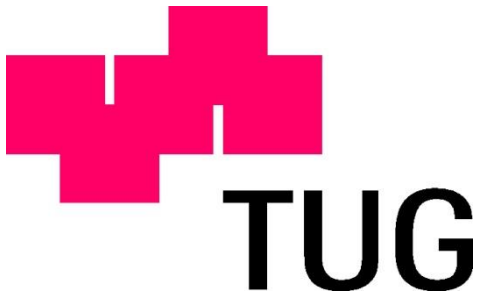


Nonequilibrium evolution of the Kondo screening cloud

Martin Nuss, Martin Ganahl, Antonius Dorda,
Hans Gerd Evertz, Enrico Arrigoni and Wolfgang von der Linden



martin.nuss@tugraz.at

DPG, Dresden, 02.04.2014



We gratefully acknowledge support from the Austrian Science Fund (**FWF**) **P24081-N16**, **ViCoM** sub projects **F04103** and **F04104** as well as the Vienna Scientific Cluster (**VSC**).

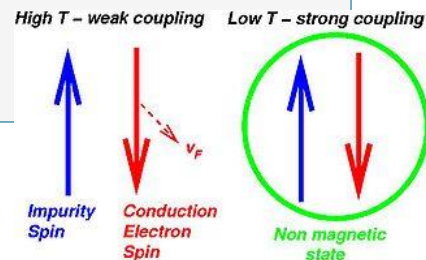
Agenda



The single impurity revisited: real time evolution

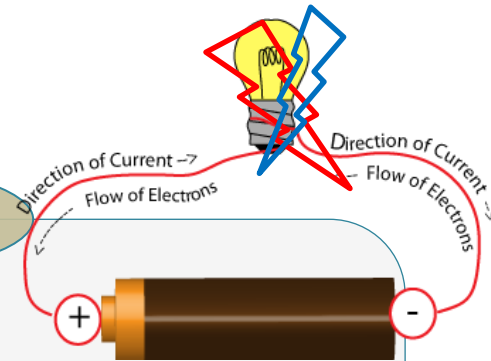
II

Non-local space-time correlations

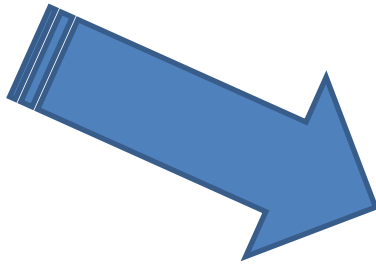


III

Correlations & bias voltage



Obtain „Gold“



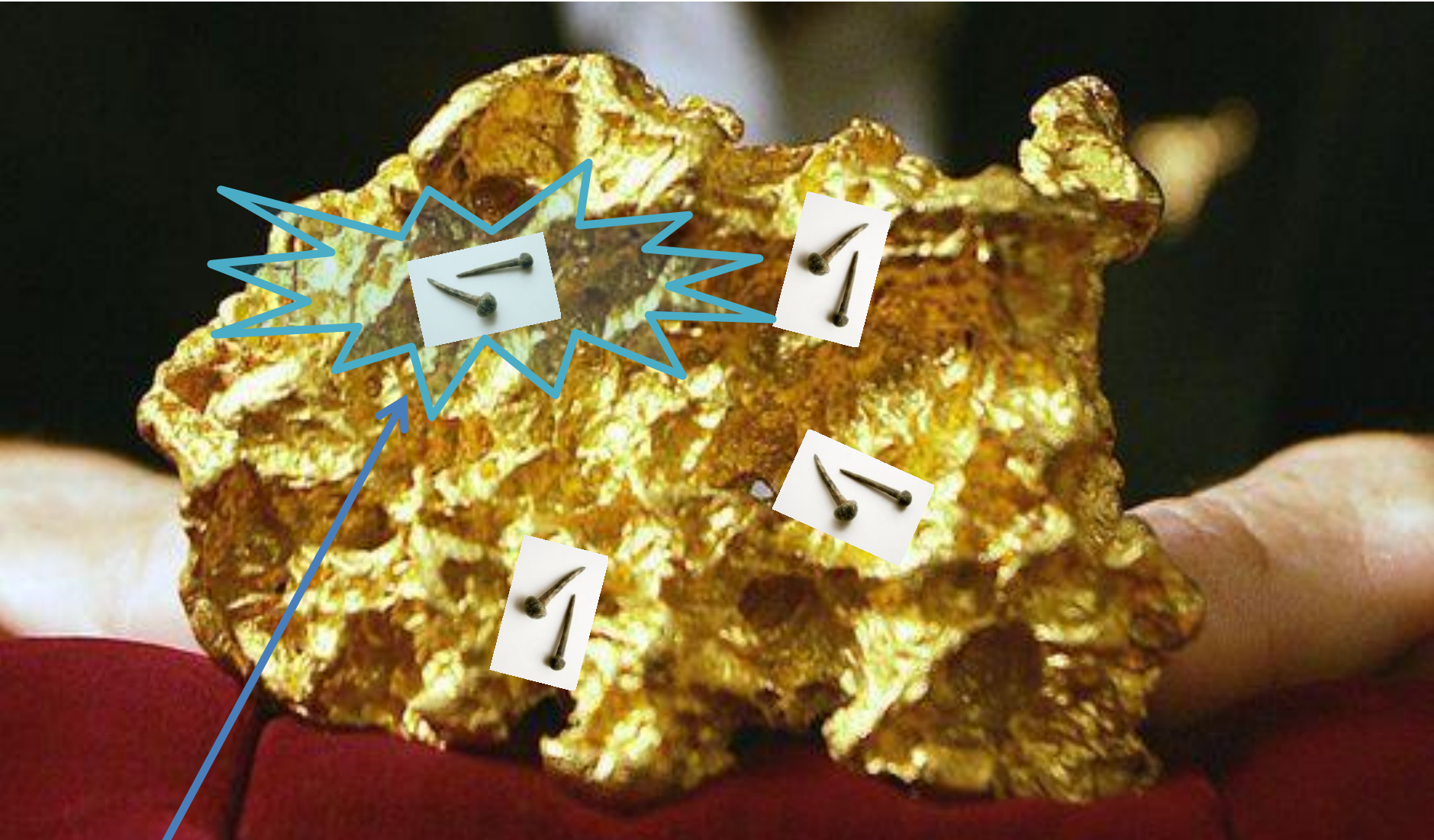
Joseph Leopold Ratinckx Der Alchemist



Put in „Iron“



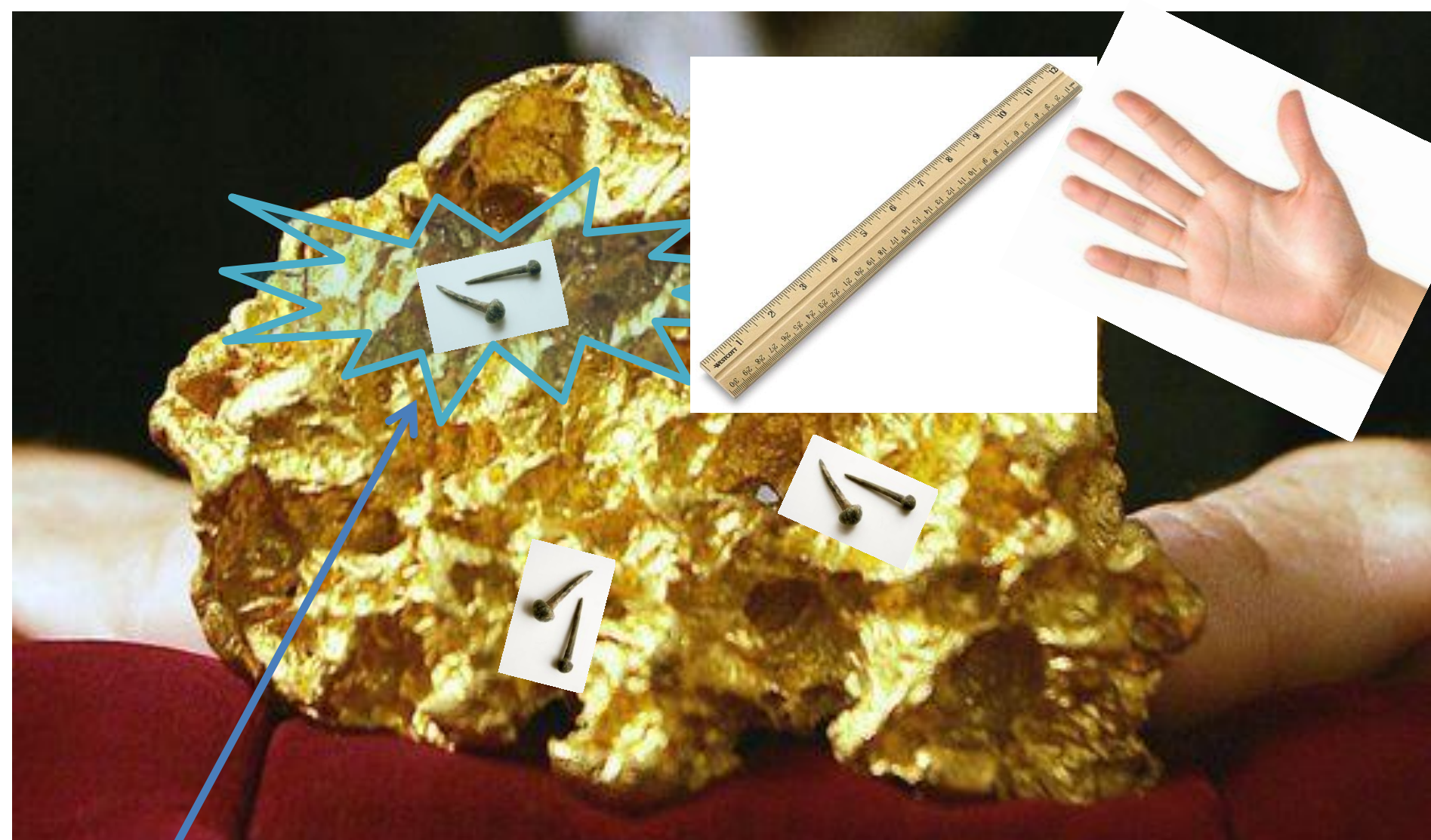
Dilute Magnetic Impurities



Singlets

A.M. Clogston, PR **125**, 541 (1962)

Measure Singlet Extent



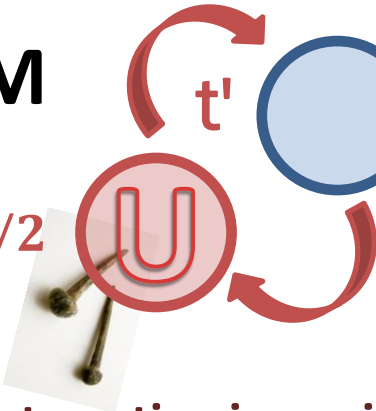
Singlets

A.M. Clogston, PR 125, 541 (1962)

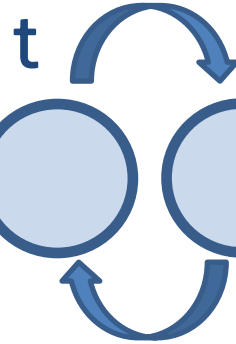
Single Impurity

P. W. Anderson, PR **124**, 41 (1961)**SIAM**

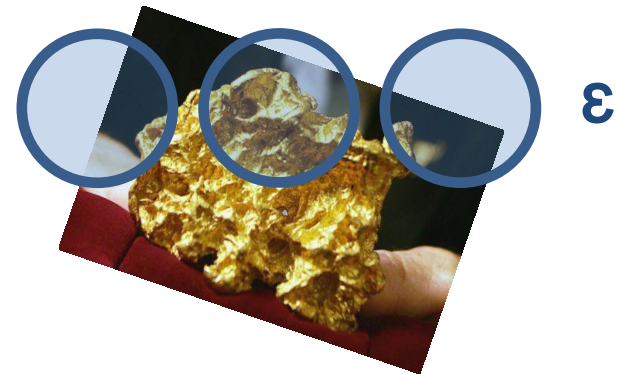
$$\varepsilon_f = -U/2$$



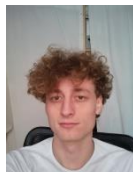
symmetric interacting impurity U



length L

finite 1D tight binding lead: ε, t
 **DMRG**

see talk by F. Heidrich-Meisner: TT2.1, Sun

○ **T=0**S. R. White, PRB **48**, 10345 (1993)
 **massively *parallel* code**
○ **Martin Ganahl**

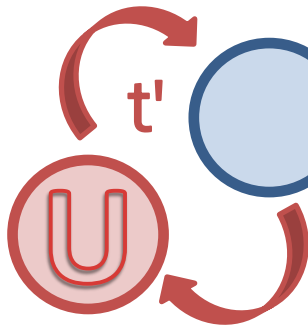


TUG

Spin-Spin Correlations

itp^{cp}P. W. Anderson, PR **124**, 41 (1961)**SIAM**

$$\varepsilon_f = -U/2$$



t

length L

 ε

symmetric interacting impurity U

r

finite 1D tight binding lead: ε, t

looking for Kondo Singlet

$$\langle S_0 S_r \rangle \propto TK$$

experimentally difficult

- NMR (bulk)

$$\chi(r) - \nu/2 \rightarrow \frac{\cos 2k_F r}{4\pi r^3} f(r/\xi_K) \chi(T)$$

- persistent currents (nano)

