Simple NOT-Gate

Decoherence & Dissipation 0000

Open Quantum System

Decoherent NOT-Gate

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# Decoherence Effects in Qubits Projektpraktikum

#### Peter Wriesnik

#### Institute of Theoretical and Computation Physics Graz University of Technology

November 28, 2012

Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System	Decoherent NOT-Gate
Outline			

# **1** A Simple Model for NOT-Gate

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Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System 00000	Decoherent NOT-Gate
Outline			

# **1** A Simple Model for NOT-Gate

Introduction to Qubits



Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System 00000	Decoherent NOT-Gate
Outline			

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# **1** A Simple Model for NOT-Gate

- Introduction to Qubits
- Interaction with Electromagnetic Field (NOT-Gate)

Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System 00000	Decoherent NOT-Gate
Outline			

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- Introduction to Qubits
- Interaction with Electromagnetic Field (NOT-Gate)
- 2 Decoherence & Dissipation

Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System	Decoherent NOT-Gate
Outline			

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# 2 Decoherence & Dissipation

The Density Matrix

Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System	Decoherent NOT-Gate
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Outline			

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- Introduction to Qubits
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# 2 Decoherence & Dissipation

- The Density Matrix
- Effect of decoherence

Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System 00000	Decoherent NOT-Gate
Outline			

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  - The Density Matrix
  - Effect of decoherence
- 3 Treatment as an Open Quantum System

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Outline			

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Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System 00000	Decoherent NOT-Gate
Outline			

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- The Lindblad equation
- Qubit in presence of dissipation

Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System 00000	Decoherent NOT-Gate
Outline			

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Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System 00000	Decoherent NOT-Gate
Outline			

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Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System 00000	Decoherent NOT-Gate
Outline			

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Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System	Decoherent NOT-Gate
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Qubits in Qua	antum Computing		

Qubits: analogon to bits in classical computing

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Qubits in Qua	antum Computing		

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# Qubits: analogon to bits in classical computing $\rightarrow$ has to have 2 distinct states

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Qubits in Qua	antum Computing		

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Qubits: analogon to bits in classical computing  $\rightarrow$  has to have 2 distinct states

Any 2-level-system could be used. Examples:

Photon polarization

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Any 2-level-system could be used. Examples:

- Photon polarization
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- Spin of an electron

Logical operations (NOT, OR,  $\ldots)$  have to be performed on the qubit

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# Mathematical Description of Qubits

#### Qubits are elements of a 2-dimensional Hilbert-space $\mathcal{H}^2$ :

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$$\ket{\psi} = a(t) \ket{\uparrow} + b(t) \ket{\downarrow}$$

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## $\rightarrow$ coefficients a(t), b(t) hold dynamics

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- $\rightarrow$  coefficients a(t), b(t) hold dynamics
- $\rightarrow$  could be denoted as  $(a(t), b(t))^T$

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2-Level-Syste	m		

# Hamiltonian for Qubits

$$\hat{H}_0 = \omega_0 \ket{\uparrow} ig< \uparrow$$

#### assigns energy $\omega_0$ to spin-up-state and 0 to spin-down

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solution: 
$$\begin{array}{c} a(t) = e^{-i\omega_0 t} \\ b(t) = 0 \end{array} 
ight\}$$
 Larmor precession

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# 1 A Simple Model for NOT-Gate

Introduction to Qubits

Interaction with Electromagnetic Field (NOT-Gate)

- 2 Decoherence & Dissipation
  - The Density Matrix
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# 4 NOT-Gate in Presence of Dissipation

- Quantum Channels
- Entropy change

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# Description of EM-Field

Hamiltonian with electromagnetic field

$$egin{aligned} \hat{H} &= \hat{H}_0 + \hat{H}_I(t) \ \hat{H}_I(t) &= f(t) \left|\uparrow\right\rangle \left\langle\downarrow\right| + f^*(t) \left|\downarrow\right\rangle \left\langle\uparrow
ight| \ ext{with } f(t) = Qe^{-i\epsilon t} \end{aligned}$$

