Decoherence Effects in Qubits

Projektpraktikum

Peter Wriesnik

Institute of Theoretical and Computation Physics
Graz University of Technology

November 28, 2012

Outline

Outline

Outline

1. A Simple Model for NOT-Gate
   - Introduction to Qubits
   - Interaction with Electromagnetic Field (NOT-Gate)

Outline

Outline

Outline

1. A Simple Model for NOT-Gate
   - Introduction to Qubits
   - Interaction with Electromagnetic Field (NOT-Gate)
2. Decoherence & Dissipation
   - The Density Matrix
   - Effect of decoherence

Outline

1. A Simple Model for NOT-Gate
   - Introduction to Qubits
   - Interaction with Electromagnetic Field (NOT-Gate)
2. Decoherence & Dissipation
   - The Density Matrix
   - Effect of decoherence
3. Treatment as an Open Quantum System

Outline

1. A Simple Model for NOT-Gate
   - Introduction to Qubits
   - Interaction with Electromagnetic Field (NOT-Gate)
2. Decoherence & Dissipation
   - The Density Matrix
   - Effect of decoherence
3. Treatment as an Open Quantum System
   - The Lindblad equation

Outline

Outline

Outline

Outline

Qubits in Quantum Computing

Qubits: analogon to bits in classical computing

Qubits in Quantum Computing

Qubits: analogon to bits in classical computing
$\rightarrow$ has to have 2 distinct states

Qubits in Quantum Computing

Qubits: analogon to bits in classical computing
$\rightarrow$ has to have 2 distinct states

Any 2-level-system could be used. Examples:

- Photon polarization

Qubits in Quantum Computing

Qubits: analogon to bits in classical computing
$\rightarrow$ has to have 2 distinct states

Any 2-level-system could be used. Examples:

- Photon polarization
- Energy levels in molecules / atoms

Qubits in Quantum Computing

Qubits: analogon to bits in classical computing
$\rightarrow$ has to have 2 distinct states

Any 2-level-system could be used. Examples:

- Photon polarization
- Energy levels in molecules / atoms
- Spin of an electron

Qubits in Quantum Computing

Qubits: analogon to bits in classical computing
$\rightarrow$ has to have 2 distinct states

Any 2-level-system could be used. Examples:

- Photon polarization
- Energy levels in molecules / atoms
- Spin of an electron

Logical operations (NOT, OR, ...) have to be performed on the qubit

Mathematical Description of Qubits

Qubits are elements of a 2-dimensional Hilbert-space $\mathcal{H}^2$:

Mathematical Description of Qubits

Qubits are elements of a 2-dimensional Hilbert-space $\mathcal{H}^2$:

$$|\psi\rangle = a(t)\,|\uparrow\rangle + b(t)\,|\downarrow\rangle$$

Mathematical Description of Qubits

Qubits are elements of a 2-dimensional Hilbert-space $\mathcal{H}^2$:

$$|\psi\rangle = a(t)\,|\uparrow\rangle + b(t)\,|\downarrow\rangle$$

$\rightarrow$ coefficients $a(t)$, $b(t)$ hold dynamics

Mathematical Description of Qubits

Qubits are elements of a 2-dimensional Hilbert-space $\mathcal{H}^2$:

$$|\psi\rangle = a(t)\,|\uparrow\rangle + b(t)\,|\downarrow\rangle$$

$\rightarrow$ coefficients $a(t)$, $b(t)$ hold dynamics

$\rightarrow$ could be denoted as $\big(a(t), b(t)\big)^T$

2-Level-System

Hamiltonian for Qubits

$$\hat{H}_0 = \omega_0 \left|\uparrow\right\rangle \left\langle\uparrow\right|$$

assigns energy ω_0 to spin-up-state and 0 to spin-down

2-Level-System

Hamiltonian for Qubits

$$\hat{H}_0 = \omega_0 \left|\uparrow\right\rangle \left\langle\uparrow\right|$$

assigns energy ω_0 to spin-up-state and 0 to spin-down

solution: $\left.\begin{aligned} a(t) &= e^{-i\omega_0 t} \\ b(t) &= 0 \end{aligned}\right\}$ Larmor precession

1. A Simple Model for NOT-Gate
   - Introduction to Qubits
   - Interaction with Electromagnetic Field (NOT-Gate)
2. Decoherence & Dissipation
   - The Density Matrix
   - Effect of decoherence
3. Treatment as an Open Quantum System
   - The Lindblad equation
   - Qubit in presence of dissipation
4. NOT-Gate in Presence of Dissipation
   - Quantum Channels
   - Entropy change

Description of EM-Field

Hamiltonian with electromagnetic field

$$\begin{aligned}\hat{H} &= \hat{H}_0 + \hat{H}_I(t) \\ \hat{H}_I(t) &= f(t)\left|\uparrow\right\rangle\left\langle\downarrow\right| + f^*(t)\left|\downarrow\right\rangle\left\langle\uparrow\right|\end{aligned}$$

with $f(t) = Qe^{-i\epsilon t}$

Description of EM-Field

Hamiltonian with electromagnetic field

$$\begin{aligned} \hat{H} &= \hat{H}_0 + \hat{H}_I(t) \\ \hat{H}_I(t) &= f(t)\left|\uparrow\right\rangle\left\langle\downarrow\right| + f^*(t)\left|\downarrow\right\rangle\left\langle\uparrow\right| \end{aligned}$$

with $f(t) = Qe^{-i\epsilon t}$

Matrix representation in the $\{\left|\uparrow\right\rangle, \left|\downarrow\right\rangle\}$-basis:

$$H(t) = \begin{pmatrix} \omega_0 & Q \cdot e^{-i\epsilon t} \\ Q \cdot e^{i\epsilon t} & 0 \end{pmatrix}$$