I. Affleck, arXiv:0911.2209 (2010)

J. Park et al. PRL **110**, 246603 (2013)

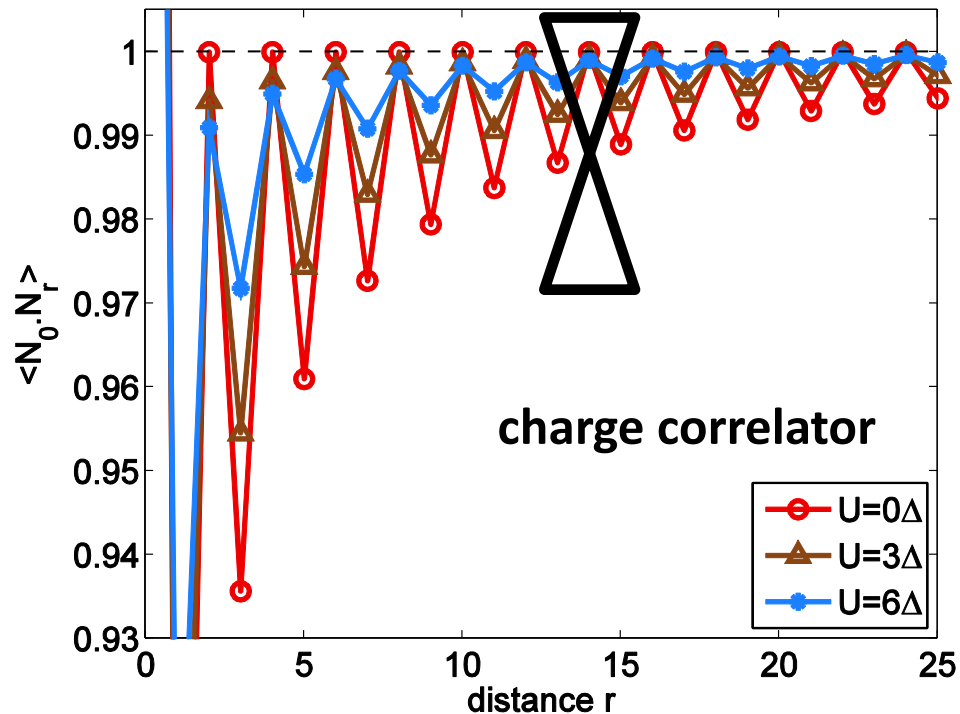
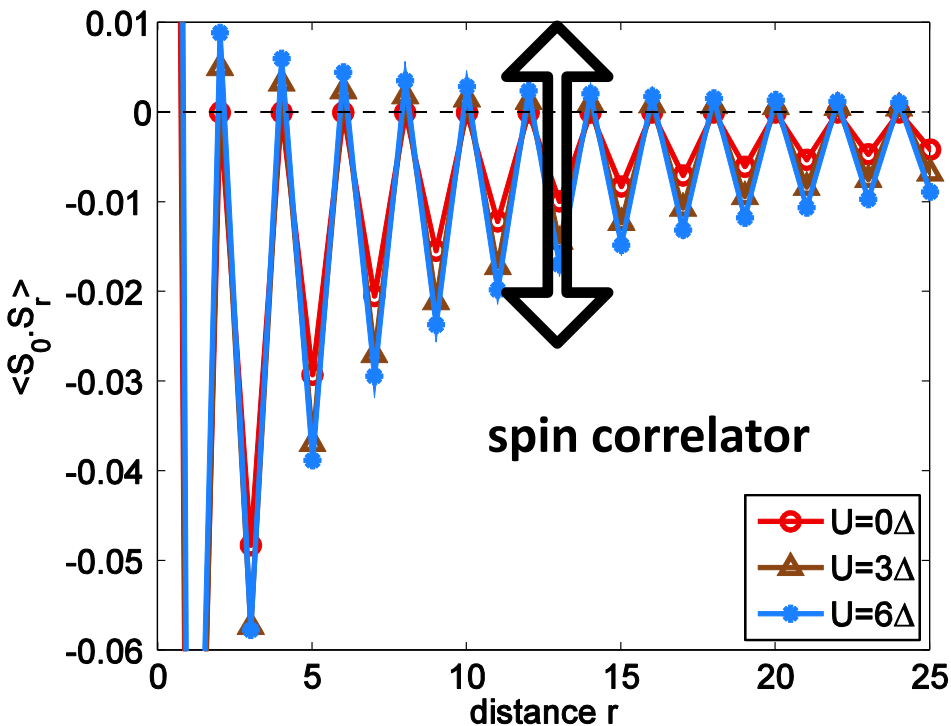
High T – weak coupling Low T – strong coupling

Impurity Spin

Conduction Electron Spin

Non magnetic state

Spin & Charge Correlators



∞ **$U=0$** H. Ishii, JLTTP **32**, 457 (1978)

- $C = \frac{4}{3}S + \langle n \rangle$
- $\xi \propto 1/\Delta$

∞ **U induces „ferromagnetic“ sites**

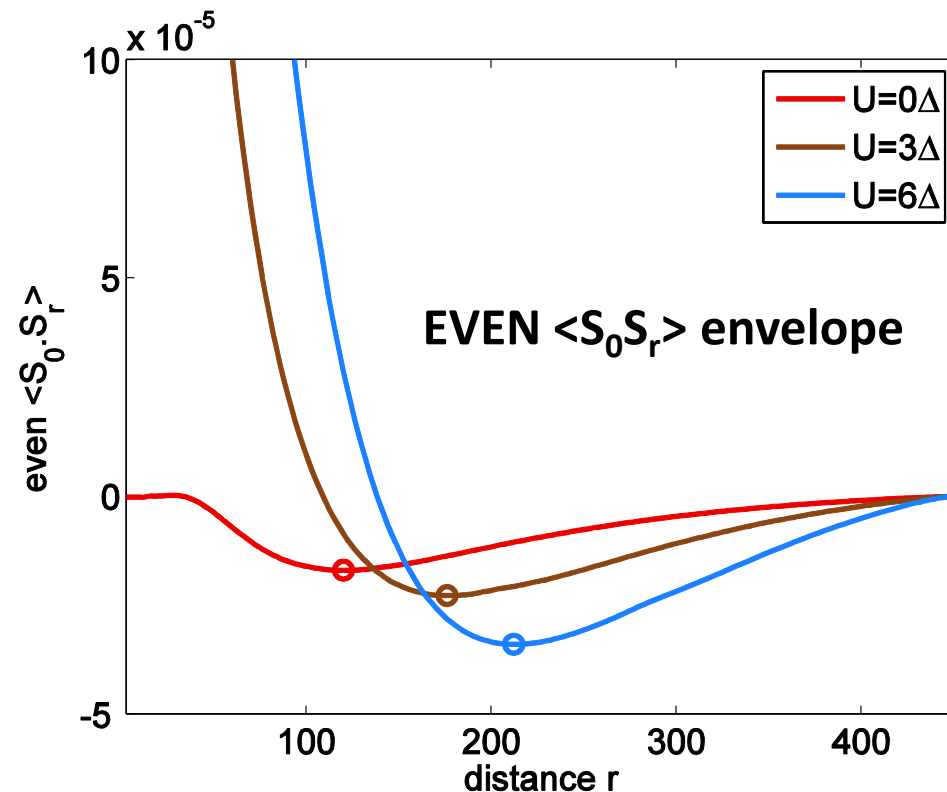
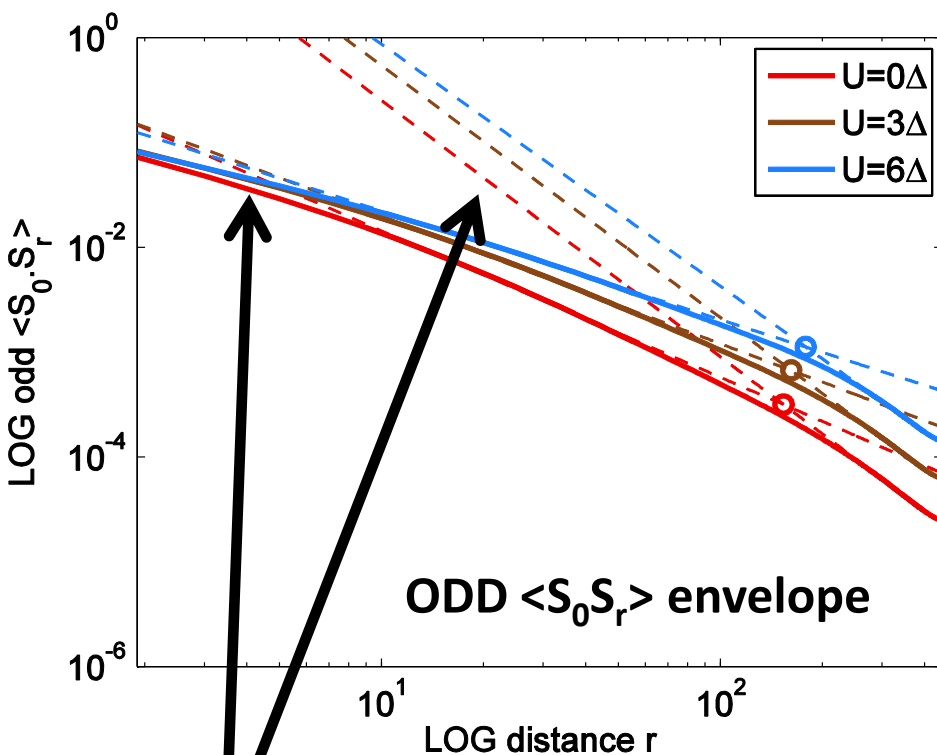
- $\xi \propto 1/T_K$

∞ **$U>0$** F. Mezei et al. PRL **29**, 1465 (1972)

- Fridel-like oscillations suppressed**
- recover at long distances**

see also QMC & tDMRG: J. E. Gubernatis, et al. RB **35**, 8478 (1987), A. Holzner et al. PRB **80**, 205114 (2009)

Characteristic Length

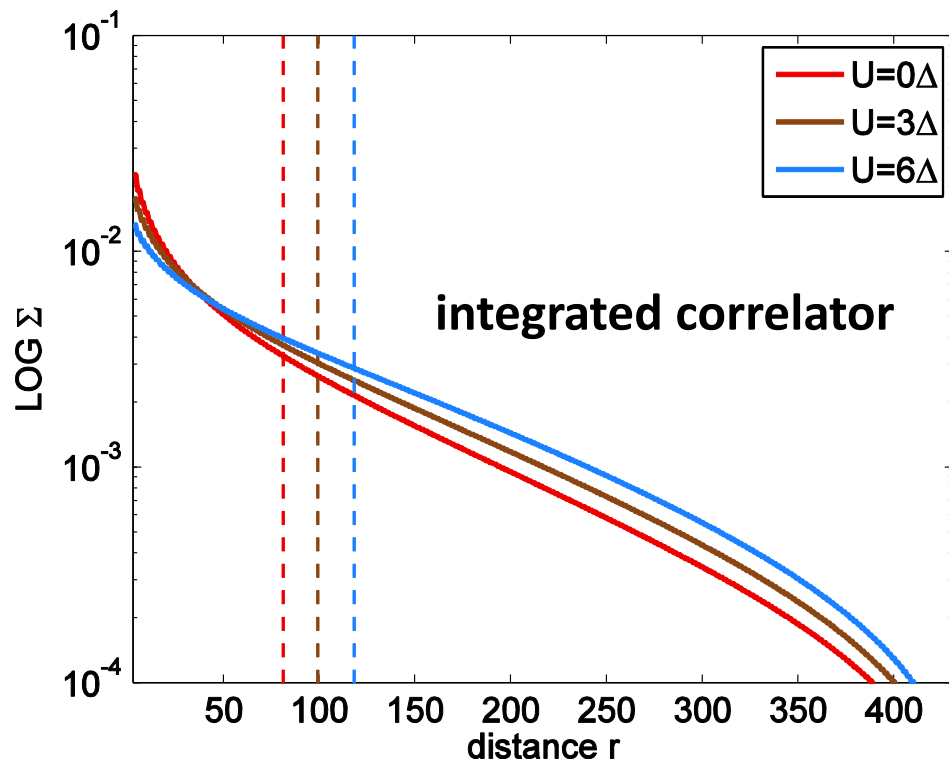


2 power laws, exponents:

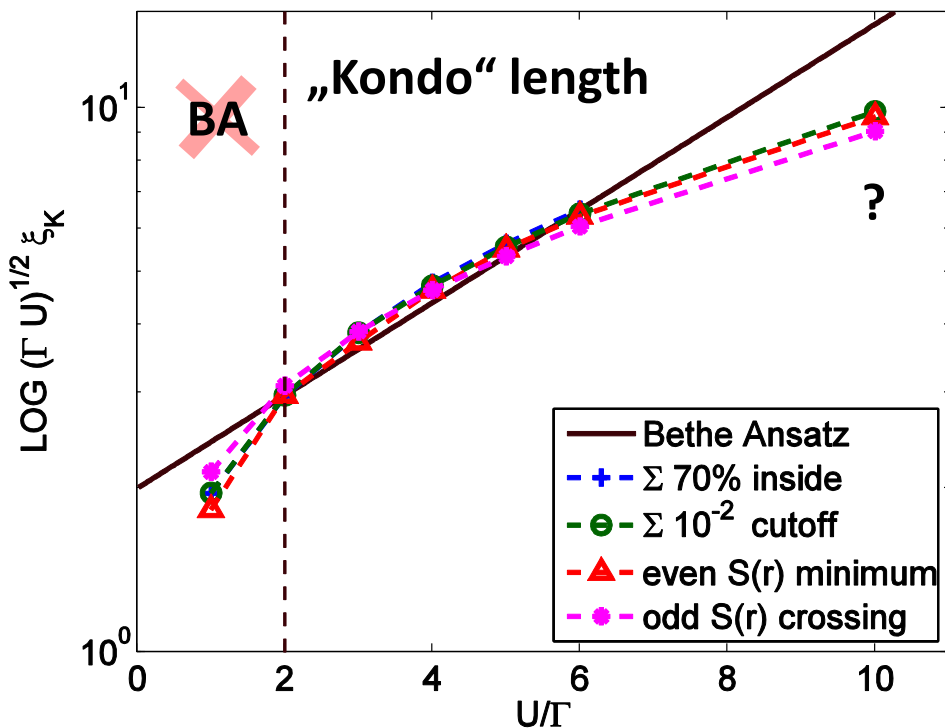
- $\sim 1.4 - U/2$
- $\sim 2.4 - U/3$

follow minima

Characteristic Length



$$\Sigma(r) = \sum_{r'=0}^r \frac{S(r')}{S(0)}$$



Bethe Ansatz

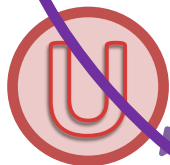
$$\xi_K \propto \frac{v_F}{k_B T_K} \propto 2t \sqrt{\frac{2}{\Delta U}} e^{\frac{\pi}{8\Delta} U}$$

Real Time Evolution

P. W. Anderson, PR **124**, 41 (1961)

SIAM

$$\varepsilon_f = -U/2$$

 t' t length L ε symmetric interacting impurity U finite 1D tight binding lead: ε, t hybridization quench: t'