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# Description of EM-Field

Hamiltonian with electromagnetic field

$$\begin{split} \hat{H} &= \hat{H}_0 + \hat{H}_I(t) \\ \hat{H}_I(t) &= f(t) \left| \uparrow \right\rangle \left\langle \downarrow \right| + f^*(t) \left| \downarrow \right\rangle \left\langle \uparrow \right| \\ & \text{with } f(t) = Q e^{-i\epsilon t} \end{split}$$

Matrix representation in the  $\{|\uparrow\rangle, |\downarrow\rangle\}$ -basis:

$${\cal H}(t) = egin{pmatrix} \omega_0 & Q \cdot e^{-i\epsilon t} \ Q \cdot e^{i\epsilon t} & 0 \end{pmatrix}$$

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# Solution to time dependent Hamiltonian

Plug into Schrödinger equation for state (a(t), b(t))':

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# Solution to time dependent Hamiltonian

Plug into Schrödinger equation for state (a(t), b(t))':

$$i rac{\mathrm{d}}{\mathrm{d}t} a(t) = \omega_0 + f(t) \cdot b(t)$$
  
 $i rac{\mathrm{d}}{\mathrm{d}t} b(t) = f^*(t) \cdot a(t)$ 

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 Decoherence & Dissipation
 Open Quantum System
 Decoherent NOT-Gate

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# Solution to time dependent Hamiltonian

Plug into Schrödinger equation for state  $(a(t), b(t))^T$ :

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Solution considering EM-field with  $|\phi(t=0)\rangle = |\uparrow\rangle$ 

$$a(t) = e^{-\frac{i}{2}(\epsilon + \omega_0)t} \cdot \left(\cos(\beta t) + i\frac{\alpha}{\beta}\sin(\beta t)\right)$$
$$b(t) = -\frac{Q \cdot i}{\beta}e^{-i\frac{\Delta}{2}t}\sin(\beta t)$$

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# <u>Resonant case:</u> $\epsilon = \omega_0 \rightarrow \Delta = \alpha = 0$ and $\beta = Q$ .

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Resonant case: 
$$\epsilon = \omega_0 \rightarrow \Delta = \alpha = 0$$
 and  $\beta = Q$ .

$$a(t) = e^{-i\omega_0 t} \cos(Qt)$$
  
 $b(t) = -i \sin(Qt)$ 

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Resonant case: 
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Choose interaction time 
$$au=rac{\pi}{2Q}
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 0,  ${\it b}( au)=-i$
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Choose interaction time  $au=rac{\pi}{2Q}
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$$\begin{array}{cc} |a|: & 1 \to 0 \\ |b|: & 0 \to 1 \end{array} \end{array} \Longrightarrow |\uparrow\rangle \to |\downarrow\rangle \\ (\text{NOT-Operation}) \end{array}$$

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Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System	Decoherent NOT-Gate
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- 1 A Simple Model for NOT-Gate
  - Introduction to Qubits
  - Interaction with Electromagnetic Field (NOT-Gate)
- Decoherence & Dissipation
   The Density Matrix
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Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System 00000	Decoherent NOT-Gate
The Density	Matrix		

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# The Density Matrix

### Definition

$$\hat{
ho} := \sum_{i=1}^{n} p_i \ket{\phi_i} ra{\phi_i} \quad ext{ with } \sum_{i=1}^{n} p_i = 1$$

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$$\hat{
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Distinguish between mixed / pure state.

