Decoherence effects in Qubits

Literatur:
A. Fisher notes (this should be enough):
www.cmmp.ucl.ac.uk/~ajf/course_notes.pdf -
(Mainly Sec. 4 there)

Book by Breuer and Petruccione (you can borrow it from me)

My quantum computer script (or references therein)
http://itp.tu-graz.ac.at/~arrigoni/vorlesungen/quantumcomputer/qucomp.pdf

Hamiltonian 1 Qubit:

$$H_0 = \varepsilon |1\rangle\langle 1|$$

Electromagnetic field: $V(t) = f(t)\, |1\rangle\langle 0| + f^*(t) |0\rangle\langle 1|$

$$H = H_0 + V(t)$$

Example: $f(t) = Q\, e^{-i\varepsilon t}$ $\Rightarrow$ resonance

Rabi oscillations (see script),
realisation of NOT and Hadamard gates

You can also use the spin basis
$$|0\rangle = |S_z = -\tfrac{1}{2}\rangle$$
$$|1\rangle = |S_z = +\tfrac{1}{2}\rangle$$

So that

$$H = \frac{\varepsilon}{2}\sigma_z + \frac{\varepsilon}{2} + f(t)\,\sigma_+ + f^*(t)\,\sigma_-$$

Decoherence and Loss : Open Quantum systems
description via density matrix ρ (2×2)

without dissipation $$\frac{d\rho}{dt} = -i\,[H, \rho]$$

(1) Evaluate explicitly time evolution of ρ with electric field

Hint: use the "interaction representation"
also called "rotating frame"

$$|\psi\rangle_R = e^{iH_0 t}\,|\psi\rangle$$

$$\rho_R = e^{iH_0 t}\,\rho\, e^{-iH_0 t}$$

write new equation for ρ_R

You can also work numerically if you want, we need it for the next part

(2) effects of dissipation (see fisher chap 4)

we consider here a very simple version with one Lindblad operator L

$$\frac{d\rho}{dt} = -i[H,\rho] + \gamma\left(L\rho L^{+} - \frac{1}{2}\{\rho, L^{+}L\}\right)$$

$$\gamma << \varepsilon$$

for dissipation: $L = \sigma^{-}$

You can work numerically (probably it still works analytically)

- Study time evolution of density matrix without E field $(V=0)$

See also fisher

- It can be convenient to parametrize ρ according to fisher, Eq. 17

- Start from a pure state $|\psi\rangle = a|0\rangle + b|1\rangle$

Plot several physical quantities

- Population of excited state $|1\rangle$

$$Tr\, \rho(t)\, |1\rangle\langle 1|$$

- Measure of coherence:

Do time evolution, then transform with Hadamard then measure the difference of population of $|1\rangle$ AND $|0\rangle$

(2A) Do the same with the phase channel $\Gamma = \sigma_z$

③

Now you probably have to work numerically

Add full hamiltonian incuding electric field

Start in $|0\rangle$

and try to produce Hadamard or Not gate in the presence of dissipation

Plot physical quantities versus time

Plot "error" i.e. "process tomography": deviation from desired result

$$\Delta\rho = \rho(t) - \rho_{Ex.}$$

Error $= \mathrm{Tr}\, \rho \rho^{+}$

All this is probably enough for your Bachelor.
If you want to do more one could do the same with two qubits and analyze, e.g. entanglement.