∞ Suzuki-Trotter *real* time evolution using MPS

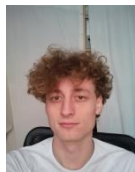
○ DMRG + TEBD

S. R. White, PRB **48**, 10345 (1993)G. Vidal, PRL **93**, 040502 (2004)

∞ *unitary*: finite time + finite size

∞ *massively parallel* code

○ Martin Ganahl

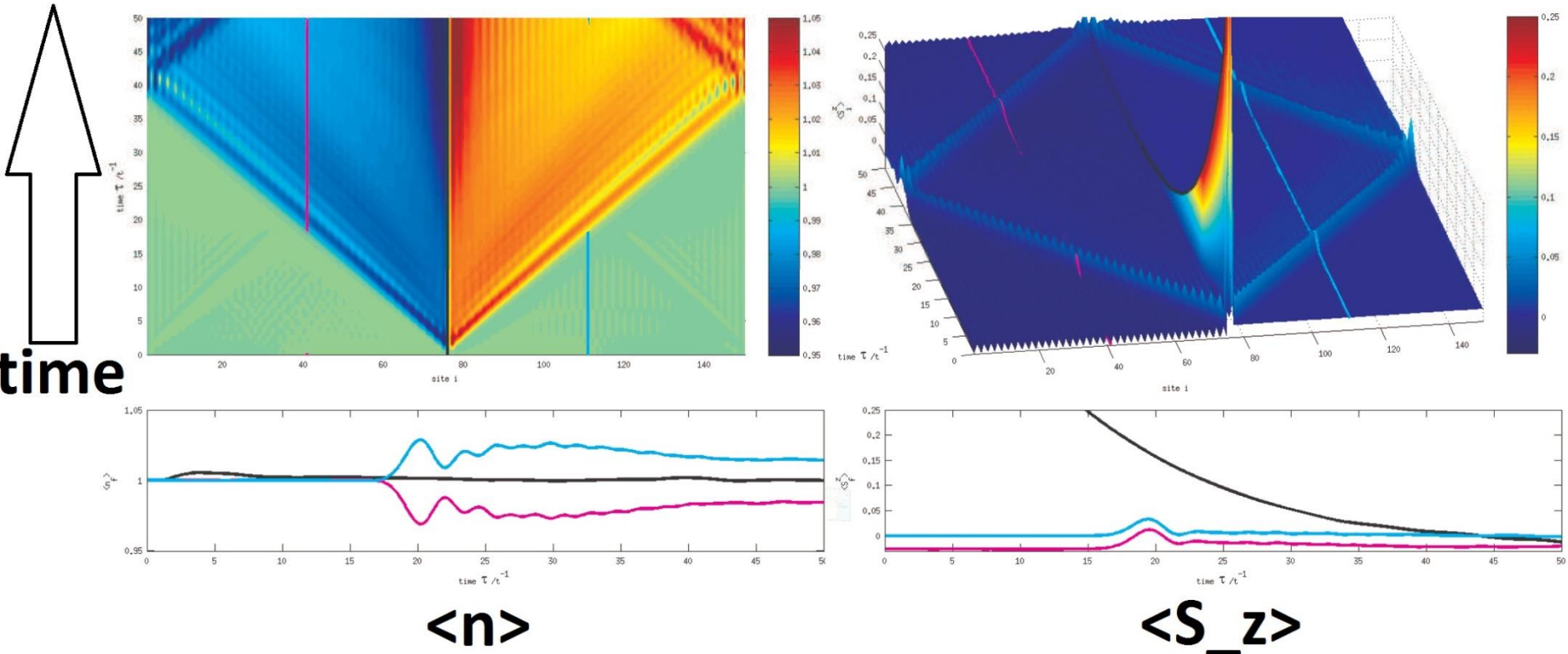


see talk by F. Heidrich-Meisner: TT2.1, Sun

see talk by P. Schmitteckert: TT9.1, Mon



Local Densities



∞ quench – transients

∞ travelling signal (Lieb Robinson)
Lieb and Robinson, CMP. **28**, 251 (1972)

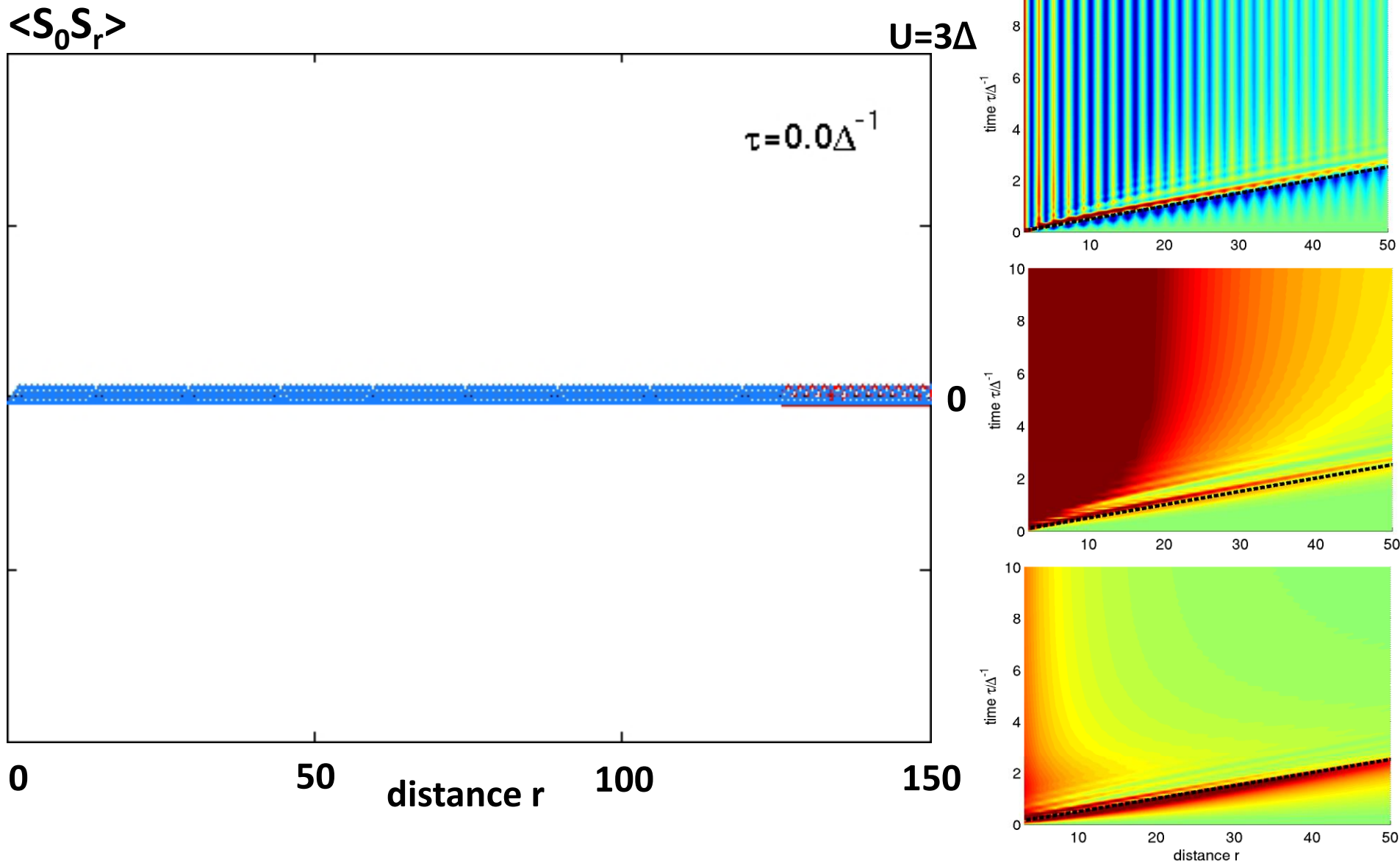
∞ eventually: (local) steady-state

∞ finite size (reflections)

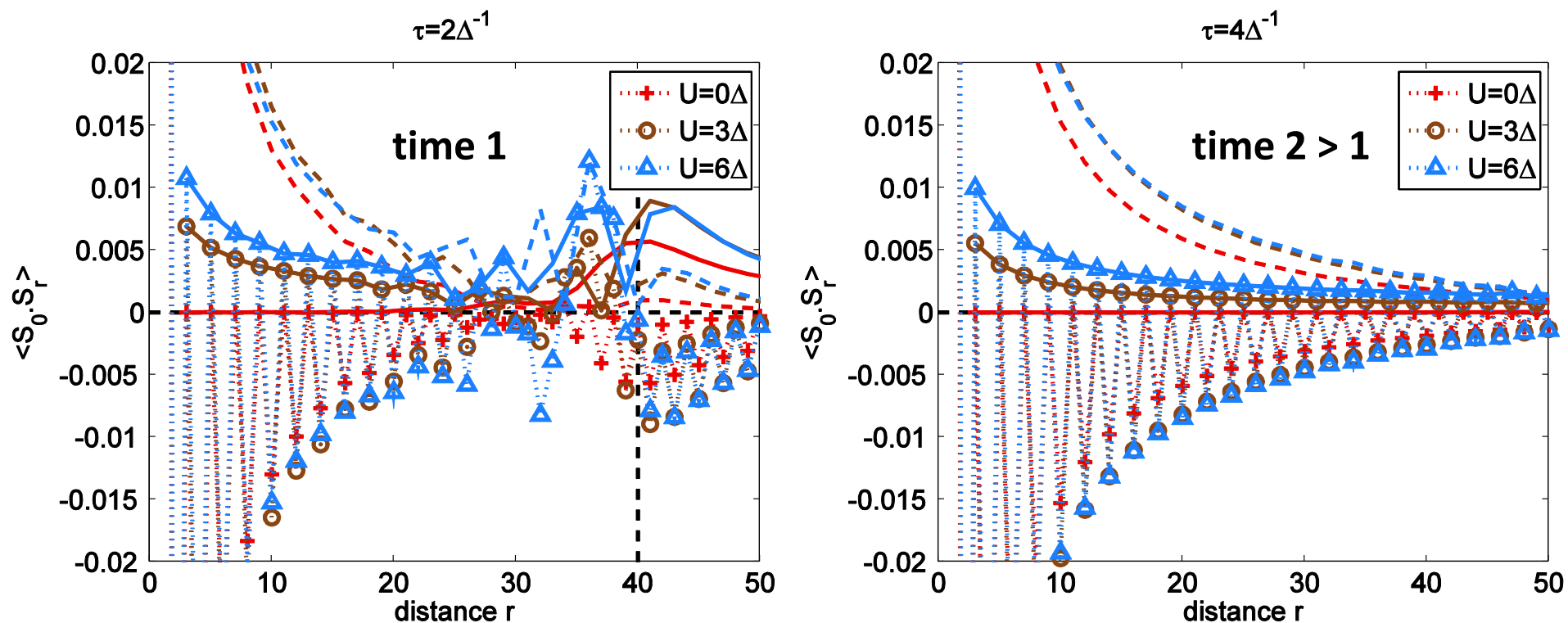
∞ finite time (entanglement growth)



$$\langle S_0 S_r \rangle \text{ (time)}$$

itp^{cp}

$\langle S_0 S_r \rangle$ (time)



time



☞ **Toulouse limit (bosonization + refermionization)**

M. Medvedyeva et al. PRB **88**, 094306 (2013)

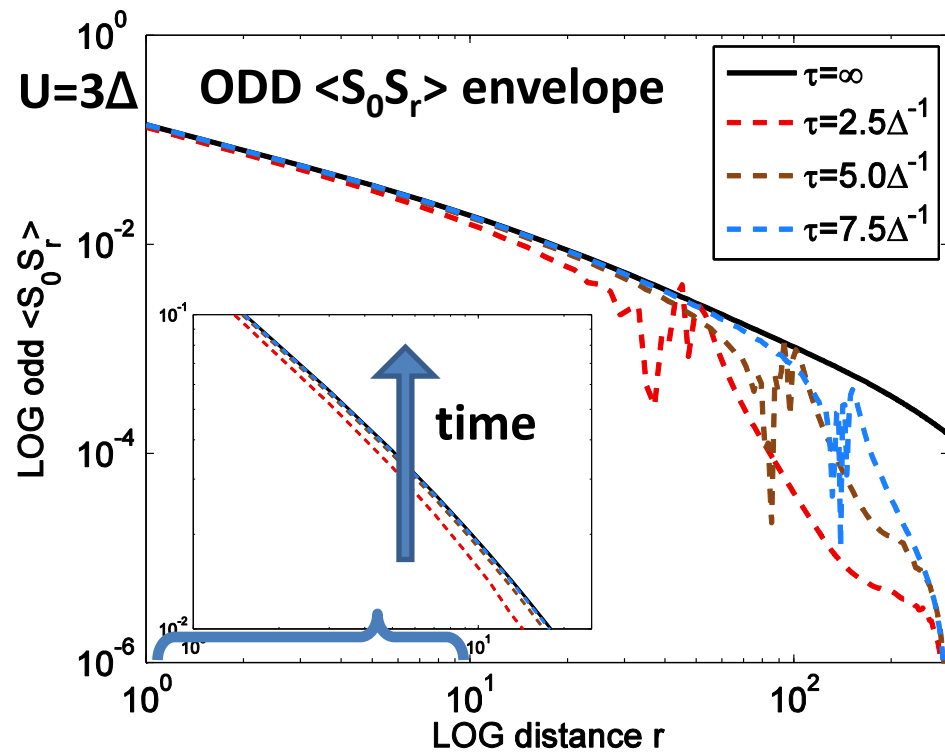
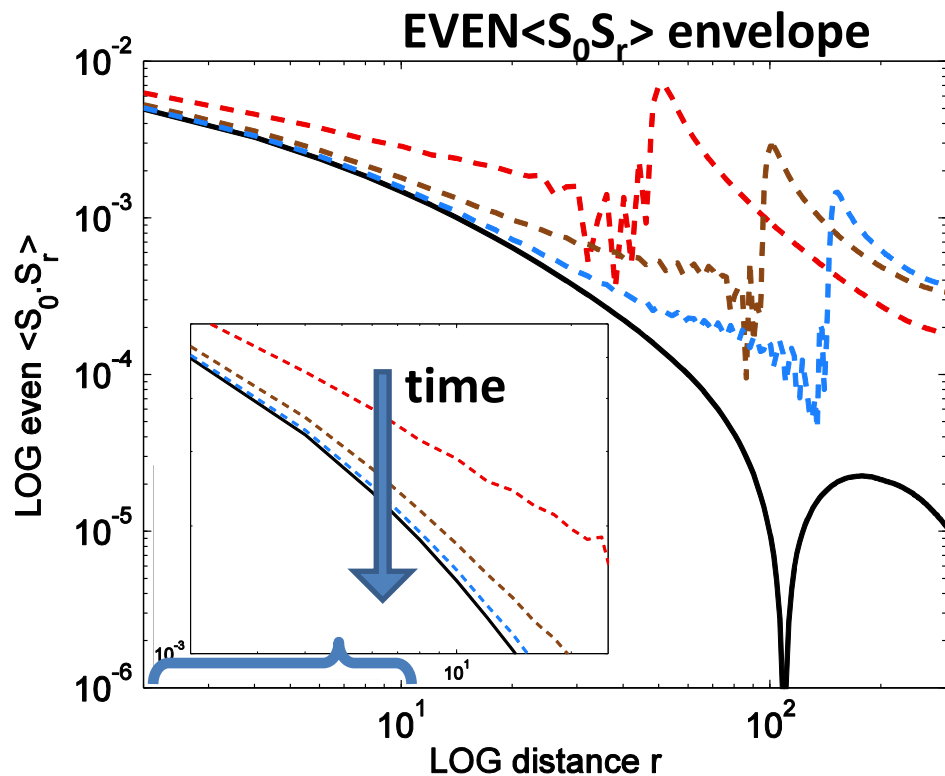
☞ **RLM (exact diagonalization)**