Simple NOT-Gate	Decoherence & Dissipation ●000	Open Quantum System 00000	Decoherent NOT-Gate
The Density	Matrix		

$$\hat{\rho} := \sum_{i=1}^{n} p_i \ket{\phi_i} ra{\phi_i} \quad \text{with } \sum_{i=1}^{n} p_i = 1$$

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$$\hat{
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Simple NOT-Gate	Decoherence & Dissipation ●000	Open Quantum System 00000	Decoherent NOT-Gate
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von Neumann-equation:  $rac{\mathrm{d}}{\mathrm{d}t}\hat{
ho}(t)=-i[\hat{H}(t),\hat{
ho}(t)]$ 

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von Neumann-equation:  $\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -i[\hat{H}(t),\hat{\rho}(t)]$ unitary time evolution:  $\hat{\rho}(t) = \hat{U}(t,t_0)\hat{\rho}(t_0)\hat{U}^{\dagger}(t,t_0)$ 

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 Simple NOT-Gate
 Decoherence & Dissipation
 Open Quantum System
 Decoherence

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# The Density Matrix for spin-1/2-systems

#### Bloch-sphere representation

$$\hat{
ho} = rac{1}{2} ig( \mathbbm{1} + ec{P} \cdot ec{\sigma} ig)$$

 $\vec{P} = (P_x, P_y, P_z)$ : expectation value of the spin  $\hat{\vec{S}} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$  $\longrightarrow$  Polarization

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$$\rho = \begin{pmatrix} \mathsf{a}\mathsf{a}^* & \mathsf{a}\mathsf{b}^* \\ \mathsf{a}^*\mathsf{b} & \mathsf{b}\mathsf{b}^* \end{pmatrix}$$

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Decoherent NOT-Gate

## The Density Matrix for spin-1/2-systems

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Note:  $\hat{
ho}$  describes a pure state if and only if  $|\vec{P}| = 1$ 

$$\rho = \begin{pmatrix} aa^* & ab^* \\ a^*b & bb^* \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 1 + P_z & P_x - iP_y \\ P_x + iP_y & 1 - P_z \end{pmatrix}$$

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	0000		

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- Introduction to Qubits
- Interaction with Electromagnetic Field (NOT-Gate)
- 2 Decoherence & Dissipation
   The Density Matrix
   Effect of decoherence

### 3 Treatment as an Open Quantum System

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### 4 NOT-Gate in Presence of Dissipation

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Simple NOT-Gate	Decoherence & Dissipation ○○●○	Open Quantum System	Decoherent NOT-Gate
Decoherence			

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Decoherence			

### A state is called decoherent, if "its interference is supressed" <sup>a</sup>.

<sup>a</sup>Michael A Nielsen and Isaac L Chuang. *Quantum Computation and Quantum Information*. 10th Anniversary Edition. Cambridge University Press, 2010. ISBN: 978-1-107-00217-3.

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$$\langle \hat{A} \rangle := \langle \psi | \hat{A} | \psi \rangle = (a^* \langle \uparrow | + b^* \langle \downarrow |) \hat{A} (a | \uparrow \rangle + b | \downarrow \rangle)$$
  
=  $aa^* A_{11} + bb^* A_{22} + \underbrace{b^* a A_{21} + ba^* A_{12}}_{\downarrow \downarrow \downarrow \downarrow}$ 

interference term

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For the density operator: off-diagonal elements vanish

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Decoherence			

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For the density operator: off-diagonal elements vanish For the Bloch-sphere:  $|\vec{P}|$  decreases

Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System 00000	Decoherent NOT-Gate
The problem	s so far		

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Simple NOT-Gate	Decoherence & Dissipation ○○○●	Open Quantum System 00000	Decoherent NOT-Gate
The problems	s so far		

Von Neumann-equation describes isolated system.