Solution to time dependent Hamiltonian

Plug into Schrödinger equation for state $\left(a(t), b(t)\right)^T$:

Solution to time dependent Hamiltonian

Plug into Schrödinger equation for state $\left(a(t), b(t)\right)^T$:

$$i\frac{\mathrm{d}}{\mathrm{d}t}a(t) = \omega_0 + f(t)\cdot b(t)$$
$$i\frac{\mathrm{d}}{\mathrm{d}t}b(t) = f^*(t)\cdot a(t)$$

Solution to time dependent Hamiltonian

Plug into Schrödinger equation for state $\big(a(t), b(t)\big)^T$:

$$i\frac{\mathrm{d}}{\mathrm{d}t}a(t) = \omega_0 + f(t)\cdot b(t)$$
$$i\frac{\mathrm{d}}{\mathrm{d}t}b(t) = f^*(t)\cdot a(t)$$

Solution considering EM-field with $|\phi(t=0)\rangle = |\uparrow\rangle$

$$a(t) = e^{-\frac{i}{2}(\epsilon+\omega_0)t}\cdot\Big(\cos(\beta t) + i\frac{\alpha}{\beta}\sin(\beta t)\Big)$$
$$b(t) = -\frac{Q\cdot i}{\beta}e^{-i\frac{\Delta}{2}t}\sin(\beta t)$$

NOT-Gate

NOT-Gate

Resonant case: $\epsilon = \omega_0 \rightarrow \Delta = \alpha = 0$ and $\beta = Q$.

NOT-Gate

Resonant case: $\epsilon = \omega_0 \rightarrow \Delta = \alpha = 0$ and $\beta = Q$.

$$a(t) = e^{-i\omega_0 t}\cos(Qt)$$
$$b(t) = -i\sin(Qt)$$

NOT-Gate

Resonant case: $\epsilon = \omega_0 \rightarrow \Delta = \alpha = 0$ and $\beta = Q$.

$$a(t) = e^{-i\omega_0 t}\cos(Qt)$$
$$b(t) = -i\sin(Qt)$$

Choose interaction time $\tau = \frac{\pi}{2Q} \rightarrow a(\tau) = 0,\ b(\tau) = -i$

NOT-Gate

Resonant case: $\epsilon = \omega_0 \to \Delta = \alpha = 0$ and $\beta = Q$.

$$a(t) = e^{-i\omega_0 t}\cos(Qt)$$
$$b(t) = -i\sin(Qt)$$

Choose interaction time $\tau = \frac{\pi}{2Q} \to a(\tau) = 0, \; b(\tau) = -i$

$$\left.\begin{array}{ll} |a| : & 1 \to 0 \\ |b| : & 0 \to 1 \end{array}\right\} \Longrightarrow |\uparrow\rangle \to |\downarrow\rangle$$

(NOT-Operation)

The Density Matrix

The Density Matrix

Definition

$$\hat{\rho} := \sum_{i=1}^{n} p_i \left|\phi_i\right\rangle \left\langle\phi_i\right| \quad \text{with} \sum_{i=1}^{n} p_i = 1$$

The Density Matrix

Definition

$$\hat{\rho} := \sum_{i=1}^{n} p_i \left|\phi_i\right\rangle \left\langle\phi_i\right| \quad \text{with} \sum_{i=1}^{n} p_i = 1$$

Distinguish between mixed / pure state.

The Density Matrix

Definition

$$\hat{\rho} := \sum_{i=1}^{n} p_i \left|\phi_i\right\rangle \left\langle\phi_i\right| \quad \text{with} \sum_{i=1}^{n} p_i = 1$$

Distinguish between mixed / pure state.

$$\hat{\rho} \text{ describes pure state } \iff \hat{\rho}^2 = \hat{\rho} \iff \mathrm{Tr}(\hat{\rho}^2) = 1$$

The Density Matrix

Definition

$$\hat{\rho} := \sum_{i=1}^{n} p_i \left|\phi_i\right\rangle \left\langle\phi_i\right| \quad \text{with} \sum_{i=1}^{n} p_i = 1$$

Distinguish between mixed / pure state.

$$\hat{\rho} \text{ describes pure state } \iff \hat{\rho}^2 = \hat{\rho} \iff \mathrm{Tr}(\hat{\rho}^2) = 1$$

von Neumann-equation: $\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -i[\hat{H}(t),\hat{\rho}(t)]$

The Density Matrix

Definition

$$\hat{\rho} := \sum_{i=1}^{n} p_i \left|\phi_i\right\rangle \left\langle\phi_i\right| \quad \text{with} \sum_{i=1}^{n} p_i = 1$$

Distinguish between mixed / pure state.