S. Ghosh et al. arXiv:1309.0027v1 (2013)



TUG

$\langle S_0 S_r \rangle(\text{time})$ envelope

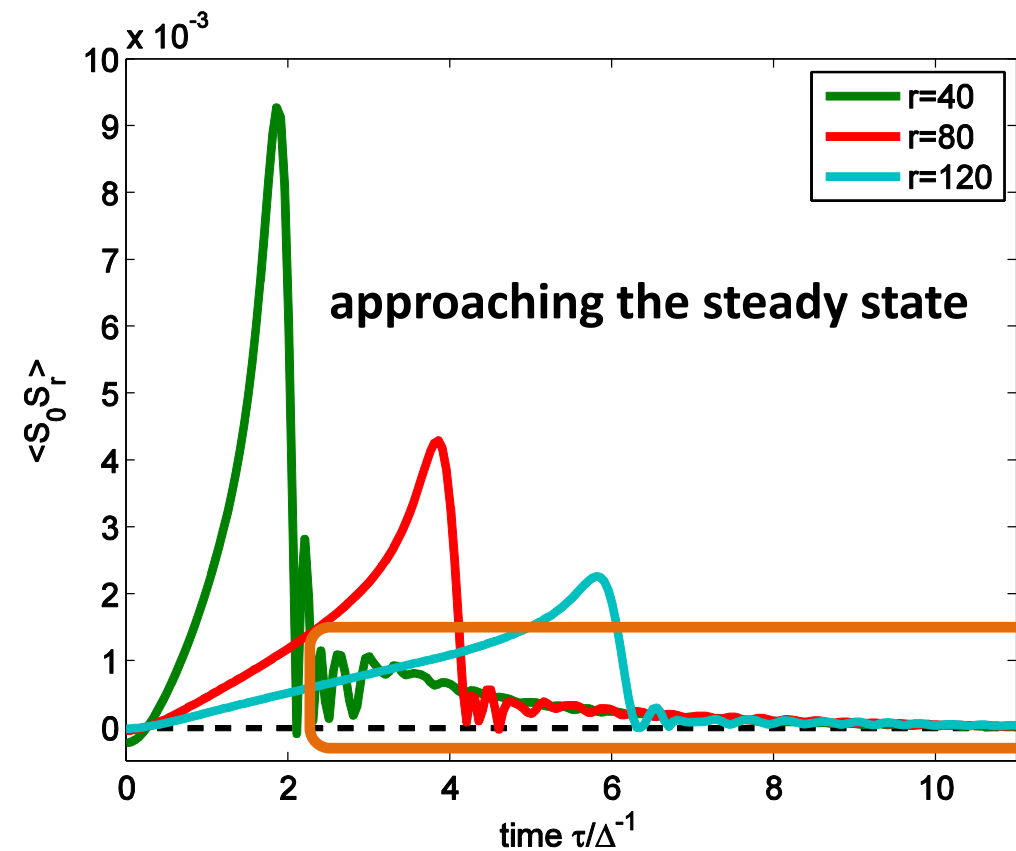
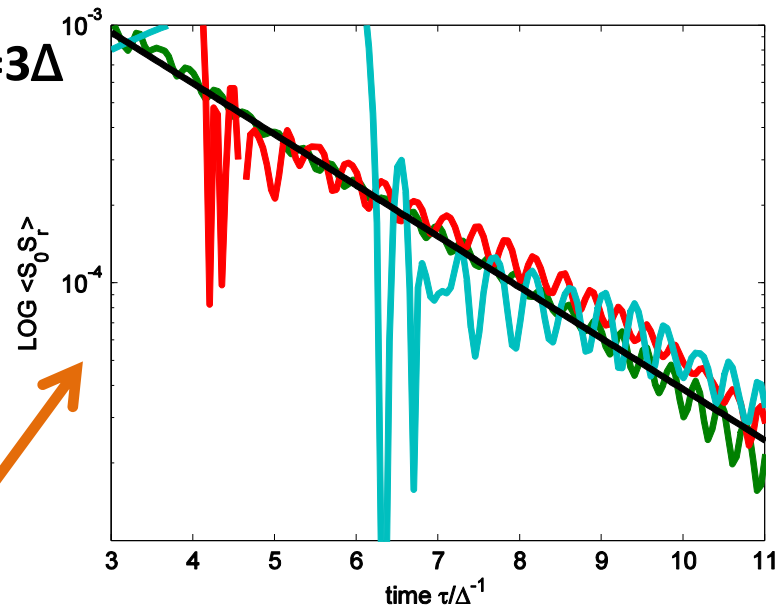
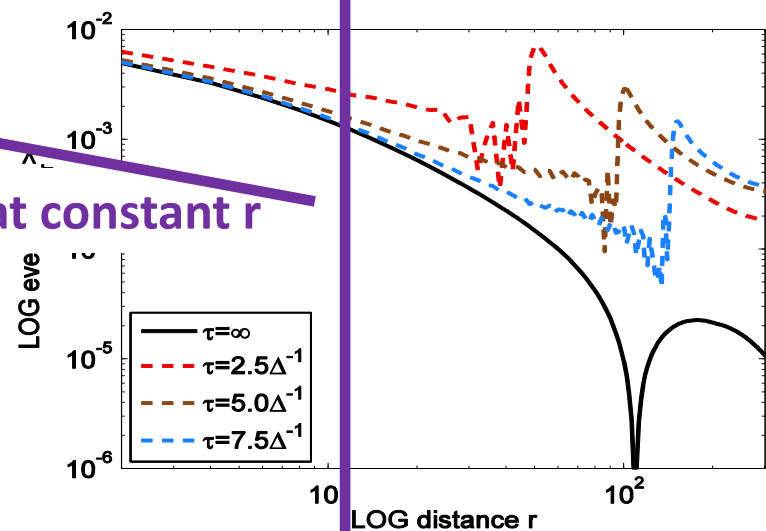
itp^{cp}

time to infinity data = equilibrium simulation!



TUG

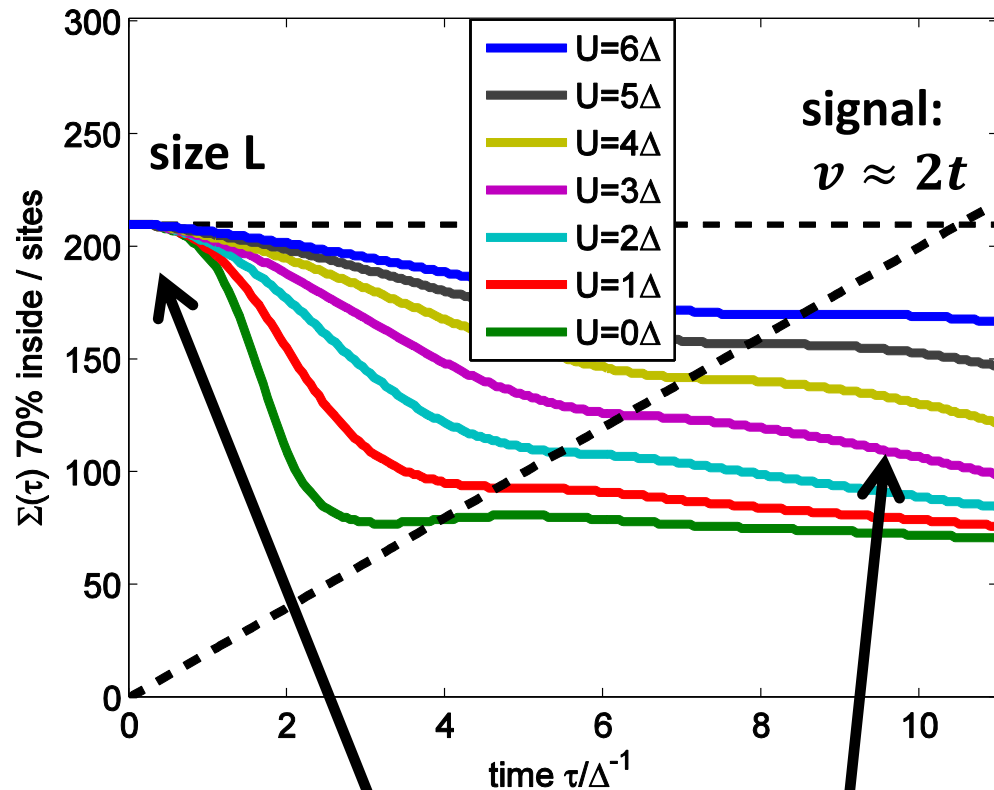
$\langle S_0 S_r \rangle(\text{time})$ envelope

itp^{cp} $U=3\Delta$ cuts at constant r 

single exponential decay:

$$\langle S_0 S_r \rangle(t) = 0.0037 e^{-0.4555 t \left[\frac{1}{\Delta} \right]}$$

Integrated Correlator



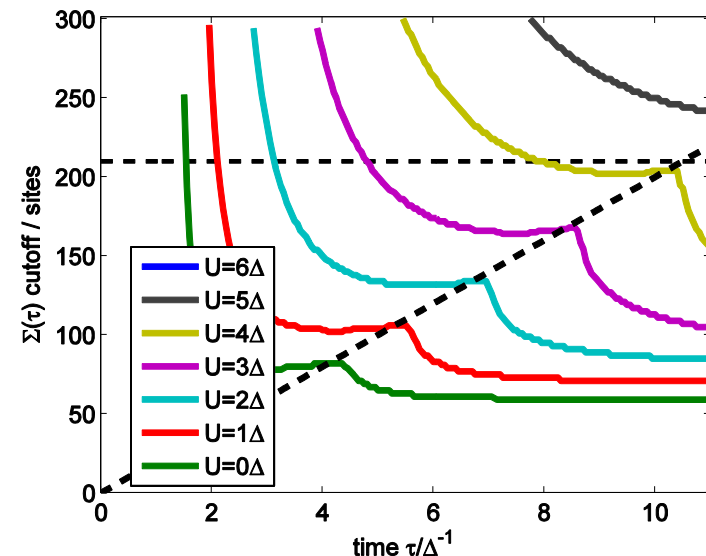
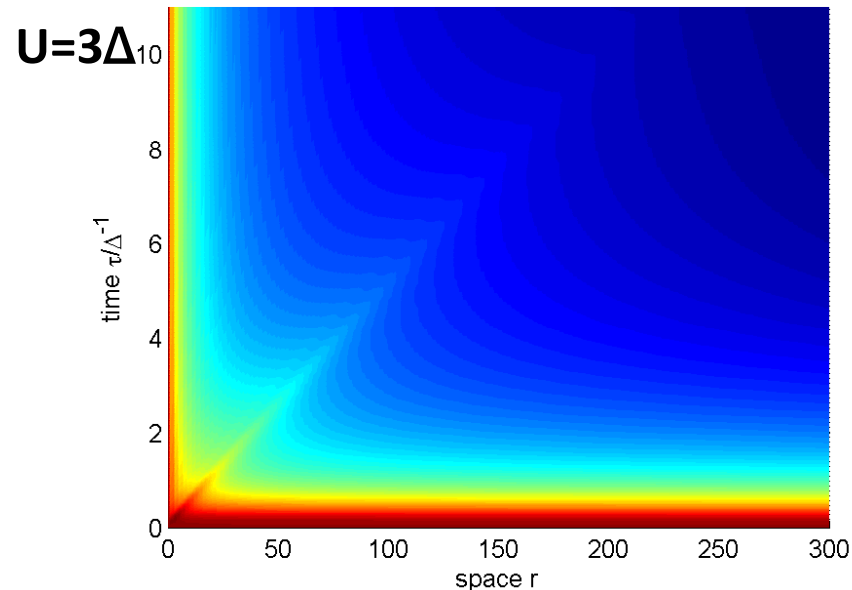
small times **X**

general trend of Kondo buildup

○ convergence at long times?

lattice velocity builds up singlet

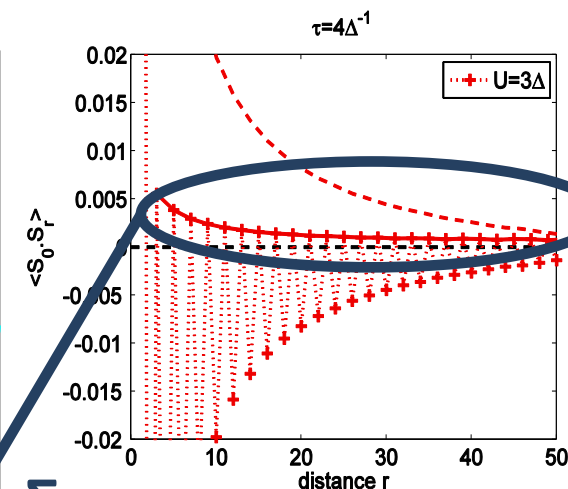
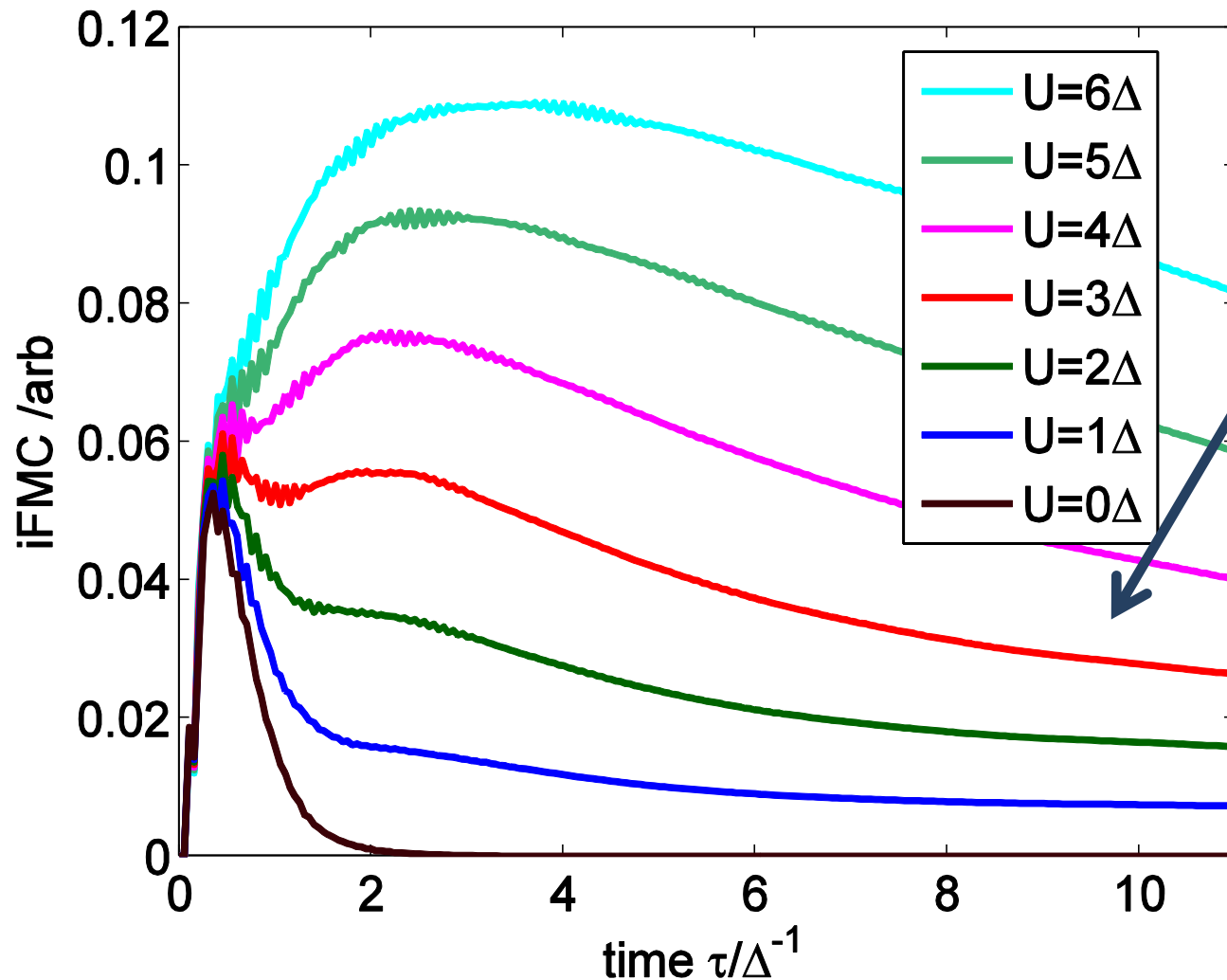
○ larger time scale?