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The problems	s so far		

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Von Neumann-equation describes isolated system. Does not produce certain effects:



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Von Neumann-equation describes isolated system. Does not produce certain effects:

Dissipation of energy (Larmor precession)

Simple NOT-Gate	Decoherence & Dissipation ○○○●	Open Quantum System 00000	Decoherent NOT-Gate
The problems	s so far		

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- Change in entropy  $(|\vec{P}| = \text{const.})$

Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System	Decoherent NOT-Gate
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 ${\rm Tr}\big(\hat{\rho}^2(t)\big)$ 

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The problems	s so far		

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$$\mathsf{Tr}(\hat{\rho}^{2}(t)) = \mathsf{Tr}(\hat{U}(t)\hat{\rho}_{0}\underbrace{\hat{U}^{\dagger}(t)\hat{U}(t)}_{=1}\hat{\rho}_{0}\hat{U}^{\dagger}(t))$$

Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System	Decoherent NOT-Gate
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The problems	s so far		

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Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System	Decoherent NOT-Gate
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 $\rightarrow$  pure state remains pure for all times

Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System	Decoherent NOT-Gate
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- 1 A Simple Model for NOT-Gate
  - Introduction to Qubits
  - Interaction with Electromagnetic Field (NOT-Gate)
- 2 Decoherence & Dissipation
  - The Density Matrix
  - Effect of decoherence
- 3 Treatment as an Open Quantum System
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  - Qubit in presence of dissipation
- 4 NOT-Gate in Presence of Dissipation
  - Quantum Channels
  - Entropy change

Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System ●0000	Decoherent NOT-Gate
Open system			

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Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System ●0000	Decoherent NOT-Gate
Open system			





Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System ●0000	Decoherent NOT-Gate
Open system			



Assume:  $t = 0: \hat{\rho} = \hat{\rho} \otimes \hat{\rho}^{(E)}$ 

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Open system			



- Assume:  $t = 0: \hat{\rho} = \hat{\rho} \otimes \hat{\rho}^{(E)}$
- Reduced density matrix:  $\hat{\rho}(t) = \operatorname{Tr}_{ENV} \hat{\hat{\rho}}$

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Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System ●0000	Decoherent NOT-Gate
Open system			



Define the Dynamical map:

Assume:  $t = 0: \hat{\rho} = \hat{\rho} \otimes \hat{\rho}^{(E)}$ 

Reduced density matrix:  $\hat{\rho}(t) = \operatorname{Tr}_{ENV} \hat{\hat{\rho}}$ 

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$$V(t): \hat{
ho}(0) 
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Open system			



- Assume:  $t = 0: \hat{\rho} = \hat{\rho} \otimes \hat{\rho}^{(E)}$
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Define the Dynamical map:  $V(t): \hat{
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Lindblad has shown<sup>a</sup> that under certain assumptions, such time evolution can be written as a quantum mechanical master equation preserving the properties of  $\hat{\rho}$  (positiveness, convexity).

<sup>a</sup>Goran Lindblad. "On the generators of quantum dynamical semigroups". In: *Communications in Mathematical Physics* 48 (1976), pp. 119–130.
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#### The Lindblad equation

$$egin{aligned} &rac{\mathrm{d}}{\mathrm{d}t}\hat{
ho}(t) = -i[\hat{H}(t),\hat{
ho}(t)] + \sum_{\mu>0} \left(\hat{L}_{\mu}\hat{
ho}(t)\hat{L}_{\mu}^{\dagger} - rac{1}{2}\{\hat{L}_{\mu}^{\dagger}\hat{L}_{\mu},\hat{
ho}(t)\}
ight) \ &= -i[\hat{H}(t),\hat{
ho}(t)] + \mathcal{D}[\hat{
ho}(t)] =: \mathcal{L}[\hat{
ho}(t)] \end{aligned}$$

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#### The Lindblad equation

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ho}(t)] \end{aligned}$$

Form: Lindblad-operator = unitary evolution + dissipation

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#### The Lindblad equation

$$\begin{split} \frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) &= -i[\hat{H}(t),\hat{\rho}(t)] + \sum_{\mu>0} \left(\hat{L}_{\mu}\hat{\rho}(t)\hat{L}_{\mu}^{\dagger} - \frac{1}{2}\{\hat{L}_{\mu}^{\dagger}\hat{L}_{\mu},\hat{\rho}(t)\}\right) \\ &= -i[\hat{H}(t),\hat{\rho}(t)] + \mathcal{D}[\hat{\rho}(t)] =: \mathcal{L}[\hat{\rho}(t)] \end{split}$$

