Title

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2-Qubits with spins

At the beginning most things here can be done analytically. You can still try to program them if you like.

Consider two spin $\frac{1}{2}$ particles a and b with Hamiltonian

$$H = H_B + H_J + H_{By} \tag{1}$$

$H_B = -B\left(S_{az} + S_{bz}\right)$

$H_{By} = -B_y S_{by}$

$H_J = J S_{az} S_{bz}$

(S_{az} is the z component of the spin of particle a, etc.)

later, we will use the spin-rotation invariant interaction

$H_J^{'} = J \boldsymbol{S}_a \cdot \boldsymbol{S}_b$

The two spins represent qubits with the rule

$$|\,0\,\rangle = \left|+\frac{1}{2}\right\rangle \qquad |\,1\,\rangle = \left|-\frac{1}{2}\right\rangle \tag{2}$$

for both particles. We work with $\hbar = 1$

<u>Realisation of C-phase gate</u>

We first want a process producing the so-called C-phase gate, which maps

$$
\begin{aligned}
|\,0\,x\,\rangle &\rightarrow e^{i\phi}\,|\,0\,x\,\rangle \\
|\,1\,x\,\rangle &\rightarrow e^{i\phi}(-1)^{x}\,|\,0\,x\,\rangle
\end{aligned}
\tag{3}
$$

here $|\,x\,y\,\rangle$ is a two-qubit register in which $x \in (0,1)$ is the state of the first spin and $y \in (0,1)$ is the one of the second spin. ϕ is an arbitrary common phase which is immaterial for a quantum state.

The transformation 3 can be realized by time evolution of the Hamiltonian 1, whereby B is switched on for a time t_B and J for a time t_J , while $B_y = 0$. Find t_B and t_J

Try the same using $H_J^{'}$ instead of H_J (I don't know the answer)

Realisation of C-not gate

Show that a C-not gate is realized by first carrying a rotation of $\pi/2$ around the y axis, which is produced by switching on B_y for a certain time t_y, then carrying out the C-phase as above, and then reversing the B_y field for the same time t_y.

The C-not maps

$|\,0\,x\,\rangle \rightarrow |\,0\,x\,\rangle \qquad |\,1\,x\,\rangle \rightarrow |\,1\,NOT(x)\,\rangle$

Reduced density operator as a measure of entanglement

Given a normalized two-spin state $|\,\Psi\,\rangle$, the reduced density operator on the space of the second spin is given by

$$\rho = Tr_1\,|\,\Psi\,\rangle\,\langle\,\Psi\,|$$

where Tr_1 is the trace over the degrees of freedom of the first spin.

The von Neumann entropy is given by

$S = -\mathrm{tr}\rho \ln \rho$

For the following initial states, write ρ as a 2×2 matrix and evaluate the entropy before and after the c-not or c-phase transformation and evaluate the entropy (be careful about normalisation!):

$|\,00\,\rangle$

$|\,10\,\rangle$

$|\,00\,\rangle + |\,10\,\rangle$

$|\,00\,\rangle + |\,01\,\rangle + |\,10\,\rangle + |\,11\,\rangle$

Next step: effect of environment on two-qubit gate, Lindblad approach

TODO