TUG

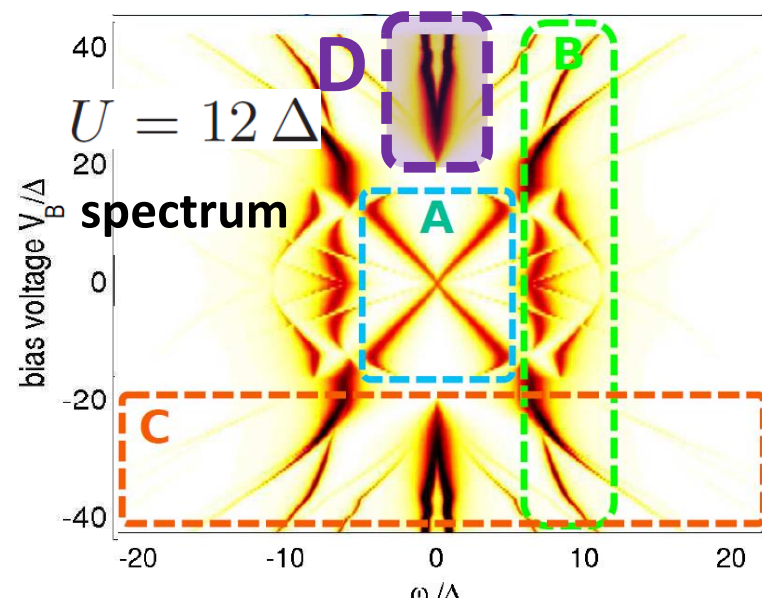
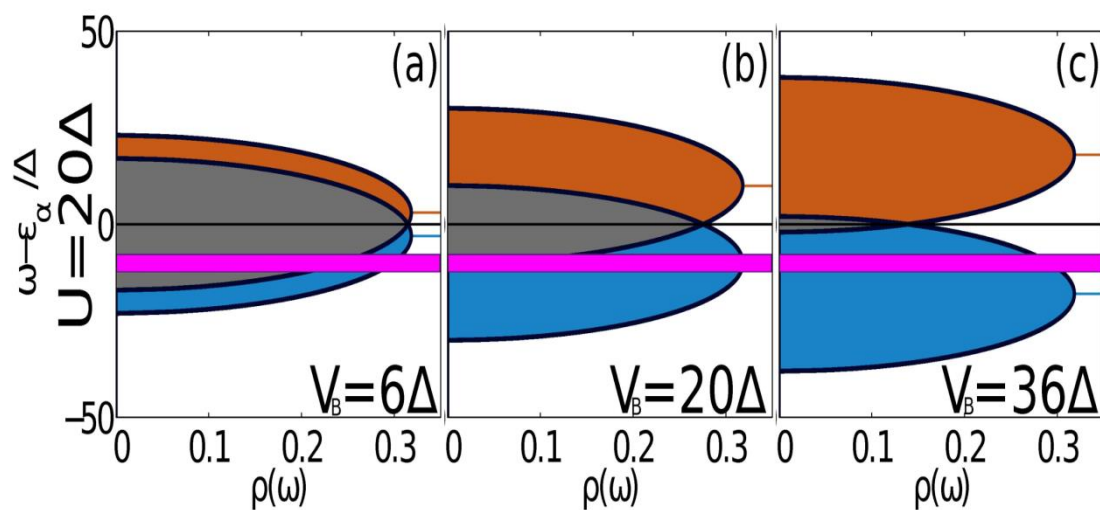
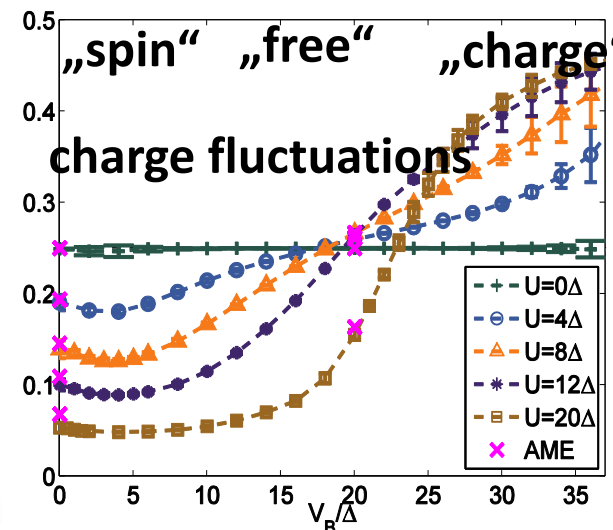
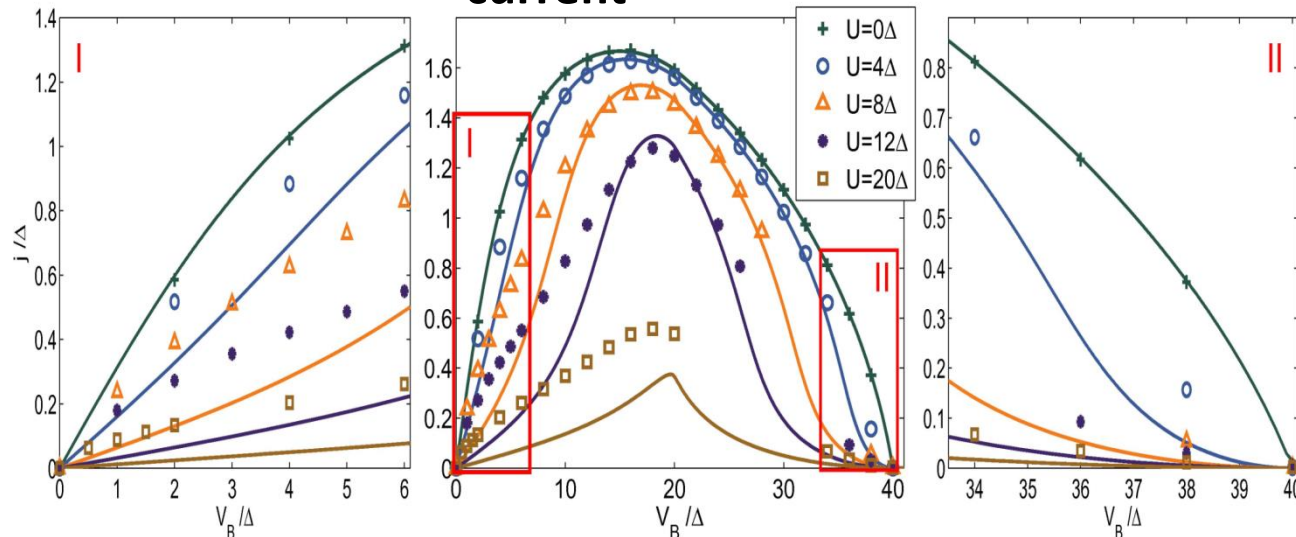
„Ferromagnetic“ Component

itp^{cp}

sum over poitive
envelope inside
light cone

Correlations & Bias

current



data from MPS, AME & nVCA



TUG

Conclusions + Outlook

itp^{cp}

• 1) real time evolution up to steady state

MN, M. Ganahl, H. G. Evertz, E. Arrigoni, W. von der Linden, PRB **88**, 045132 (2013)

MN, E. Arrigoni, W. von der Linden, AIP Conf. Proc. 1485, ISBN: 978-0-7354-1097-8 (2012)

• 2) correlations + bias voltage

MN, C. Heil, M. Ganahl, M. Knap, H. G. Evertz, E. Arrigoni, W. von der Linden, PRB **86**, 245119 (2012)

• 3) Kondo singlet

MN, M. Ganahl, A. Dorda, H. G. Evertz, E. Arrigoni, W. von der Linden, (2014)

• whats to come

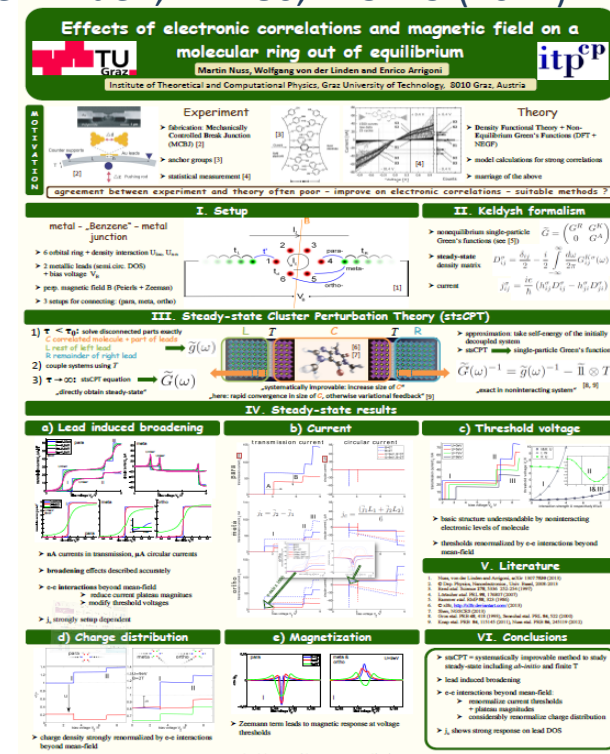
- Kondo singlet under bias
- Reverse quench

our work on molecular junctions:

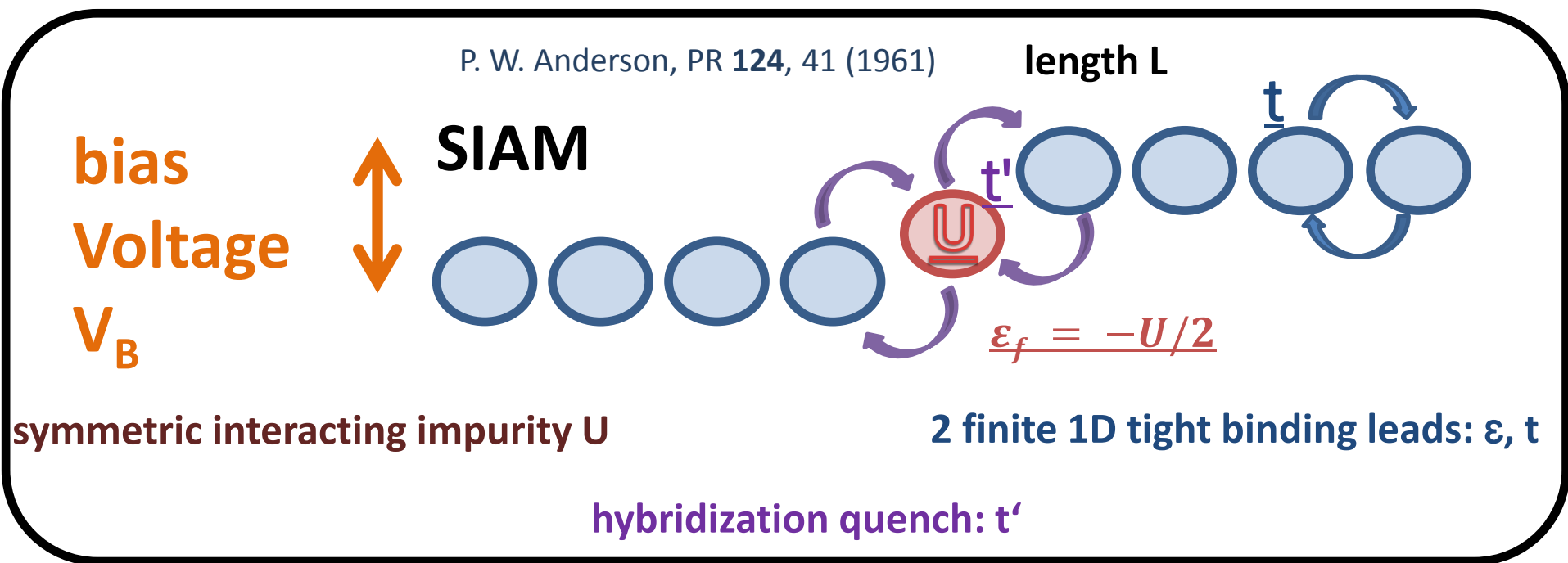
martin.nuss@tugraz.at

TT 101.14 „Transport“, Thu 15:00

MN, W. von der Linden, E. Arrigoni, arXiv:1307.7530, PRB in press (2014)



Real Time Evolution



∞ Suzuki-Trotter *real* time evolution using MPS

○ DMRG + TEBD

S. R. White, PRB **48**, 10345 (1993)

G. Vidal, PRL **93**, 040502 (2004)

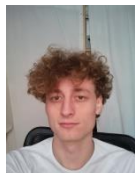
see talk by F. Heidrich-Meisner: TT2.1, Sun