Form: Lindblad-operator = unitary evolution + dissipation  $\hat{L}_{\mu}$  ... Lindblad operators

Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System 00●00	Decoherent NOT-Gate
Lindblad oper	rators		

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Lindblad oper	rators		

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Lindblad oper	rators		

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$$\hat{L}_{\mu} \sim \hat{\sigma}^{-}$$
 produce *amplitude* damping

Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System	Decoherent NOT-Gate
Lindblad oper	rators		

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$$\hat{L}_{\mu} \sim \hat{\sigma}_{z}$$

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Lindblad or	perators		

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#### What is the meaning of the Lindblad operators $\hat{L}_{\mu}$ ?

 $\hat{L}_{\mu} \sim \hat{\sigma}^{-}$  produce *amplitude* damping  $\hat{L}_{\mu} \sim \hat{\sigma}_{z}$  produce *phase* damping

Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System 00●00	Decoherent NOT-Gate
Lindblad or	erators		

# What is the meaning of the Lindblad operators $\hat{L}_{\mu}$ ?

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Note: one can consider more than one Lindblad operator

Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System	Decoherent NOT-Gate
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- 1 A Simple Model for NOT-Gate
  - Introduction to Qubits
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- 2 Decoherence & Dissipation
  - The Density Matrix
  - Effect of decoherence
- 3 Treatment as an Open Quantum System
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  - Qubit in presence of dissipation
- 4 NOT-Gate in Presence of Dissipation
  - Quantum Channels
  - Entropy change

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# Amplitude damped Qubit

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# Amplitude damped Qubit

Choose 
$$\hat{L}_{\mu} = \sqrt{\gamma} \hat{\sigma}^{-}$$



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# Amplitude damped Qubit

Choose 
$$\hat{L}_{\mu} = \sqrt{\gamma} \hat{\sigma}^{-1}$$

$$\dot{P}_{z} = -\gamma(1 + P_{z})$$
$$\dot{P}_{x} = -\omega_{0}P_{y} - \frac{\gamma}{2}P_{x}$$
$$\dot{P}_{y} = \omega_{0}P_{x} - \frac{\gamma}{2}P_{y}$$

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# Amplitude damped Qubit

Choose 
$$\hat{L}_{\mu} = \sqrt{\gamma} \hat{\sigma}^{-1}$$

$$\dot{P}_{z} = -\gamma(1 + P_{z})$$
$$\dot{P}_{x} = -\omega_{0}P_{y} - \frac{\gamma}{2}P_{x}$$
$$\dot{P}_{y} = \omega_{0}P_{x} - \frac{\gamma}{2}P_{y}$$

$$P_{x}(t) = (-y_{0}\sin(\omega_{0}t) + x_{0}\cos(\omega_{0}t))e^{-\frac{\gamma}{2}t}$$
$$P_{y}(t) = (y_{0}\cos(\omega_{0}t) + x_{0}\sin(\omega_{0}t))e^{-\frac{\gamma}{2}t}$$
$$P_{z}(t) = -1 + (z_{0}+1)e^{-\gamma t}$$

 $\longrightarrow$  amplitude damping occurs  $(P_z(t \rightarrow \infty) = -1)$ 

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# Amplitude damped Qubit



Figure : Plot of the time evolution obtained before.  $\gamma = 0.05$ ,  $\omega_0 = 1$ . At t = 0, the system was prepared with a polarization  $1/\sqrt{3} \cdot (1, 1, 1)^T$ 