$$\hat{\rho} \text{ describes pure state } \iff \hat{\rho}^2 = \hat{\rho} \iff \mathrm{Tr}(\hat{\rho}^2) = 1$$

von Neumann-equation: $\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -i[\hat{H}(t),\hat{\rho}(t)]$

unitary time evolution: $\hat{\rho}(t) = \hat{U}(t, t_0)\hat{\rho}(t_0)\hat{U}^\dagger(t, t_0)$

The Density Matrix for spin-1/2-systems

The Density Matrix for spin-1/2-systems

Bloch-sphere representation

$$\hat{\rho} = \frac{1}{2}\left(\mathbb{1} + \vec{P} \cdot \vec{\hat{\sigma}}\right)$$

$\vec{P} = (P_x, P_y, P_z)$: expectation value of the spin $\vec{\hat{S}} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$
$\longrightarrow$ Polarization

The Density Matrix for spin-1/2-systems

Bloch-sphere representation

$$\hat{\rho} = \frac{1}{2}\left(\mathbb{1} + \vec{P}\cdot\vec{\hat{\sigma}}\right)$$

$\vec{P} = (P_x, P_y, P_z)$: expectation value of the spin $\vec{\hat{S}} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$
$\longrightarrow$ Polarization

Note: $\hat{\rho}$ describes a pure state if and only if $|\vec{P}| = 1$

The Density Matrix for spin-1/2-systems

Bloch-sphere representation

$$\hat{\rho} = \frac{1}{2}\left(\mathbb{1} + \vec{P} \cdot \hat{\vec{\sigma}}\right)$$

$\vec{P} = (P_x, P_y, P_z)$: expectation value of the spin $\hat{\vec{S}} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$
$\longrightarrow$ Polarization

Note: $\hat{\rho}$ describes a pure state if and only if $|\vec{P}| = 1$

$$\rho = \begin{pmatrix} aa^* & ab^* \\ a^*b & bb^* \end{pmatrix}$$

The Density Matrix for spin-1/2-systems

Bloch-sphere representation

$$\hat{\rho} = \frac{1}{2}\left(\mathbb{1} + \vec{P} \cdot \vec{\hat{\sigma}}\right)$$

$\vec{P} = (P_x, P_y, P_z)$: expectation value of the spin $\vec{\hat{S}} = (\hat{S}_x, \hat{S}_y, \hat{S}_z)$
$\longrightarrow$ Polarization

Note: $\hat{\rho}$ describes a pure state if and only if $|\vec{P}| = 1$

$$\rho = \begin{pmatrix} aa^* & ab^* \\ a^*b & bb^* \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 1 + P_z & P_x - iP_y \\ P_x + iP_y & 1 - P_z \end{pmatrix}$$

Decoherence

Decoherence

A state is called decoherent, if "its interference is supressed" [a].

---

[a] Michael A Nielsen and Isaac L Chuang. *Quantum Computation and Quantum Information*. 10th Anniversary Edition. Cambridge University Press, 2010. ISBN: 978-1-107-00217-3.

Decoherence

A state is called decoherent, if "its interference is supressed"[a].

[a] Michael A Nielsen and Isaac L Chuang. *Quantum Computation and Quantum Information*. 10th Anniversary Edition. Cambridge University Press, 2010. ISBN: 978-1-107-00217-3.

$$\langle \hat{A} \rangle := \langle \psi | \hat{A} | \psi \rangle = (a^* \langle \uparrow | + b^* \langle \downarrow |) \hat{A} (a | \uparrow \rangle + b | \downarrow \rangle)$$
$$= aa^* A_{11} + bb^* A_{22} + \underbrace{b^* a A_{21} + ba^* A_{12}}_{\text{interference term}}$$

Decoherence

A state is called decoherent, if "its interference is supressed"[a].

[a] Michael A Nielsen and Isaac L Chuang. *Quantum Computation and Quantum Information*. 10th Anniversary Edition. Cambridge University Press, 2010. ISBN: 978-1-107-00217-3.

$$\langle \hat{A} \rangle := \langle \psi | \hat{A} | \psi \rangle = (a^* \langle \uparrow | + b^* \langle \downarrow |) \hat{A} (a | \uparrow \rangle + b | \downarrow \rangle)$$
$$= aa^* A_{11} + bb^* A_{22} + \underbrace{b^* a A_{21} + ba^* A_{12}}_{\text{interference term}}$$

For the density operator: off-diagonal elements vanish

Decoherence

A state is called decoherent, if "its interference is supressed"[a].

[a] Michael A Nielsen and Isaac L Chuang. *Quantum Computation and Quantum Information*. 10th Anniversary Edition. Cambridge University Press, 2010. ISBN: 978-1-107-00217-3.

$$\langle \hat{A} \rangle := \langle \psi | \hat{A} | \psi \rangle = (a^* \langle \uparrow | + b^* \langle \downarrow |) \hat{A} (a | \uparrow \rangle + b | \downarrow \rangle)$$
$$= aa^* A_{11} + bb^* A_{22} + \underbrace{b^* a A_{21} + ba^* A_{12}}_{\text{interference term}}$$

For the density operator: off-diagonal elements vanish
For the Bloch-sphere: $|\vec{P}|$ decreases

The problems so far

The problems so far

Von Neumann-equation describes isolated system.

