L1

(1) The CPHASE(i,j) $\forall$ PAIR OF i,j MUTUALLY COMMUTE, AS THE CONFIGURATION BASIS

$$|m_1, m_2 \ldots m_N\rangle$$

IS A COMMON EIGENBASIS

(2) CPHASE(i,j) COMMUTES WITH ANY OPERATOR NOT ACTING (I.E. EQUIVALENT TO THE IDENTITY) ON i AND j.

MORE GENERALLY ~~THIS IS BECAUSE ONE CAN~~ OPERATORS ACTING ON DIFFERENT QUBITS COMMUTE

(3) DEFINE (for $i<j$)

$$S^{(i-j)} = \text{CPHASE}(i,i+1)\,\text{CPHASE}(i+1,i+2)\cdots\text{CPHASE}(j-1,j)$$

(THE ORDER IS NOT IMPORTANT DUE TO (1))

(4) START FROM THE STATE

$$|\varphi\rangle = S^{(1-5)}\,|\psi_1\rangle \otimes |\psi_2\rangle \otimes |\psi_3\rangle \otimes |\psi_4\rangle \otimes |\psi_5\rangle$$

WITH $|\psi_1\rangle = |\psi_{IN}\rangle$

$|\psi_j\rangle = |+\rangle$ FOR $j = 2, \ldots 5$

(a) ~~MEASURE $|\psi_1\rangle$ IN THE $\mathcal{B}(0)$ BASIS~~

(5) CONSIDER THE TWO ELEMENTS OF THE $\mathcal{B}(\alpha)$ BASIS

$$\left|{}^{0}_{1}, \alpha\right\rangle \equiv \frac{1}{\sqrt{2}}\left(|0\rangle \pm e^{i\alpha}|1\rangle\right) = e^{i\frac{\alpha}{2}} R_z(\alpha) H \left|{}^{0}_{1}\right\rangle$$

THE INVERSION

$$\left|{}^{0}_{1}\right\rangle = \frac{1}{\sqrt{2}}\left(|0,\alpha\rangle \pm |1,\alpha\rangle\right) \; {}^{1}_{e^{-i\alpha}}$$

(6) THE EFFECT OF A MEASURE IN THE BASIS $\mathcal{B}(\alpha)$ CORRESPONDS TO

(a) WRITE THE STATE IN TERMS OF EIGENSTATES OF THIS BASIS

(b) PROJECT ON ONE OF THE TWO EIGENSTATES.

THIS ACTION APPLIED ON QUBIT $|\psi_i\rangle$ COMMUTES WITH ALL CPHASE(j,ℓ) WITH $j,\ell \neq i$ (BECAUSE OF (2))

(7) FOR THE LINEAR CLUSTER WE CAN THUS CONSIDER THE EFFECT ON THE PRODUCT OF TWO NEIGHBORING ~~QUBITS~~ ENTANGLED IN THE FORM:

$C_{PHASE}^{(i,i+1)} |\psi_i\rangle \otimes |+\rangle_{i+1}$

TAKING $|\psi\rangle = a|0\rangle + b|1\rangle$

$= a|0\rangle|+\rangle + b|1\rangle|-\rangle =$ (USING (5))

$$= \frac{a}{\sqrt{2}}\left(|0,\alpha\rangle + |1,\alpha\rangle\right)|+\rangle + e^{-i\alpha}\frac{b}{\sqrt{2}}\left(|0,\alpha\rangle - |1,\alpha\rangle\right)|-\rangle$$

$$= |0,\alpha\rangle\left(a|+\rangle + e^{-i\alpha} b|-\rangle\right) + |1,\alpha\rangle\left(a|+\rangle - e^{-i\alpha} b|-\rangle\right)$$

~~GLOBALE KONSTANTEN WERDEN VERNACHLÄSSIGT, WEIL UNWICHTIG~~

$$= |0,\alpha\rangle e^{-i\frac{\alpha}{2}} H R_z(-\alpha)\left(a|0\rangle + b|1\rangle\right)$$

$$+ |1,\alpha\rangle e^{-i\frac{\alpha}{2}} H R_z(-\alpha)\left(a|0\rangle - b|1\rangle\right)$$

$$= |0,\alpha\rangle \otimes H R_z(-\alpha)|\psi\rangle + |1,\alpha\rangle \otimes H R_z(-\alpha)\sigma_z|\psi\rangle$$

THEREFORE, AFTER THE MEASURE THE ADJACENT STATE TO THE RIGHT IS TRANSFORMED TO

$$H R_z(-\alpha)\,\sigma_z^m|\psi\rangle = \sigma_x^m H R_z(-\alpha)|\psi\rangle$$