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- **1** A Simple Model for NOT-Gate
  - Introduction to Qubits
  - Interaction with Electromagnetic Field (NOT-Gate)
- 2 Decoherence & Dissipation
  - The Density Matrix
  - Effect of decoherence
- **3** Treatment as an Open Quantum System
  - The Lindblad equation
  - Qubit in presence of dissipation

#### 4 NOT-Gate in Presence of Dissipation

- Quantum Channels
- Entropy change

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Open Quantum System

Decoherent NOT-Gate

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# NOT-Gate with Amplitude Damping Channel

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 Open Quantum System
 Decoherent NOT-Gate

 NOT-Gate with Amplitude Damping Channel

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Again, use 
$$\hat{L}_{\mu}=\sqrt{\gamma}\hat{\sigma}^{-}.$$

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Again, use  $\hat{L}_{\mu} = \sqrt{\gamma} \hat{\sigma}^{-}$ . Full Hamiltonian  $\hat{H} = \hat{H}_{0} + \hat{H}_{I}(t)$ , plug into Lindblad equation

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Again, use  $\hat{L}_{\mu} = \sqrt{\gamma} \hat{\sigma}^{-}$ . Full Hamiltonian  $\hat{H} = \hat{H}_0 + \hat{H}_I(t)$ , plug into Lindblad equation

$$\frac{\mathrm{d}}{\mathrm{d}t}P_{x} = 2QP_{z}\sin(\epsilon t) - \omega_{0}P_{y} - \frac{\gamma}{2}P_{x}$$
$$\frac{\mathrm{d}}{\mathrm{d}t}P_{y} = -2QP_{z}\cos(\epsilon t) + \omega_{0}P_{x} - \frac{\gamma}{2}P_{y}$$
$$\frac{\mathrm{d}}{\mathrm{d}t}P_{z} = 2Q(P_{y}\cos(\epsilon t) + P_{x}\cos(\epsilon t)) - \gamma(1 + P_{z})$$

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 $\rightarrow$  numerical solution

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 Open Quantum System
 Decoherent NOT-Gate

 NOT-Gate with Amplitude Damping Channel



Figure : Time evolution of a NOT-Gate with  $\hat{L}_{\mu} = \sqrt{\gamma}\hat{\sigma}^-$  (amplitude damping).  $\omega_0 = \epsilon = 1$  (resonance), Q = 1,  $\gamma = 0.05$ .

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### NOT-Gate with Phase Damping Channel

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# NOT-Gate with Phase Damping Channel

Use different Lindblad operator:  $\hat{L}_{\mu} = \sqrt{\lambda} \hat{\sigma}_z$ 

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# NOT-Gate with Phase Damping Channel

Use different Lindblad operator:  $\hat{L}_{\mu} = \sqrt{\lambda} \hat{\sigma}_z$ 

$$\frac{\mathrm{d}}{\mathrm{d}t}P_{x} = 2QP_{z}\sin(\epsilon t) - \omega_{0}y - 2\lambda x$$
$$\frac{\mathrm{d}}{\mathrm{d}t}P_{y} = -2QP_{z}\cos(\epsilon t) + \omega_{0}x - 2\lambda y$$
$$\frac{\mathrm{d}}{\mathrm{d}t}P_{z} = 2Q(P_{y}\cos(\epsilon t) + x\cos(\epsilon t))$$

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# NOT-Gate with Phase Damping Channel

Use different Lindblad operator:  $\hat{L}_{\mu} = \sqrt{\lambda} \hat{\sigma}_z$ 

$$\frac{\mathrm{d}}{\mathrm{d}t}P_{x} = 2QP_{z}\sin(\epsilon t) - \omega_{0}y - 2\lambda x$$
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$$\frac{\mathrm{d}}{\mathrm{d}t}P_{z} = 2Q(P_{y}\cos(\epsilon t) + x\cos(\epsilon t))$$

Difference to amplitude damping: no (direct) damping of  $P_z$  occurs

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Open Quantum System

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# NOT-Gate with Phase Damping Channel