The problems so far

Von Neumann-equation describes isolated system.
Does not produce certain effects:

The problems so far

Von Neumann-equation describes isolated system.
Does not produce certain effects:

- Dissipation of energy (Larmor precession)

The problems so far

Von Neumann-equation describes isolated system.
Does not produce certain effects:

- Dissipation of energy (Larmor precession)
- Change in entropy ($|\vec{P}| = \text{const.}$)

The problems so far

Von Neumann-equation describes isolated system.
Does not produce certain effects:

- Dissipation of energy (Larmor precession)
- Change in entropy ($|\vec{P}| = \text{const.}$)
- Statistical physics: there are no isolated systems $\rightarrow$ assume (random) interactions of the system with the environment

The problems so far

Von Neumann-equation describes isolated system.
Does not produce certain effects:

- Dissipation of energy (Larmor precession)
- Change in entropy ($|\vec{P}| = \text{const.}$)
- Statistical physics: there are no isolated systems $\rightarrow$ assume (random) interactions of the system with the environment

$$\mathrm{Tr}\left(\hat{\rho}^2(t)\right)$$

The problems so far

Von Neumann-equation describes isolated system.
Does not produce certain effects:

- Dissipation of energy (Larmor precession)
- Change in entropy ($|\vec{P}| = \text{const.}$)
- Statistical physics: there are no isolated systems $\rightarrow$ assume (random) interactions of the system with the environment

$$\text{Tr}\big(\hat{\rho}^2(t)\big) = \text{Tr}\Big(\hat{U}(t)\hat{\rho}_0 \underbrace{\hat{U}^\dagger(t)\hat{U}(t)}_{=\mathbb{1}} \hat{\rho}_0 \hat{U}^\dagger(t)\Big)$$

The problems so far

Von Neumann-equation describes isolated system.
Does not produce certain effects:

- Dissipation of energy (Larmor precession)
- Change in entropy ($|\vec{P}| = \text{const.}$)
- Statistical physics: there are no isolated systems $\rightarrow$ assume (random) interactions of the system with the environment

$$\mathrm{Tr}\left(\hat{\rho}^2(t)\right) = \mathrm{Tr}\left(\hat{U}(t)\hat{\rho}_0 \underbrace{\hat{U}^\dagger(t)\hat{U}(t)}_{=\mathbb{1}} \hat{\rho}_0 \hat{U}^\dagger(t)\right) = \mathrm{Tr}(\hat{\rho}_0^2)$$

The problems so far

Von Neumann-equation describes isolated system.
Does not produce certain effects:

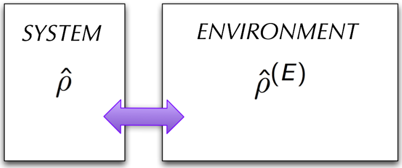
- Dissipation of energy (Larmor precession)
- Change in entropy ($|\vec{P}| = \text{const.}$)
- Statistical physics: there are no isolated systems $\rightarrow$ assume (random) interactions of the system with the environment

$$\text{Tr}\left(\hat{\rho}^2(t)\right) = \text{Tr}\left(\hat{U}(t)\hat{\rho}_0 \underbrace{\hat{U}^\dagger(t)\hat{U}(t)}_{=\mathbb{1}} \hat{\rho}_0 \hat{U}^\dagger(t)\right) = \text{Tr}(\hat{\rho}_0^2)$$

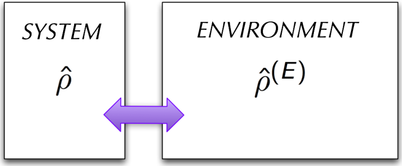
$\rightarrow$ pure state remains pure for all times

Open system

Open system

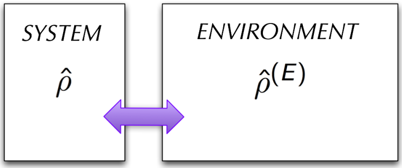


Open system



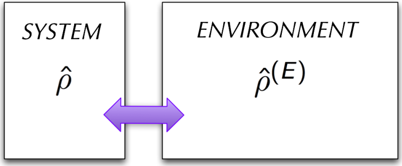
- Assume:
  $t = 0$: $\hat{\tilde{\rho}} = \hat{\rho} \otimes \hat{\rho}^{(E)}$

Open system



- Assume:
  $t = 0$: $\hat{\tilde{\rho}} = \hat{\rho} \otimes \hat{\rho}^{(E)}$
- Reduced density matrix:
  $\hat{\rho}(t) = \mathrm{Tr}_{ENV}\, \hat{\tilde{\rho}}$

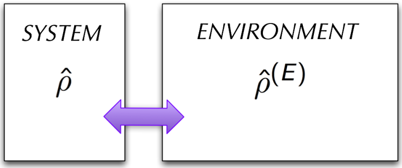
Open system



- Assume:
  $t = 0$: $\hat{\hat{\rho}} = \hat{\rho} \otimes \hat{\rho}^{(E)}$
- Reduced density matrix:
  $\hat{\rho}(t) = \mathrm{Tr}_{ENV}\, \hat{\hat{\rho}}$

Define the Dynamical map: $V(t): \ \hat{\rho}(0) \rightarrow \hat{\rho}(t)$

Open system



- Assume:
  $t = 0$: $\hat{\hat{\rho}} = \hat{\rho} \otimes \hat{\rho}^{(E)}$
- Reduced density matrix:
  $\hat{\rho}(t) = \mathrm{Tr}_{ENV}\, \hat{\hat{\rho}}$

Define the Dynamical map: $V(t): \ \hat{\rho}(0) \to \hat{\rho}(t)$

Lindblad has shown[a] that under certain assumptions, such time evolution can be written as a quantum mechanical master equation preserving the properties of $\hat{\rho}$ (positiveness, convexity).