(SINCE $H\sigma_z = \sigma_x H$) WHERE $m = 0, 1$ IS THE RESULT OF THE MEASURE

(8a) AFTER THE MEASURE OF $|\psi_1\rangle$ IN $B(\alpha_1)$ WITH $\alpha_1 = 0$, $|\psi_0\rangle$ IS TRANSFORMED TO

$$|\psi_1\rangle = |m_1, \alpha_1\rangle \otimes S^{(2-5)} |\tilde{\psi}_2\rangle \otimes |\psi_3\rangle \otimes |\psi_4\rangle \otimes |\psi_5\rangle$$

m_1 BEING THE RESULT OF THE MEASUREMENT

~~$|\tilde{\psi}_1\rangle = S^{(2-5)} |\tilde{\psi}_2\rangle$~~

WITH $|\tilde{\psi}_2\rangle = \sigma_x^{m_1} H |\psi_{IN}\rangle$

WHERE WE HAVE USED THE RESULTS OF (7)

(b) FROM (7)

THE MEASURE OF $|\tilde{\psi}_2\rangle$ IN $B(\alpha)$ WITH $\alpha = \alpha_2 = -\xi(-1)^{m_1}$

YIELDS: (ASSUMING m_2 THE RESULT)

$$|\psi_2\rangle = |m_1, \alpha_1\rangle \otimes |m_2, \alpha_2\rangle \otimes S^{(3-5)} |\tilde{\psi}_3\rangle \otimes |\psi_4\rangle \otimes |\psi_5\rangle$$

WITH
$$|\tilde{\psi}_3\rangle = \sigma_x^{m_2} H R_z\left(-\xi(-1)^{m_1}\right) |\tilde{\psi}_2\rangle$$

$$= \sigma_x^{m_2} H \sigma_x^{m_1} R_z(-\xi) H |\psi_{IN}\rangle$$

$$= \sigma_x^{m_2} \sigma_z^{m_1} R_x(-\xi) |\psi_{IN}\rangle$$

USEFUL RELATIONS

$H = \frac{1}{\sqrt{2}}(\sigma_x + \sigma_z)$

$\sigma_x R_z(\alpha) \sigma_x = R_z(-\alpha)$

$H R_z(\alpha) H = R_x(\alpha)$

$H \sigma_z = \sigma_x H$

$H \sigma_x = \sigma_z H$

$H^2 = I$

$\sigma_x \sigma_z = -\sigma_z \sigma_x$

$\sigma_z R_z(\alpha) = R_z(\alpha) \sigma_z$

RESULS BE m_3

MEASURING $|\tilde{\psi}_3\rangle$ IN $B(\alpha_3)$ $\alpha_3 = -\eta(-1)^{m_2}$

YIELDS

$$|\psi_3\rangle = \ldots \otimes S^{(4\cdot 5)} |\tilde{\psi}_4\rangle \otimes |\psi_5\rangle$$

$$|\tilde{\psi}_4\rangle = \tilde{\sigma}_x^{m_3} H R_z(-\eta(-1)^{m_2}) |\tilde{\psi}_3\rangle$$

$$= \tilde{\sigma}_x^{m_3} \tilde{\sigma}_z^{m_2} \tilde{\sigma}_x^{m_1} H R_z(-\eta) R_x(-\xi) |\psi_{IN}\rangle$$

$$= \text{CONST} \cdot \tilde{\sigma}_x^{m_1+m_3} \tilde{\sigma}_z^{m_2} H R_z(-\eta) R_x(-\xi) |\psi_{IN}\rangle$$

CAN BE DROPPED

MEASURING $|\tilde{\psi}_4\rangle$ FINALLY YIELDS

$$|\psi_4\rangle = \ldots \quad |\tilde{\psi}_5\rangle \quad \left(\begin{array}{l} \alpha_4 = -\gamma(-1)^{m_1+m_3} \\ m_4 \text{ RESULT} \end{array} \right)$$

$$|\tilde{\psi}_5\rangle = H R_z(-\alpha_4) \tilde{\sigma}_z^{m_4} |\tilde{\psi}_4\rangle$$

$$= \tilde{\sigma}_x^{m_4+m_2} \tilde{\sigma}_z^{m_1+m_3} \underbrace{H^2}_{=1} R_x(-\gamma) R_z(-\eta) R_x(-\xi) |\psi_{IN}\rangle$$

MORE OR LESS THE REQUIRED RESULT