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$$\frac{\mathrm{d}}{\mathrm{d}t}P_{z} = 2Q(P_{y}\cos(\epsilon t) + x\cos(\epsilon t))$$

Difference to amplitude damping: no (direct) damping of  $P_z$ occurs  $\longrightarrow$  Phase Damping Channel

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#### NOT-Gate with Phase Damping Channel



Figure : Time evolution of a qubit considering a noisy phase damping channel.  $\omega_0 = \epsilon = 1$  (resonance), Q = 1,  $\lambda = 0.05$ .

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Simple NOT-Gate	Decoherence & Dissipation	Open Quantum System	Decoherent NOT-Gate
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- 1 A Simple Model for NOT-Gate
  - Introduction to Qubits
  - Interaction with Electromagnetic Field (NOT-Gate)
- 2 Decoherence & Dissipation
  - The Density Matrix
  - Effect of decoherence
- **3** Treatment as an Open Quantum System
  - The Lindblad equation
  - Qubit in presence of dissipation

#### 4 NOT-Gate in Presence of Dissipation

- Quantum Channels
- Entropy change

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Decoherent NOT-Gate

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# Entropy in Quantum Systems

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# Entropy in Quantum Systems

# From probability theory: $S = -\sum_i p_i \cdot \ln(p_i)$

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# Entropy in Quantum Systems

#### From probability theory: $S = -\sum_{i} p_i \cdot \ln(p_i)$ $\longrightarrow$ measure information

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Decoherent NOT-Gate

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# Entropy in Quantum Systems

#### From probability theory: $S = -\sum_{i} p_i \cdot \ln(p_i)$ $\longrightarrow$ measure information

#### von Neumann-entropy

$$S = -\mathsf{Tr}(\hat{
ho} \cdot \mathsf{In}(\hat{
ho}))$$

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Decoherent NOT-Gate

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# Entropy in Quantum Systems

#### From probability theory: $S = -\sum_{i} p_i \cdot \ln(p_i)$ $\longrightarrow$ measure information

#### von Neumann-entropy

$$S = -\mathsf{Tr}(\hat{
ho} \cdot \mathsf{In}(\hat{
ho}))$$

Note: S = 0 if and only if  $\hat{\rho}$  describes a pure state

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Decoherent NOT-Gate

# Entropy in Quantum Systems

#### From probability theory: $S = -\sum_{i} p_i \cdot \ln(p_i)$ $\longrightarrow$ measure information

#### von Neumann-entropy

$$S = -\mathsf{Tr}(\hat{
ho} \cdot \mathsf{In}(\hat{
ho}))$$

Note: S = 0 if and only if  $\hat{\rho}$  describes a pure state maximally entangled state:  $\hat{\rho} = \frac{1}{2}\mathbb{1} \longrightarrow S = \ln(2)$
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## Long time evolution, phase damped



Figure : NOT-Gate under consideration of a phase damping  $(\hat{L}_{\mu} = \sqrt{\lambda}\hat{\sigma}_z)$ . Again,  $\omega_0 = \epsilon = 1$  (resonance), Q = 1,  $\lambda = 0.05$ 

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## Long time evolution, amplitude damped



Figure : NOT-Gate under consideration of an amplitude damping. Again,  $\omega_0 = \epsilon = 1$  (resonance), Q = 1,  $\lambda = 0.05$ 

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## Entropy change



Figure : Long time evolution of a NOT-operation under consideration of an amplitude damping. Again,  $\omega_0 = \epsilon = 1$  (resonance), Q = 1,  $\lambda = 0.05$ .

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## Entropy of amplitude damped qubit



Figure : Change in the von Neumann-entropy of a simple qubit (no EM interaction) considering amplitude damping as discussed before. Parameters are Q = 1,  $\gamma = 0.05$ .

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Thank you for your attention.