[a] Goran Lindblad. "On the generators of quantum dynamical semigroups". In: *Communications in Mathematical Physics* 48 (1976), pp. 119–130.

The Lindblad equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -i[\hat{H}(t),\hat{\rho}(t)] + \sum_{\mu>0}\left(\hat{L}_\mu\hat{\rho}(t)\hat{L}_\mu^\dagger - \frac{1}{2}\left\{\hat{L}_\mu^\dagger\hat{L}_\mu, \hat{\rho}(t)\right\}\right)$$

$$= -i[\hat{H}(t),\hat{\rho}(t)] + \mathcal{D}[\hat{\rho}(t)] =: \mathcal{L}[\hat{\rho}(t)]$$

The Lindblad equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -i[\hat{H}(t),\hat{\rho}(t)] + \sum_{\mu>0}\left(\hat{L}_\mu\hat{\rho}(t)\hat{L}_\mu^\dagger - \frac{1}{2}\{\hat{L}_\mu^\dagger\hat{L}_\mu, \hat{\rho}(t)\}\right)$$

$$= -i[\hat{H}(t),\hat{\rho}(t)] + \mathcal{D}[\hat{\rho}(t)] =: \mathcal{L}[\hat{\rho}(t)]$$

Form: Lindblad-operator = unitary evolution + dissipation

The Lindblad equation

$$\frac{\mathrm{d}}{\mathrm{d}t}\hat{\rho}(t) = -i[\hat{H}(t),\hat{\rho}(t)] + \sum_{\mu>0}\left(\hat{L}_\mu\hat{\rho}(t)\hat{L}_\mu^\dagger - \frac{1}{2}\{\hat{L}_\mu^\dagger\hat{L}_\mu, \hat{\rho}(t)\}\right)$$
$$= -i[\hat{H}(t),\hat{\rho}(t)] + \mathcal{D}[\hat{\rho}(t)] =: \mathcal{L}[\hat{\rho}(t)]$$

Form: Lindblad-operator = unitary evolution + dissipation
$\hat{L}_\mu$ ... Lindblad operators

Lindblad operators

Lindblad operators

What is the meaning of the Lindblad operators $\hat{L}_\mu$?

Lindblad operators

What is the meaning of the Lindblad operators $\hat{L}_\mu$?

- $\hat{L}_\mu \sim \hat{\sigma}^-$

Lindblad operators

What is the meaning of the Lindblad operators $\hat{L}_\mu$?

- $\hat{L}_\mu \sim \hat{\sigma}^-$ produce *amplitude* damping

Lindblad operators

What is the meaning of the Lindblad operators $\hat{L}_\mu$?

- $\hat{L}_\mu \sim \hat{\sigma}^-$ produce *amplitude* damping
- $\hat{L}_\mu \sim \hat{\sigma}_z$

Lindblad operators

What is the meaning of the Lindblad operators $\hat{L}_\mu$?

- $\hat{L}_\mu \sim \hat{\sigma}^-$ produce *amplitude* damping
- $\hat{L}_\mu \sim \hat{\sigma}_z$ produce *phase* damping

Lindblad operators

What is the meaning of the Lindblad operators $\hat{L}_\mu$?

- $\hat{L}_\mu \sim \hat{\sigma}^-$ produce *amplitude* damping
- $\hat{L}_\mu \sim \hat{\sigma}_z$ produce *phase* damping

Note: one can consider more than one Lindblad operator

Amplitude damped Qubit

Amplitude damped Qubit

Choose $\hat{L}_\mu = \sqrt{\gamma}\hat{\sigma}^-$

Amplitude damped Qubit

Choose $\hat{L}_\mu = \sqrt{\gamma}\hat{\sigma}^-$

$$\dot{P}_z = -\gamma(1 + P_z)$$
$$\dot{P}_x = -\omega_0 P_y - \frac{\gamma}{2} P_x$$
$$\dot{P}_y = \omega_0 P_x - \frac{\gamma}{2} P_y$$

Amplitude damped Qubit

Choose $\hat{L}_\mu = \sqrt{\gamma}\hat{\sigma}^-$

$$\dot{P}_z = -\gamma(1 + P_z)$$
$$\dot{P}_x = -\omega_0 P_y - \frac{\gamma}{2} P_x$$
$$\dot{P}_y = \omega_0 P_x - \frac{\gamma}{2} P_y$$

$$P_x(t) = \big(- y_0 \sin(\omega_0 t) + x_0 \cos(\omega_0 t)\big) e^{-\frac{\gamma}{2} t}$$
$$P_y(t) = \big(y_0 \cos(\omega_0 t) + x_0 \sin(\omega_0 t)\big) e^{-\frac{\gamma}{2} t}$$
$$P_z(t) = -1 + (z_0 + 1) e^{-\gamma t}$$

