flow back to the ferromagnet, leading to this additional resistance of the junction, which is also called the *spin bottleneck effect*.

### 2.1.5 the conductivity mismatch problem

There are two interesting cases regarding the proportions of the three different region's resistances:

$R_c \ll R_N, R_F$ : transparent contact

$R_c \gg R_N, R_F$ : tunnel contact

This can be considered the ordinary case of a contact between a ferromagnetic and a non-ferromagnetic material.

Here the contact dominates the whole junction.

spin injection efficiency

(Eq. 56) evaluates for the two cases to

$$P_j = \frac{R_F}{R_N + R_F} P_{\sigma,F}$$

$$P_j = P_\Sigma$$

nonequilibrium resistance  $\delta R$  (Eq. 63)

$$\delta R = \frac{R_N R_F}{R_N + R_F} P_{\sigma,F}^2$$

$$\delta R = \frac{R_N R_c P_\Sigma^2 + R_F R_c (P_{\sigma,F} - P_\Sigma)^2}{R_c}$$

But here it is essential to consider also the difference between  $R_N$  and  $R_F$ . If  $R_N \approx R_F$  the efficiency is high:

$$P_j \approx P_{\sigma,F}$$

This is the case for injection from ferromagnetic metals(semiconductors) to non-ferrom. metals(semiconductors).

The more interesting case is the one in which the non-magnetic material has a higher resistance, for instance a ferromagnet. metal - nonmagnet. semiconductor junction:

$R_c \ll R_N \gg R_F$ : transparent contact

$R_c \gg R_N \gg R_F$  tunnel contact



$$P_j \approx \frac{R_F}{R_N} P_{\sigma,F} \ll P_{\sigma,F}$$

$$\delta R \approx R_F P_{\sigma,F}^2 \approx R_F (\ll R_N)$$

Spin injection efficiency is highly reduced, this is called the *conductivity mismatch problem*, because this is the case if the conductivities of the two materials are greatly different.

$$P_j = P_\Sigma$$

$$\delta R \approx R_N P_\Sigma^2$$

Here, there is no such problem. Injection efficiency is determined by the contact conductance polarization, and the nonequilibrium resistance is about the same magnitude of the nonmagnetic region's resistance.

## 2.2 spin relaxation and dephasing

The following explanations are taken from [12], Sec IV A. and are intended for giving some basic idea of spin decay.

The process of equilibration of spin is one of the most important to understand when building devices. Nonequilibrium spin must exist long enough in order to obtain measurable effects.

Given an ensemble of particles with spin density  $s$ , exposed to a magnetic field  $\mathbf{B} = B_0 \hat{\mathbf{e}}_z + \mathbf{B}_1(t)$ . The  $z$ -component of  $s$  then has a non-zero equilibrium value  $s_{0,z}$ . In this case,  $s$  obeys the following equations:

BLOCH EQUATIONS

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$$\frac{\partial s_x}{\partial t} = \gamma(\mathbf{s} \times \mathbf{B})_x - \frac{s_x}{T_2}$$

$$\frac{\partial s_y}{\partial t} = \gamma(\mathbf{s} \times \mathbf{B})_y - \frac{s_y}{T_2}$$

$$\frac{\partial s_z}{\partial t} = \gamma(\mathbf{s} \times \mathbf{B})_z - \frac{s_z - s_{0,z}}{T_1}$$

with

$$\gamma = \frac{g\mu_B}{\hbar} \dots \text{electron gyromagnetic ratio}$$

There are two characteristic times regarding equilibration: the *spin relaxation time*<sup>10</sup>  $T_1$  and the *spin dephasing time*<sup>11</sup>  $T_2$ .

**relaxation time** time it takes for the spin along the static field to decay. If the  $z$ -component of the  $\mathbf{B}$ -field is non-zero, the electron ensemble must exchange its

<sup>10</sup>longitudinal time

<sup>11</sup>transversal time

energy with the surrounding particles. For instance, this can happen by phonon generation in the lattice, referred to as thermal equilibration.

**dephasing time** time for the decay of the transversal component. The spin precession about the longitudinal axis initially occurs in phase. Over the time, spatial and temporal fluctuations of the precession's frequency lead to decay of this time.

### 3 giant- and tunnel magneto resistive effects

There are a number of different effects in special junctions where spin plays an important role, whereas the GMR and TMR effects are the most common ones. In fact, there is not a universal well-grounded theory for any of these phenomena, rather than adjusted models for given particular junctions.

Therefore this section aims at giving an idea of what is going on in the fields of GMR and TMR, in which the explanations in principle follow the ones of [6], with additional support of [11] and [8].

#### 3.1 giant magneto resistance (GMR)

GMR is famous because of the already developed devices e.g. used in magnetic hard disks read-heads. Moreover, it is the phenomena which caused the kick-off of the new spintronic field of research.