∞ *unitary*: finite time + finite size

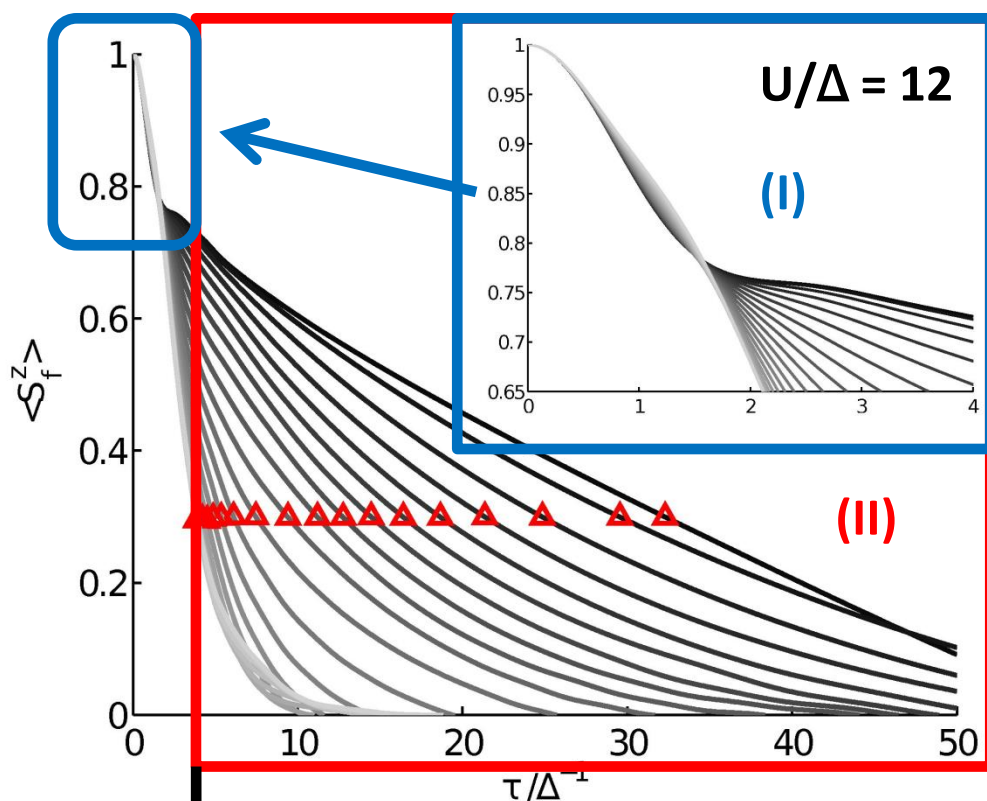
see talk by P. Schmitteckert: TT9.1, Mon

∞ massively *parallel* code

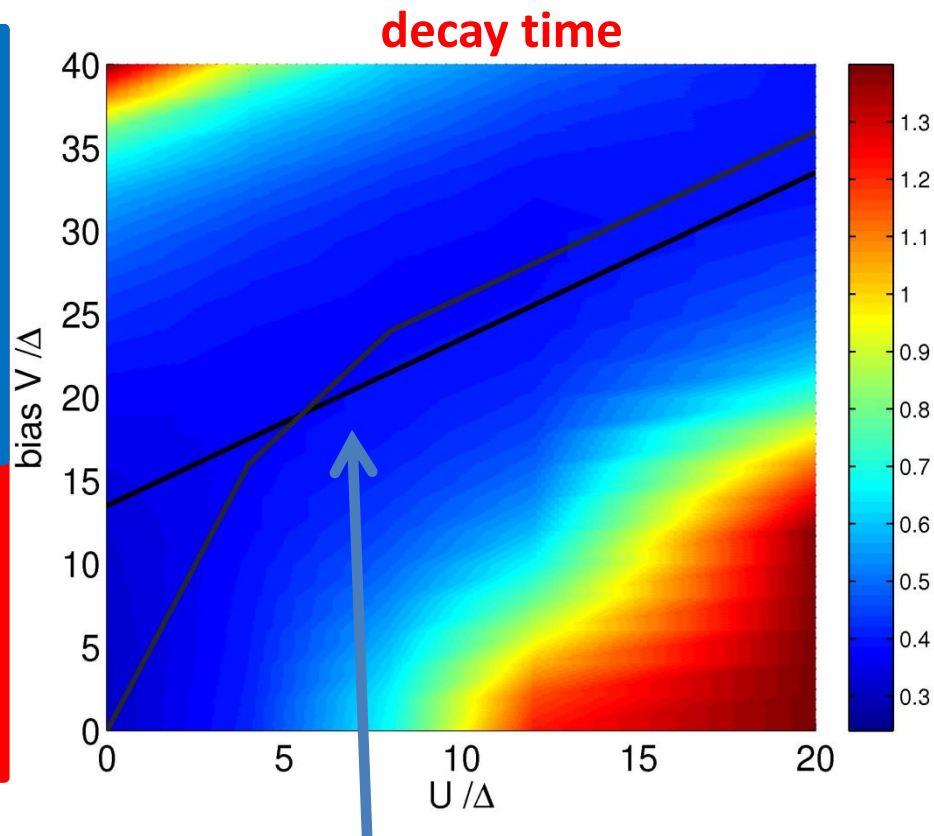
○ Martin Ganahl



Spin Relaxation



(II) $\tau_s \approx 1.7 \Delta^{-1}$ (III)
 complex single exponential



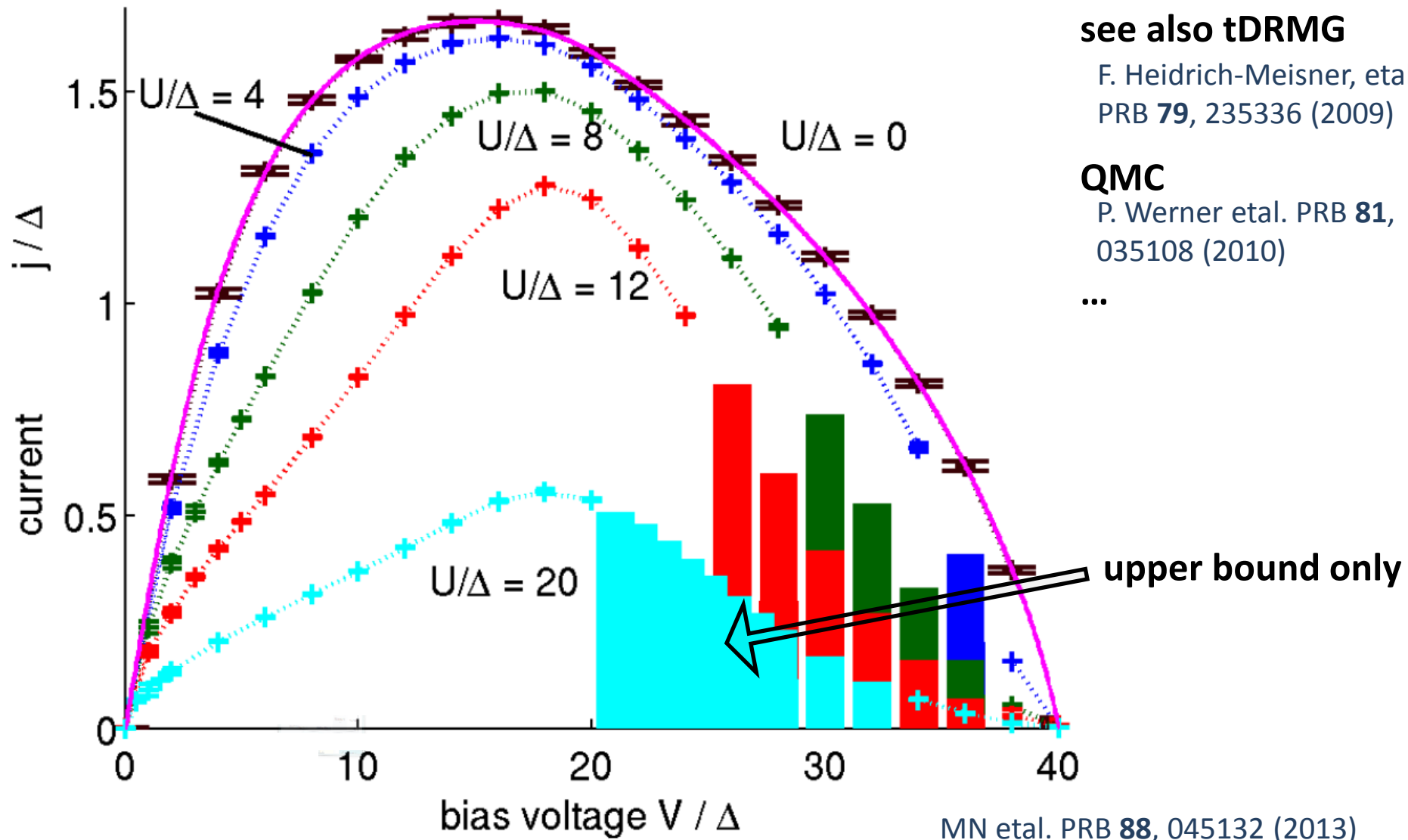
optimal relaxation: $V_c[\Delta] \approx 1.35(U[\Delta] + 1)$
 $\tau \approx (V - U - c)^4$

low bias voltage $\longrightarrow$ high bias voltage

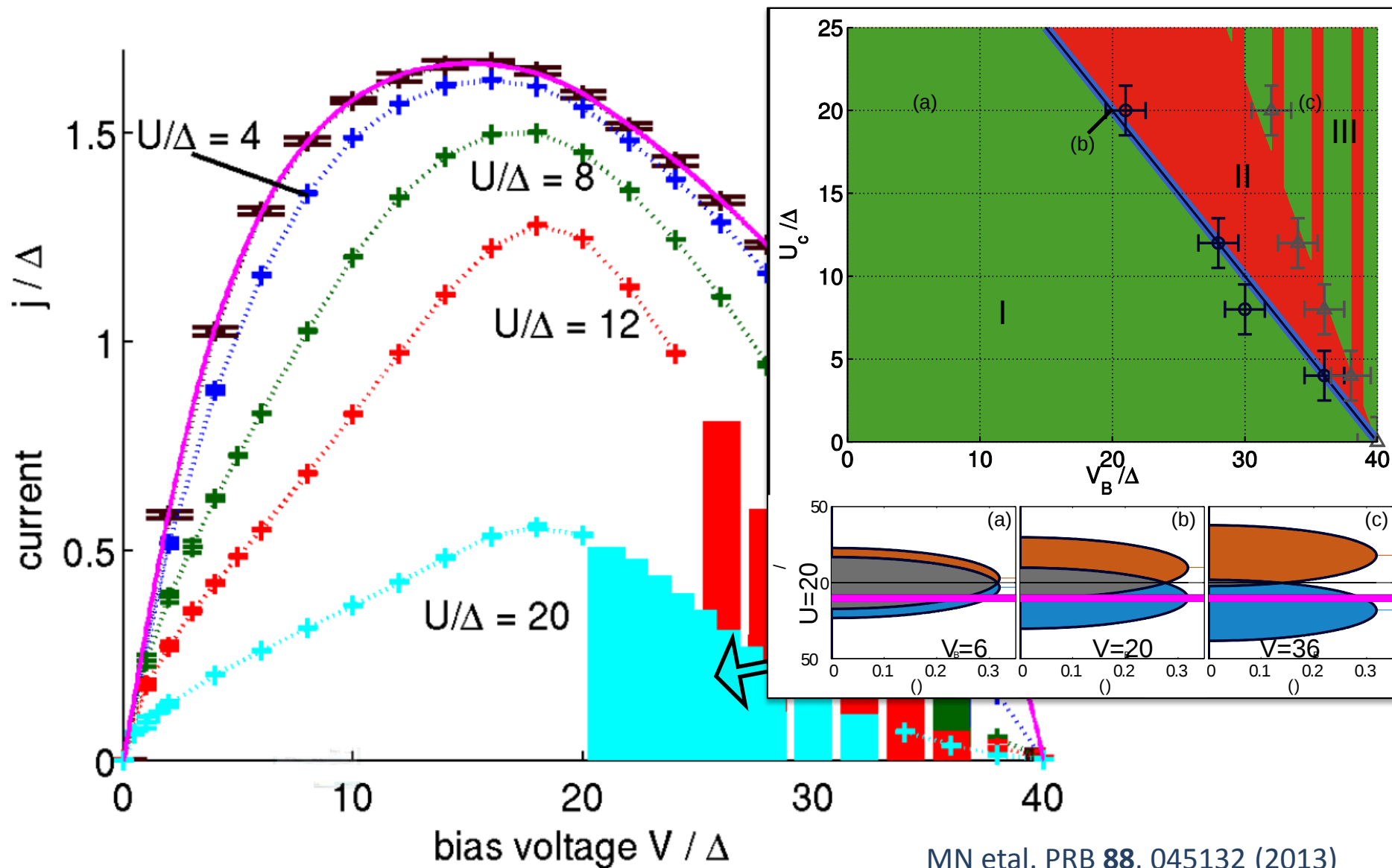
see also bold-line QMC:

G. Cohen, et al. PRB **87**, 195108 (2013)

Current - Voltage



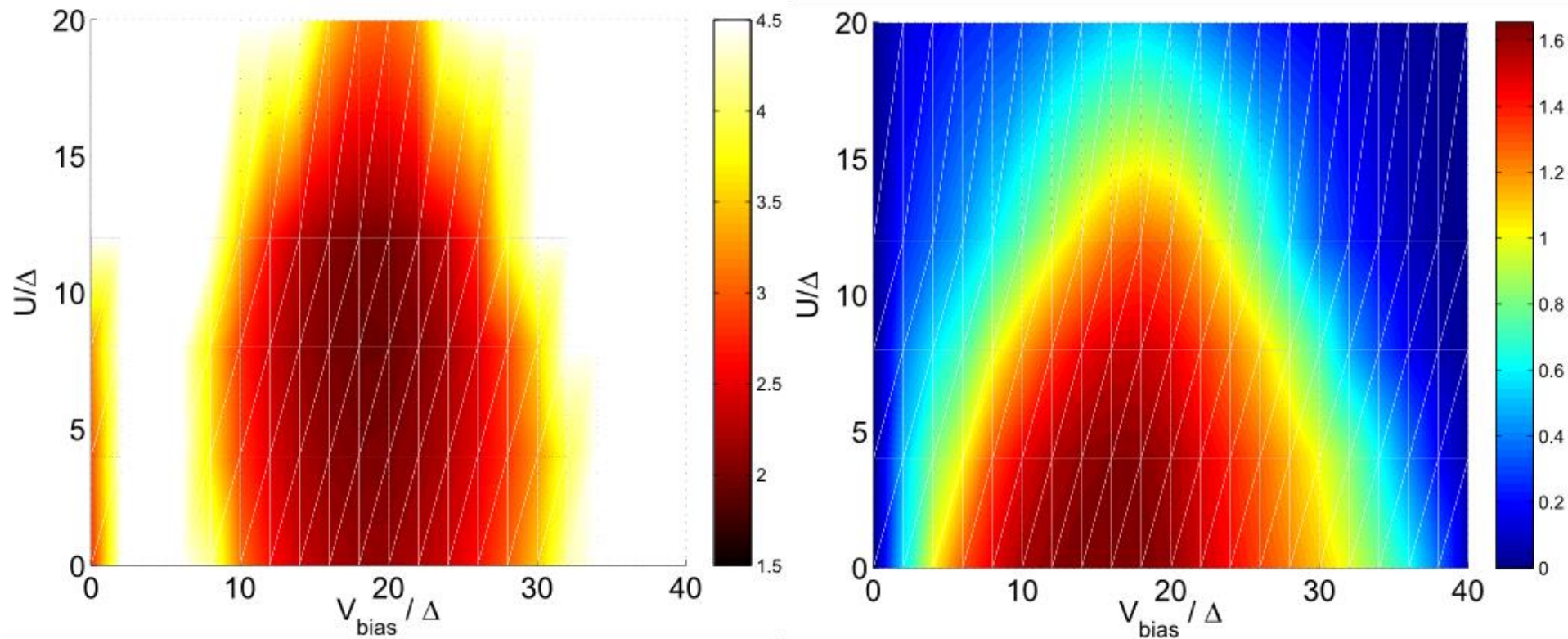
Current - Voltage





TUG

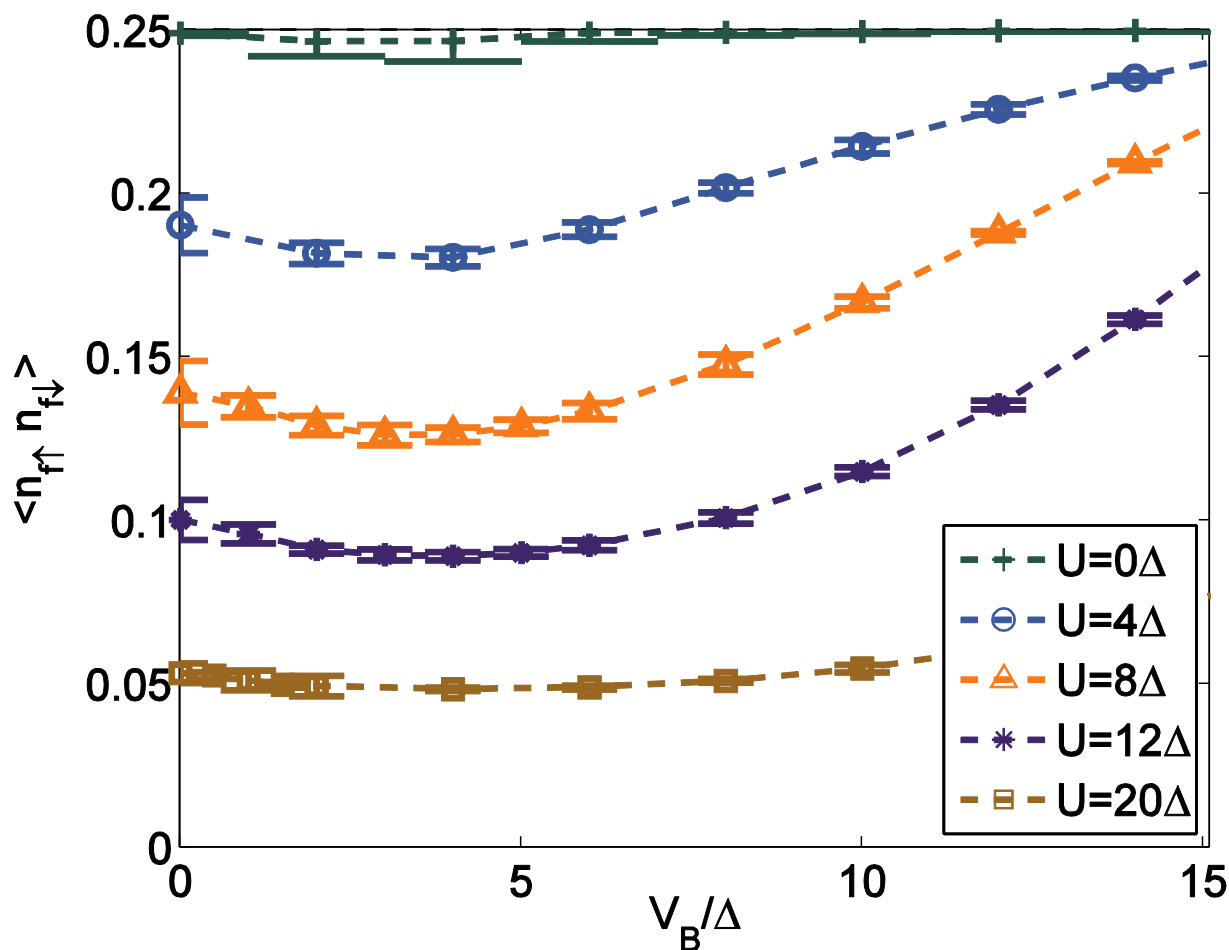
Entanglement vs Current

itp^{cp}

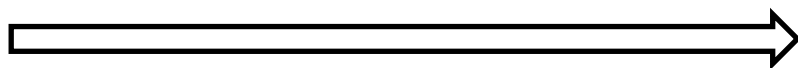
$$S_i = -\text{tr}(\hat{\rho}_L \ln(\hat{\rho}_L))$$

$$\hat{j}_{ij}(\tau) = it_{ij} \sum_{\sigma} \left(a_{i\sigma}^{\dagger} a_{j\sigma} - a_{i\sigma} a_{j\sigma}^{\dagger} \right)$$

Double Occupancy



bias voltage



charge fluctuations

$$\langle n_f \rangle = 1 \quad \langle m_f \rangle = 0$$

$$D = \langle n_{f\uparrow} n_{f\downarrow} \rangle$$

$$= \frac{1}{2} \left(\langle n_f^2 \rangle - \langle n_f \rangle^2 \right)$$

$$M = \frac{1}{4} (1 - 2D)$$

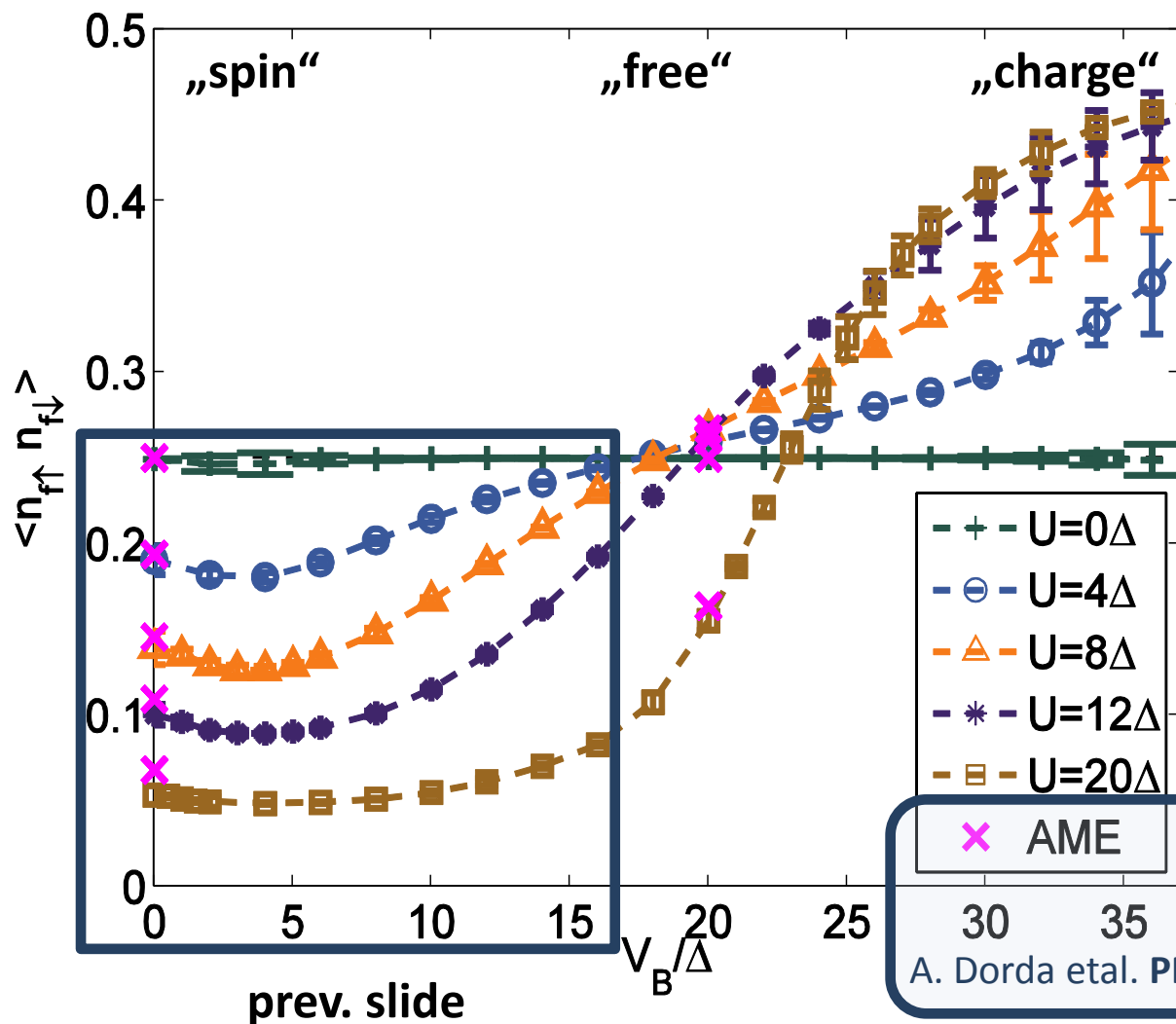
noninteracting limit

$\circ$ $D = 0.25$

for MV-QMC, RT-QMC,
SNRG see

A. Dirks et al EPL **102** 37011 (2013)

Double Occupancy



correl.	D	2M
„spin“	0 ... ¼	½ ... ¼
„free“	¼	¼
„charge“	¼ ... ½	¼ ... 0

implications for transport?

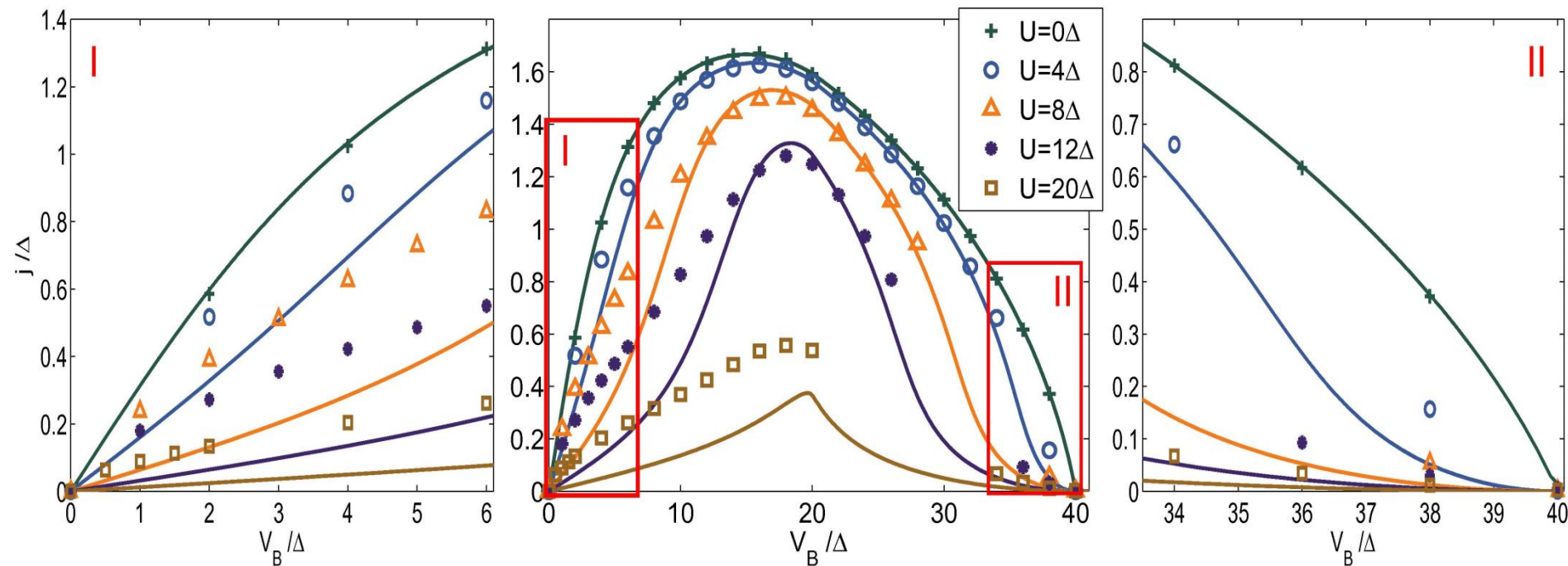
$$|\Psi\rangle \sim \sum (c_{\uparrow\downarrow} |\uparrow\downarrow\rangle_f - c_0 |0\rangle_f) \otimes |L\rangle \otimes |R\rangle$$

see talk by A. Dorda

TT96.5, Thu 16:00

A. Dorda et al. PRB in press (2014), arXiv:1312.4586

Current - Voltage



non interacting



interacting

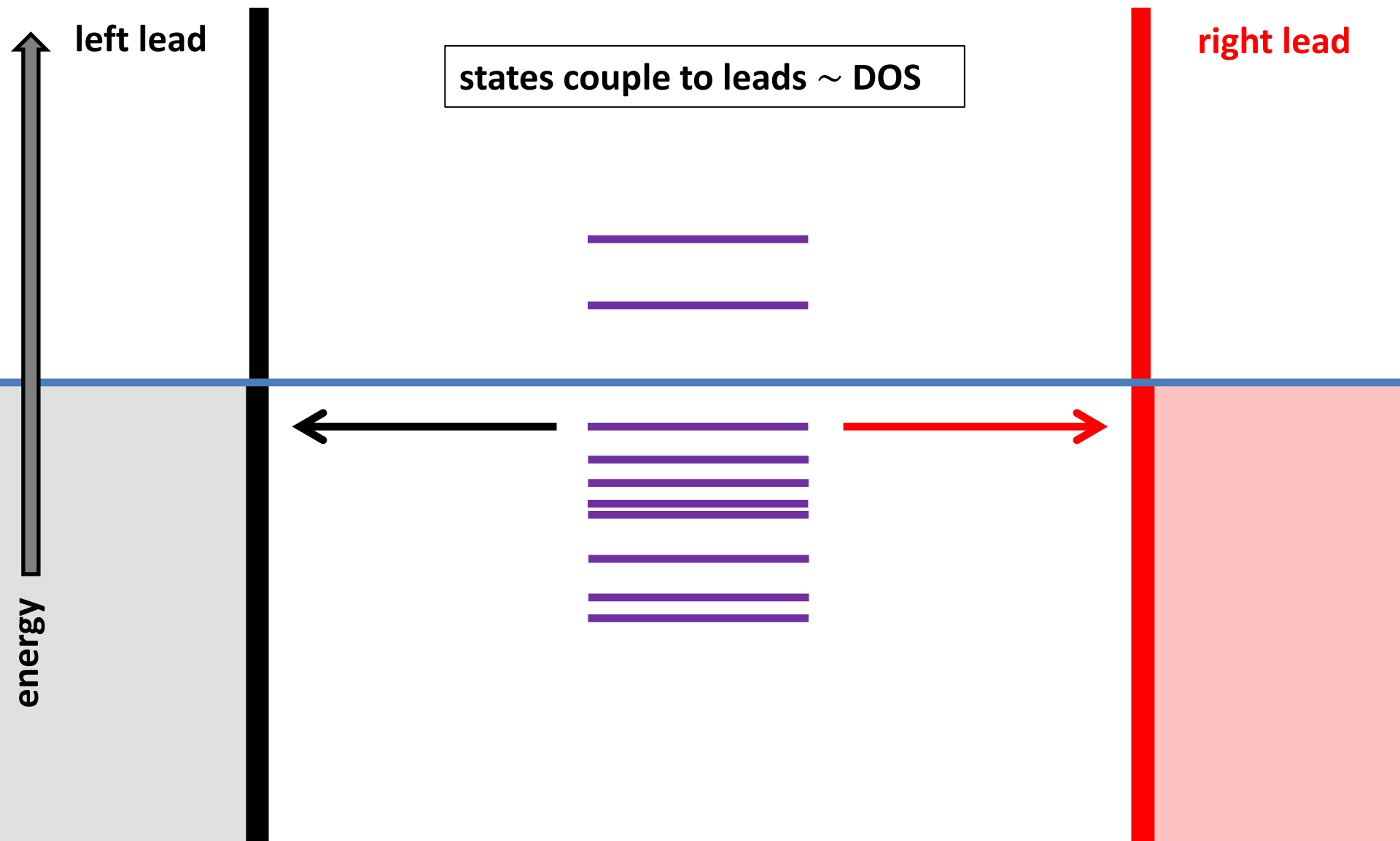


$$j = i \int_{-\infty}^{\infty} \frac{d\omega}{2\pi} ((\gamma_L(\omega) - \gamma_R(\omega)) G^<(\omega))$$