$\longrightarrow$ amplitude damping occurs $(P_z(t \to \infty) = -1)$

Amplitude damped Qubit

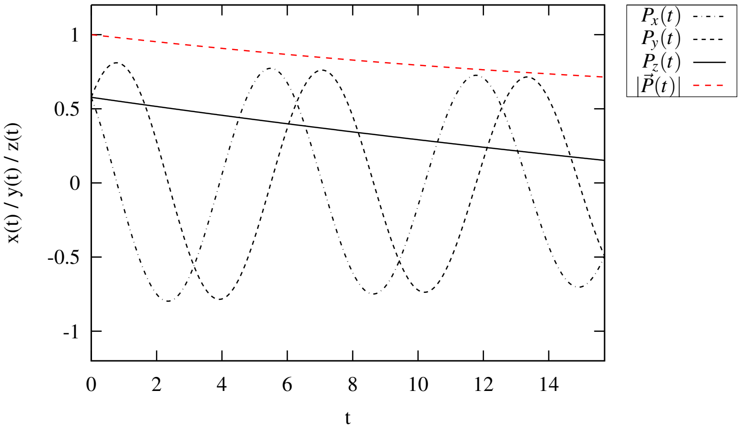


Figure : Plot of the time evolution obtained before. $\gamma = 0.05$, $\omega_0 = 1$. At $t = 0$, the system was prepared with a polarization $1/\sqrt{3} \cdot (1, 1, 1)^T$

NOT-Gate with Amplitude Damping Channel

NOT-Gate with Amplitude Damping Channel

Again, use $\hat{L}_\mu = \sqrt{\gamma}\hat{\sigma}^-$.

NOT-Gate with Amplitude Damping Channel

Again, use $\hat{L}_\mu = \sqrt{\gamma}\hat{\sigma}^-$.
Full Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_I(t)$, plug into Lindblad equation

NOT-Gate with Amplitude Damping Channel

Again, use $\hat{L}_\mu = \sqrt{\gamma}\hat{\sigma}^-$.
Full Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_I(t)$, plug into Lindblad equation

$$\frac{\mathrm{d}}{\mathrm{d}t}P_x = 2QP_z\sin(\epsilon t) - \omega_0 P_y - \frac{\gamma}{2}P_x$$

$$\frac{\mathrm{d}}{\mathrm{d}t}P_y = -2QP_z\cos(\epsilon t) + \omega_0 P_x - \frac{\gamma}{2}P_y$$

$$\frac{\mathrm{d}}{\mathrm{d}t}P_z = 2Q\big(P_y\cos(\epsilon t) + P_x\cos(\epsilon t)\big) - \gamma(1 + P_z)$$

NOT-Gate with Amplitude Damping Channel

Again, use $\hat{L}_\mu = \sqrt{\gamma}\hat{\sigma}^-$.
Full Hamiltonian $\hat{H} = \hat{H}_0 + \hat{H}_I(t)$, plug into Lindblad equation

$$\frac{\mathrm{d}}{\mathrm{d}t}P_x = 2QP_z\sin(\epsilon t) - \omega_0 P_y - \frac{\gamma}{2}P_x$$
$$\frac{\mathrm{d}}{\mathrm{d}t}P_y = -2QP_z\cos(\epsilon t) + \omega_0 P_x - \frac{\gamma}{2}P_y$$
$$\frac{\mathrm{d}}{\mathrm{d}t}P_z = 2Q\big(P_y\cos(\epsilon t) + P_x\cos(\epsilon t)\big) - \gamma(1 + P_z)$$

→ numerical solution

NOT-Gate with Amplitude Damping Channel

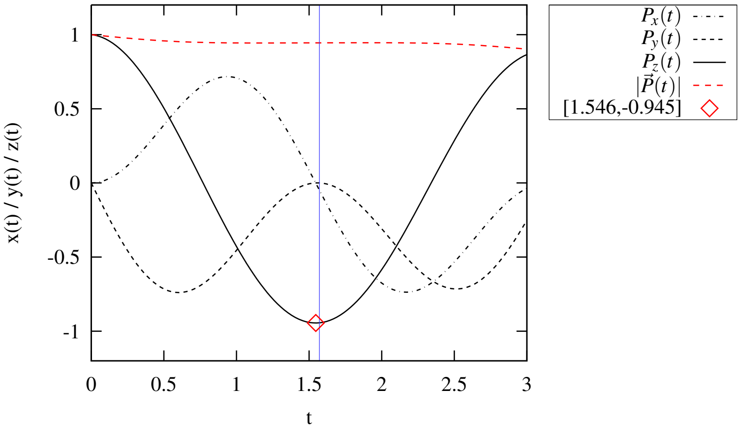


Figure : Time evolution of a NOT-Gate with $\hat{L}_\mu = \sqrt{\gamma}\hat{\sigma}^-$ (amplitude damping). $\omega_0 = \epsilon = 1$ (resonance), $Q = 1$, $\gamma = 0.05$.