The way a GMR-device basically behaves is that resistance is low for parallel-aligned magnetizations, and high for the anti-parallel case. Exchange interaction splits the two spin subbands, resulting in a different DOS for each of the possible spin orientations. Making one of the two magnetic layers fixed, so that no external field can change the orientation of magnetization, the resistance of the junction can be controlled by applying an external field either parallel to the fixed layer or anti-parallel.

Unfortunately, this very simplified description gives just an idea of what is happening or what is supposed to happen in GMR circuits. However, there are two common geometries, current in plane (CIP) and current perpendicular to plane (CPP) (see Fig. 9). The CPP structure (used in Fig. 10) is the one which is easier to analyze theoretically, but the CIP method was first used in devices because of easier fabrication [6].

For the following descriptions, the term *majority(minority) electron* refers to a particle with spin parallel(antiparallel) to the magnetization of the ferromagnet.

##### 3.1.1 CIP - current in plane GMR

Because of the splitted spin subbands, there is a difference between the conductivities of the two spin orientations ( $\sigma_{\uparrow} \neq \sigma_{\downarrow}$ , see Sec. 2.1.2). Usually, the majority electrons carry the current primarily and scatterings are weaker than for the others. If the two layers are anti-aligned, an electron with the majority spin magnetic moment in one layer would be a minority electron in the other one. Consequently, for this case, resistance

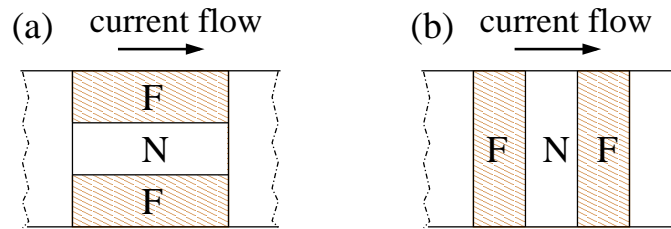


Figure 9: principal layout of current (a) in plane and (b) perpendicular to plane GMR devices ([12], Fig.3)

does not differ between spin-up and spin down particles.

But if the layers are aligned, going through the junction is “easier” for one of the orientations. In fact, overall resistance is greater than for the anti-aligned state.

For the CIP geometry it is essential that the size layers fulfill special conditions: The *electron mean free path* is the critical length scale, which is different for majority- and minority electrons. The nonmagnetic layer must be smaller than the electron mean free path, the ferromagnetic layer smaller than the majority electron’s mean free path.

### 3.1.2 CPP - current perpendicular to plane GMR

Figure 10 shows the situation in a FNF-junction for the CPP case. This geometry can be modeled greatly by the approach used for the F/N junction (see sec. 2.1.3). Spin accumulation for the injecting part has in principle the same structure as drawn in Fig. 7. For the detecting part, it makes a difference if the two FM-layers are aligned or anti-aligned: the polarization of the nonmagnetic region leads to more electrons in the spin-up(down) state as there would be in equilibrium. So if the ferromagnet at the detector is magnetized in the other direction, the probability to scatter is higher for electrons, as there are not enough states for them. As long as the length scale of the nonmagnetic region is about the same as the *spin relaxation length*  $L_s$ , one can expect a measureable difference in resistance.

## 3.2 tunnel magneto resistance (TMR)

In this geometry, between two magnetic materials, there is an insulating thin barrier, forming the *magnetic tunnel junction* (see Fig. 11). The principles of tunnel magneto resistance can be understood by again looking at the density of states of electrons. The spin subbands are splitted, namely the DOS at the Fermi level is different for the two electron species. Under the constraint that spin is conserved during the tunneling process, an electron cannot switch the subband. The difference in resistance between the aligned- and antialigned state comes from the different tunneling rate of the two cases. The commonly cited model is the one of Jullière ([4]), who derived an expression

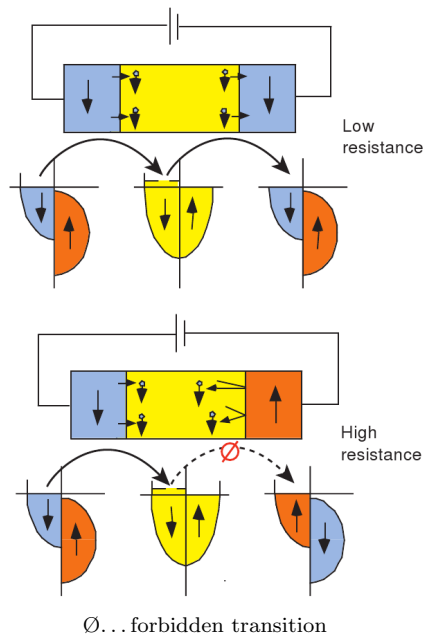


Figure 10: Basic concept of a spin valve: spin polarized transport in a FM-NFM-FM CPP junction. Note that the nonmagnetic region is quite large in contrast to MTJs (see TMR). ([8], Fig. 2)

