

Dichtematrix

S bezeichnet Spin der gemessen werden soll = Qubit (System)

T bezeichnet Spin, mit dem S wechselwirkt (Environment)

$$|\Psi_S(t)\rangle = a(t)|0\rangle + b(t)|1\rangle \quad ; \quad |\Psi_T(t)\rangle = c(t)|0\rangle + d(t)|1\rangle$$

Keine Interaktion zu Zeitpunkt $t=0$

$$|\Psi(t=0)\rangle = (a|0\rangle + b|1\rangle)(c|0\rangle + d|1\rangle)$$

$$\begin{aligned} \hat{\rho} &= |\Psi\rangle\langle\Psi| = (a|0\rangle + b|1\rangle)(c|0\rangle + d|1\rangle)(c^*\langle 0| + d^*\langle 1|)(a^*\langle 0| + b^*\langle 1|) \\ &= (ac|0\rangle\langle 0| + bc|1\rangle\langle 0| + ad|0\rangle\langle 1| + bd|1\rangle\langle 1|)(c^*\langle 0| + d^*\langle 1|)(a^*\langle 0| + b^*\langle 1|) \\ &= (ac|0\rangle\langle 0|\tilde{0}\rangle + bc|1\rangle\langle 0|\tilde{0}\rangle + ad|0\rangle\langle 1|\tilde{1}\rangle + bd|1\rangle\langle 1|\tilde{1}\rangle)(c^*\langle \tilde{0}| + d^*\langle \tilde{1}|)(a^*\langle 0| + b^*\langle 1|) \end{aligned}$$

$$\begin{aligned} \hat{\rho} &= |ac|^2|0\rangle\langle 0|\tilde{0}\rangle\langle \tilde{0}| + |bc|^2|1\rangle\langle 0|\tilde{0}\rangle\langle \tilde{0}| + |ad|^2|0\rangle\langle 1|\tilde{1}\rangle\langle \tilde{1}| + |bd|^2|1\rangle\langle 1|\tilde{1}\rangle\langle \tilde{1}| \\ &+ ab^*|c|^2|0\rangle\langle 0|\tilde{0}\rangle\langle \tilde{0}| + |a|^2cd^*|0\rangle\langle 1|\tilde{1}\rangle\langle \tilde{0}| + acd^*b^*|0\rangle\langle 0|\tilde{0}\rangle\langle \tilde{1}| \\ &+ ba^*|c|^2|1\rangle\langle 0|\tilde{0}\rangle\langle \tilde{0}| + bcd^*a^*|1\rangle\langle 0|\tilde{0}\rangle\langle \tilde{1}| + |b|^2cd^*|1\rangle\langle 0|\tilde{0}\rangle\langle \tilde{1}| \\ &+ |a|^2dc^*|0\rangle\langle 1|\tilde{1}\rangle\langle \tilde{0}| + adc^*b^*|0\rangle\langle 1|\tilde{1}\rangle\langle \tilde{0}| + ab^*|d|^2|0\rangle\langle 1|\tilde{1}\rangle\langle \tilde{1}| \\ &+ bdc^*a^*|1\rangle\langle 1|\tilde{1}\rangle\langle \tilde{0}| + dc^*|b|^2|1\rangle\langle 1|\tilde{1}\rangle\langle \tilde{0}| + b^*|d|^2|1\rangle\langle 1|\tilde{1}\rangle\langle \tilde{1}| \end{aligned}$$

$$\hat{\rho}_A = \text{Tr}_T(\rho) = \sum_e \langle \phi_e | \rho | \phi_e \rangle \quad \text{mit } \phi_e = |\tilde{0}\rangle, |\tilde{1}\rangle$$

$$\hat{\rho}_A^{(0)} = |ac|^2|0\rangle\langle 0| + |bc|^2|1\rangle\langle 1| + ab^*|c|^2|0\rangle\langle 1| + ba^*|c|^2|1\rangle\langle 0|$$

$$\hat{\rho}_A^{(1)} = |ad|^2|0\rangle\langle 0| + |bd|^2|1\rangle\langle 1| + ab^*|d|^2|0\rangle\langle 1| + ba^*|d|^2|1\rangle\langle 0|$$

$$\hat{\rho}_A = \hat{\rho}_A^{(0)} + \hat{\rho}_A^{(1)}$$

$$\rho_A = \begin{pmatrix} |ad|^2 & ab^*|d|^2 \\ ba^*|d|^2 & |bd|^2 \end{pmatrix} + \begin{pmatrix} |ac|^2 & ab^*|c|^2 \\ ba^*|c|^2 & |bc|^2 \end{pmatrix}$$

$$= \underline{\underline{\begin{pmatrix} |ad|^2 + |ac|^2 & ab^*(|d|^2 + |c|^2) \\ ba^*(|d|^2 + |c|^2) & |bd|^2 + |bc|^2 \end{pmatrix}}}$$