NOT-Gate with Phase Damping Channel

NOT-Gate with Phase Damping Channel

Use different Lindblad operator: $\hat{L}_\mu = \sqrt{\lambda}\hat{\sigma}_z$

NOT-Gate with Phase Damping Channel

Use different Lindblad operator: $\hat{L}_\mu = \sqrt{\lambda}\hat{\sigma}_z$

$$\frac{\mathrm{d}}{\mathrm{d}t} P_x = 2QP_z \sin(\epsilon t) - \omega_0 y \; -2\lambda x$$

$$\frac{\mathrm{d}}{\mathrm{d}t} P_y = -2QP_z \cos(\epsilon t) + \omega_0 x \; -2\lambda y$$

$$\frac{\mathrm{d}}{\mathrm{d}t} P_z = 2Q\left(P_y \cos(\epsilon t) + x \cos(\epsilon t)\right)$$

NOT-Gate with Phase Damping Channel

Use different Lindblad operator: $\hat{L}_\mu = \sqrt{\lambda}\hat{\sigma}_z$

$$\frac{\mathrm{d}}{\mathrm{d}t}P_x = 2QP_z\sin(\epsilon t) - \omega_0 y \; -2\lambda x$$
$$\frac{\mathrm{d}}{\mathrm{d}t}P_y = -2QP_z\cos(\epsilon t) + \omega_0 x \; -2\lambda y$$
$$\frac{\mathrm{d}}{\mathrm{d}t}P_z = 2Q\left(P_y\cos(\epsilon t) + x\cos(\epsilon t)\right)$$

Difference to amplitude damping: no (direct) damping of P_z occurs

NOT-Gate with Phase Damping Channel

Use different Lindblad operator: $\hat{L}_\mu = \sqrt{\lambda}\hat{\sigma}_z$

$$\frac{\mathrm{d}}{\mathrm{d}t}P_x = 2QP_z \sin(\epsilon t) - \omega_0 y - 2\lambda x$$
$$\frac{\mathrm{d}}{\mathrm{d}t}P_y = -2QP_z \cos(\epsilon t) + \omega_0 x - 2\lambda y$$
$$\frac{\mathrm{d}}{\mathrm{d}t}P_z = 2Q\left(P_y \cos(\epsilon t) + x \cos(\epsilon t)\right)$$

Difference to amplitude damping: no (direct) damping of P_z occurs $\longrightarrow$ *Phase Damping Channel*

NOT-Gate with Phase Damping Channel

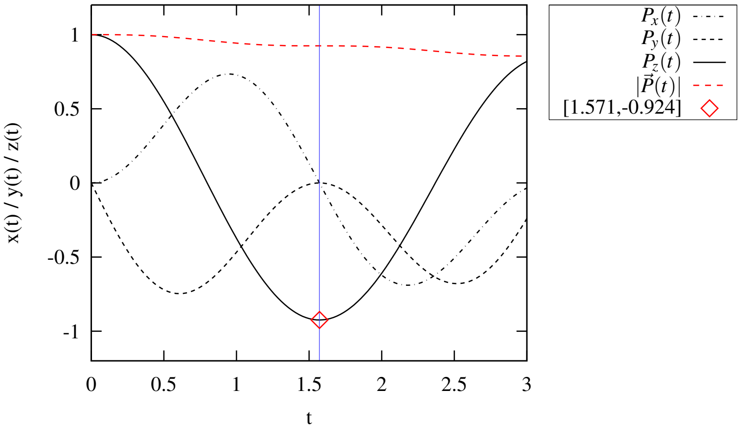


Figure : Time evolution of a qubit considering a noisy phase damping channel. $\omega_0 = \epsilon = 1$ (resonance), $Q = 1$, $\lambda = 0.05$.

Entropy in Quantum Systems

Entropy in Quantum Systems

From probability theory: $S = -\sum_i p_i \cdot \ln(p_i)$

Entropy in Quantum Systems

From probability theory: $S = -\sum_i p_i \cdot \ln(p_i)$
$\longrightarrow$ measure information

Entropy in Quantum Systems

From probability theory: $S = -\sum_i p_i \cdot \ln(p_i)$
$\longrightarrow$ measure information

von Neumann-entropy

$$S = -\mathrm{Tr}\left(\hat{\rho} \cdot \ln(\hat{\rho})\right)$$

Entropy in Quantum Systems

From probability theory: $S = -\sum_i p_i \cdot \ln(p_i)$
$\longrightarrow$ measure information

von Neumann-entropy

$$S = -\mathrm{Tr}\big(\hat{\rho} \cdot \ln(\hat{\rho})\big)$$

Note: $S = 0$ if and only if $\hat{\rho}$ describes a pure state

Entropy in Quantum Systems

From probability theory: $S = -\sum_i p_i \cdot \ln(p_i)$
$\longrightarrow$ measure information

von Neumann-entropy

$$S = -\mathrm{Tr}\big(\hat{\rho} \cdot \ln(\hat{\rho})\big)$$

Note: $S = 0$ if and only if $\hat{\rho}$ describes a pure state
maximally entangled state: $\hat{\rho} = \frac{1}{2}\mathbb{1} \longrightarrow S = \ln(2)$