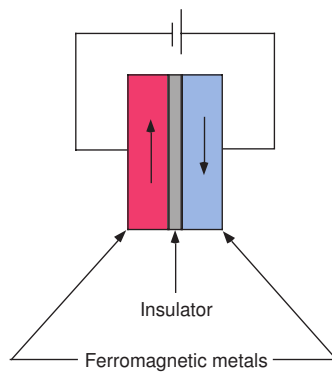


Figure 11: Geometry of a magnetic tunnel junction ([8], fig. 6)

for the

TMR RATIO

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$$TMR = \frac{R_{AP} - R_P}{R_P} = \frac{G_P - G_{AP}}{G_{AP}} \quad (64)$$

with

$R$  ... resistance

$G$  ... conductance

$P$  ... parallel measurement

$AP$  ... anti-parallel measurement

In fact, this model is not accurate concerning various effects regarding temperature dependence and the size of the barriers. That is the reason why the derivation will not be mentioned here.

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## References

- [1] C. JACOBONIA, G. O. ; QUARANTAA, A. A.: A review of some charge transport properties of silicon. In: *Solid-State Electronics* 20 (1977), Nr. 2, S. 77–89
- [2] FABIAN, J. ; MATOS-ABIAGUE, A. ; ERTLER, C. ; STANO, P. ; ZUTIC, I.: Semiconductor Spintronics. In: *ACTA PHYSICA SLOVACA* 57 (2007), S. 565. – URL <http://www.citebase.org/abstract?id=oai:arXiv.org:0711.1461>
- [3] FOUNDATION, The N.: *Physics 2007*. Oct. 2008. – URL [http://nobelprize.org/nobel\\_prizes/physics/laureates/2007/index.html](http://nobelprize.org/nobel_prizes/physics/laureates/2007/index.html)
- [4] JULLIERE, M.: Tunneling between ferromagnetic films. In: *Physics Letters A* 54 (1975), September, Nr. 3, S. 225–226. – URL [http://dx.doi.org/10.1016/0375-9601\(75\)90174-7](http://dx.doi.org/10.1016/0375-9601(75)90174-7). – ISSN 03759601
- [5] MISIAKOS, Konstantinos ; TSAMAKIS, Dimitris: Accurate measurements of the silicon intrinsic carrier density from 78 to 340 K. In: *Journal of Applied Physics* 74 (1993), Nr. 5, S. 3293–3297. – URL <http://link.aip.org/link/?JAP/74/3293/1>
- [6] NABER, W. J. M. ; FAEZ, S. ; WIEL, W. G. van d.: Organic Spintronics. In: *APPL.PHYS.* 40 (2007), S. –205. – URL <http://www.citebase.org/abstract?id=oai:arXiv.org:cond-mat/0703455>
- [7] PRAMANIK, S. ; BANDYOPADHYAY, S. ; CAHAY, M.: The inequality of charge and spin diffusion coefficients. In: *Journal of Applied Physics* 104 (2008), S. 014304. – URL <http://www.citebase.org/abstract?id=oai:arXiv.org:0807.2229>
- [8] PRINZ, Gary A.: Magneto-electronics. In: *Science Mag* 282 (1998), November, S. 1661. – URL <http://www.sciencemag.org/cgi/content/full/282/5394/1660>. – ISSN 0036-8075
- [9] RASHBA, E. I.: Theory of electrical spin injection: Tunnel contacts as a solution of the conductivity mismatch problem. In: *Phys. Rev. B* 62 (2000), Dec, Nr. 24, S. –16267
- [10] REGGIANI, Marion: *Hot-electron transport in semiconductors*. 58. Springer, 1985 (Topics in applied physics)
- [11] WOLF, S. A. ; AWSCHALOM, D. D. ; BUHRMAN, R. A. ; DAUGHTON, J. M. ; MOLNAR, S. v. ; ROUKES, M. L. ; CHTCHELKOVA, A. Y. ; TREGGER, D. M.: Spintronics: A Spin-Based Electronics Vision for the Future. In: *Science Mag* 294 (2001), November, S. 1488. – URL <http://www.sciencemag.org/cgi/content/full/294/5546/1488>. – ISSN 0036-8075

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- [12] ZUTIC, Igor ; FABIAN, Jaroslav ; SARMA, S. D.: Spintronics: Fundamentals and applications. In: *Reviews of Modern Physics* 76 (2004), S. 323. – URL <http://www.citebase.org/abstract?id=oai:arXiv.org:cond-mat/0405528>