Long time evolution, phase damped

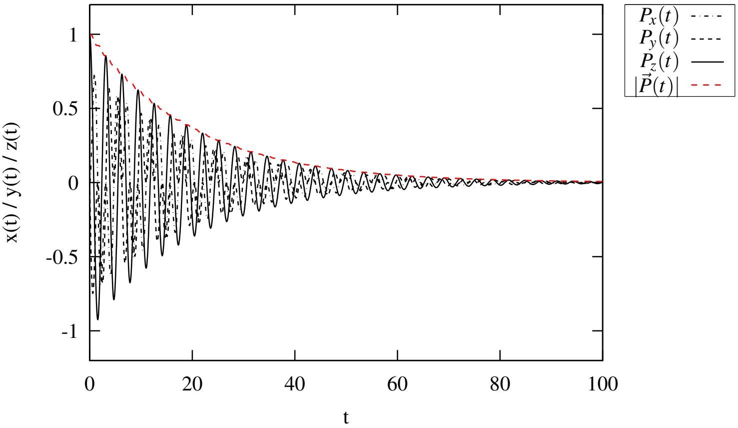


Figure : NOT-Gate under consideration of a phase damping ($\hat{L}_\mu = \sqrt{\lambda}\hat{\sigma}_z$). Again, $\omega_0 = \epsilon = 1$ (resonance), $Q = 1$, $\lambda = 0.05$

Long time evolution, amplitude damped

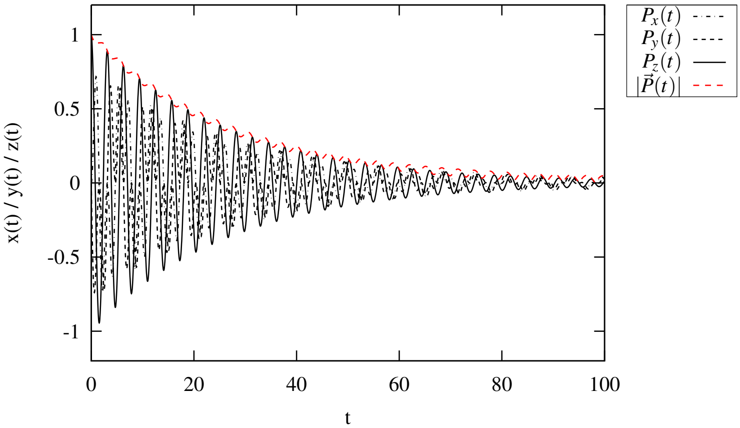


Figure : NOT-Gate under consideration of an amplitude damping. Again, $\omega_0 = \epsilon = 1$ (resonance), $Q = 1$, $\lambda = 0.05$

Entropy change

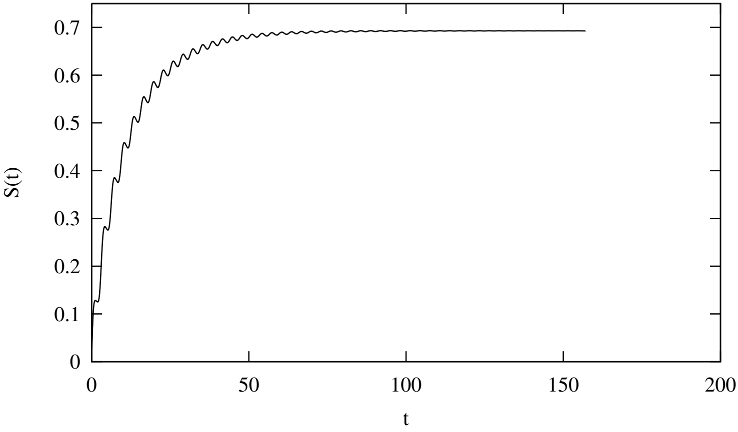


Figure : Long time evolution of a NOT-operation under consideration of an amplitude damping. Again, $\omega_0 = \epsilon = 1$ (resonance), $Q = 1$, $\lambda = 0.05$.

Entropy of amplitude damped qubit

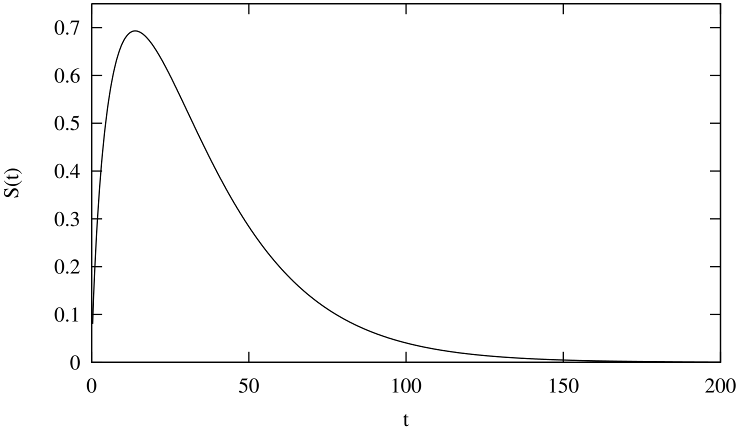


Figure : Change in the von Neumann-entropy of a simple qubit (no EM interaction) considering amplitude damping as discussed before. Parameters are $Q = 1$, $\gamma = 0.05$.

Thank you for your attention.