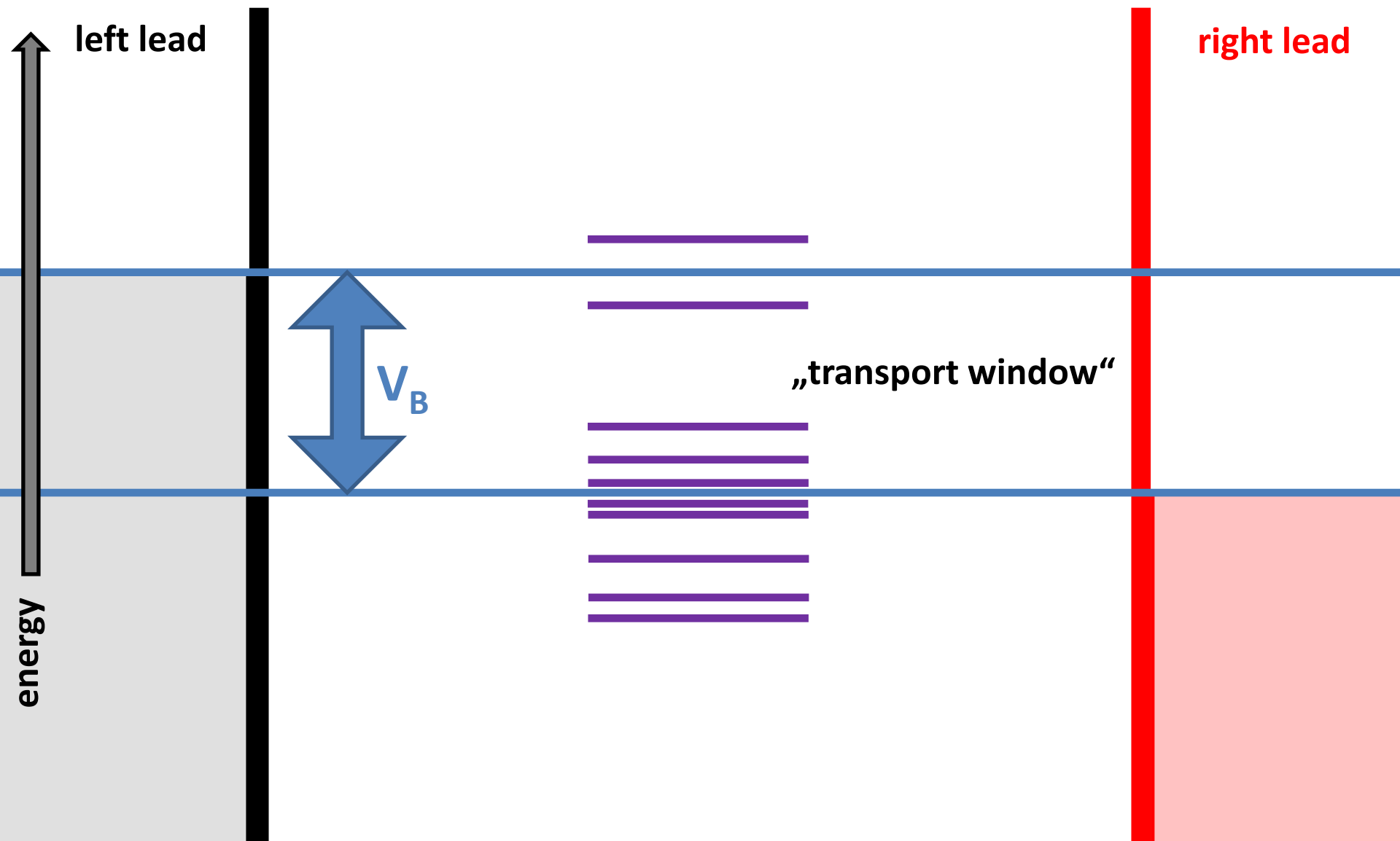
$$+ (p_{F,L}(\omega)\gamma_L(\omega) - p_{F,R}(\omega)\gamma_R(\omega)) (G^R(\omega) - G^A(\omega))$$

Meir and Wingreen, PRL **68**, 2512 (1992)Haug and Jauho, **Springer**, (1998)**Dynamics ?**

Wide Band



Wide Band



„Wide Band“ Limit

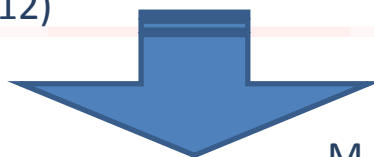


~ effective temperature T

L. Mühlbacher et al. PRB **83**, 075107 (2011)
 M. Pletyukhov et al, PRL **108**, 260601 (2012)
 A. V. Kretinin et al. PRB **85**, 201301 (2012)

~ effective magnetic field B

A. V. Kretinin et al. PRB **84**, 245316 (2011)



M. Pletyukhov et al. PRL **104**, 106801 (2010)

suppressing **electronic correlations**

correlations

NDR
 Kondo interact. med. QI

low bias

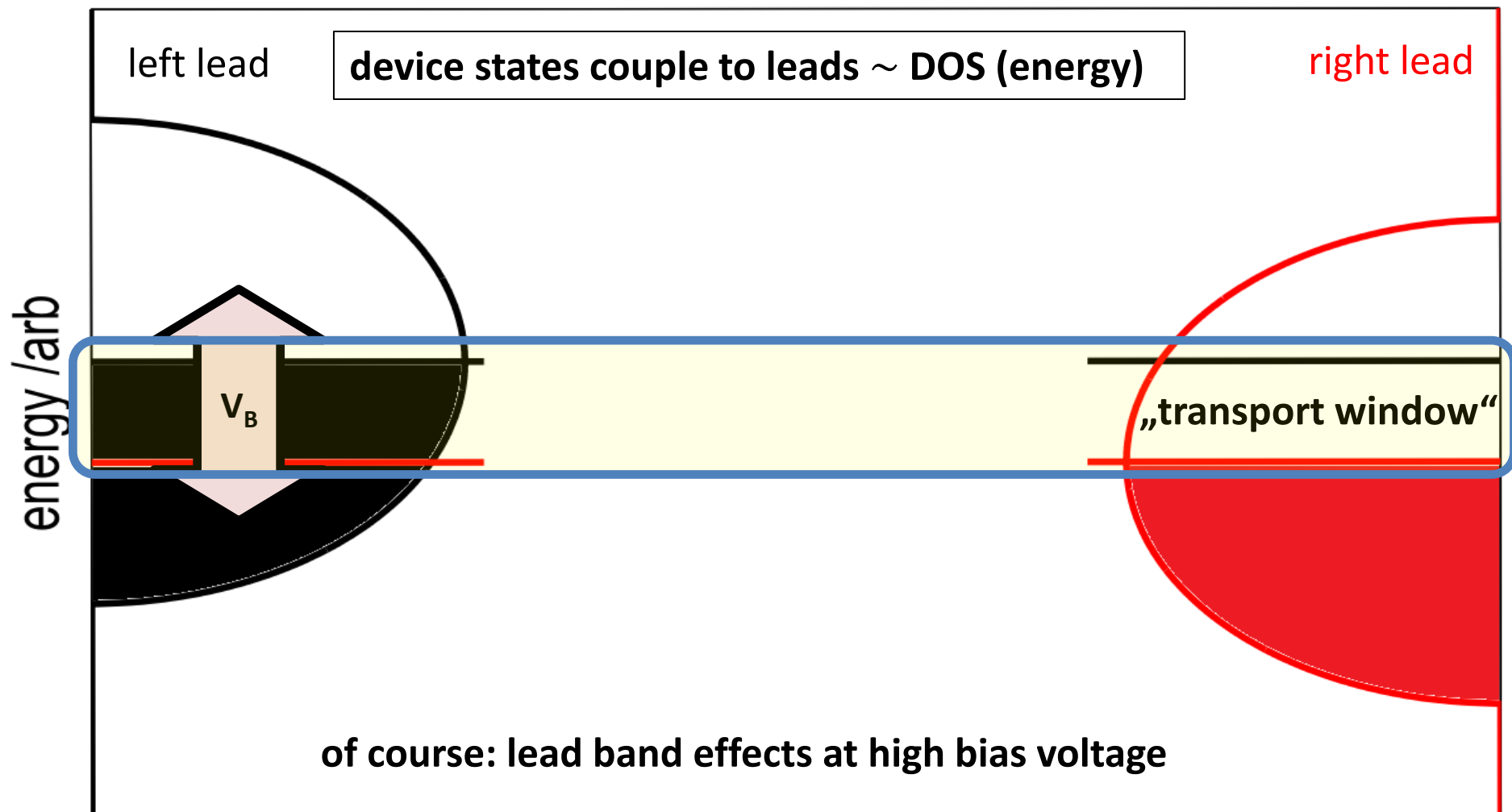
high bias

eff. free

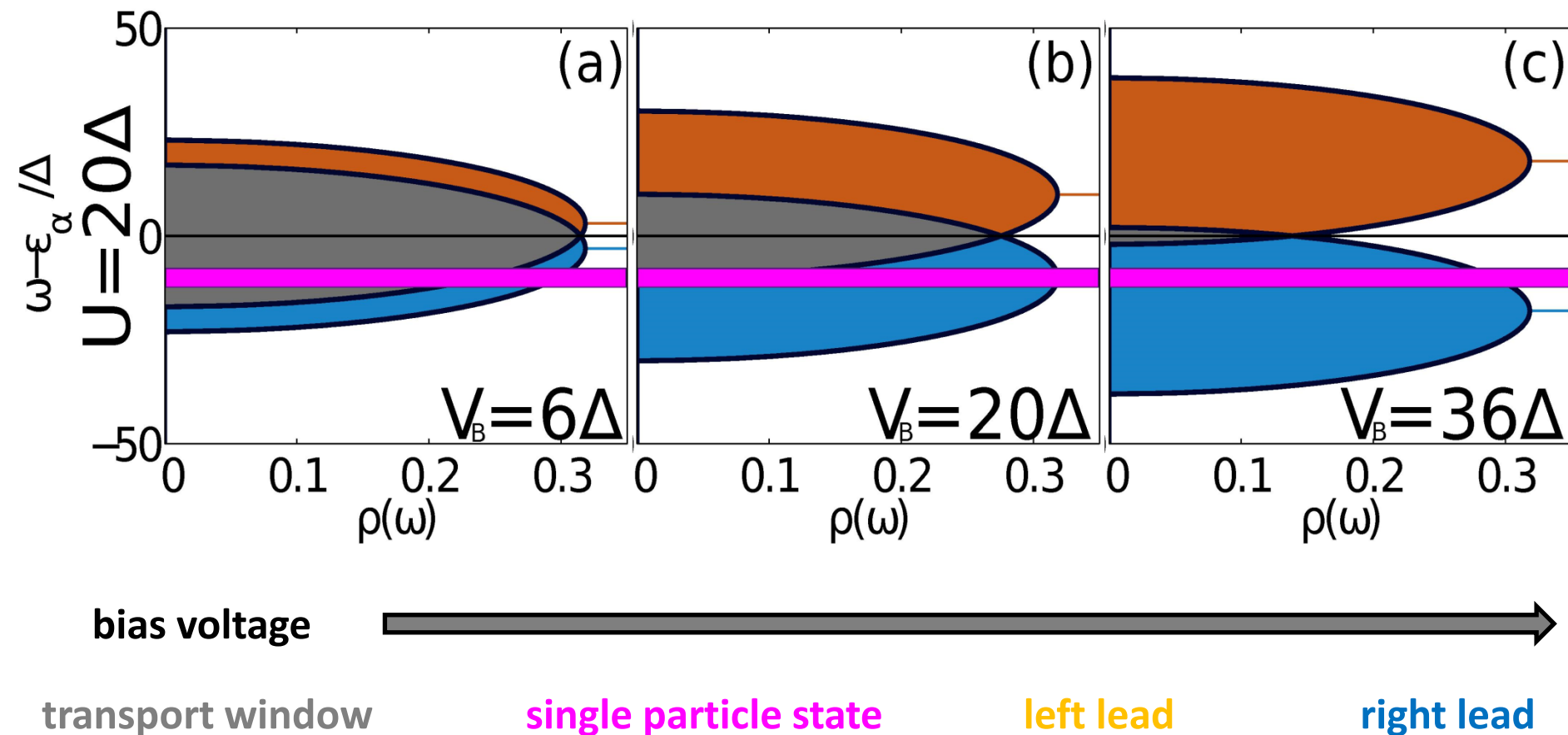
nothing interesting at high bias voltages? NO: Lead induced NDR:

Bâldea and Köppel, PRB **81**, 193401 (2010)

Lead DOS effects



Lead DOS effects

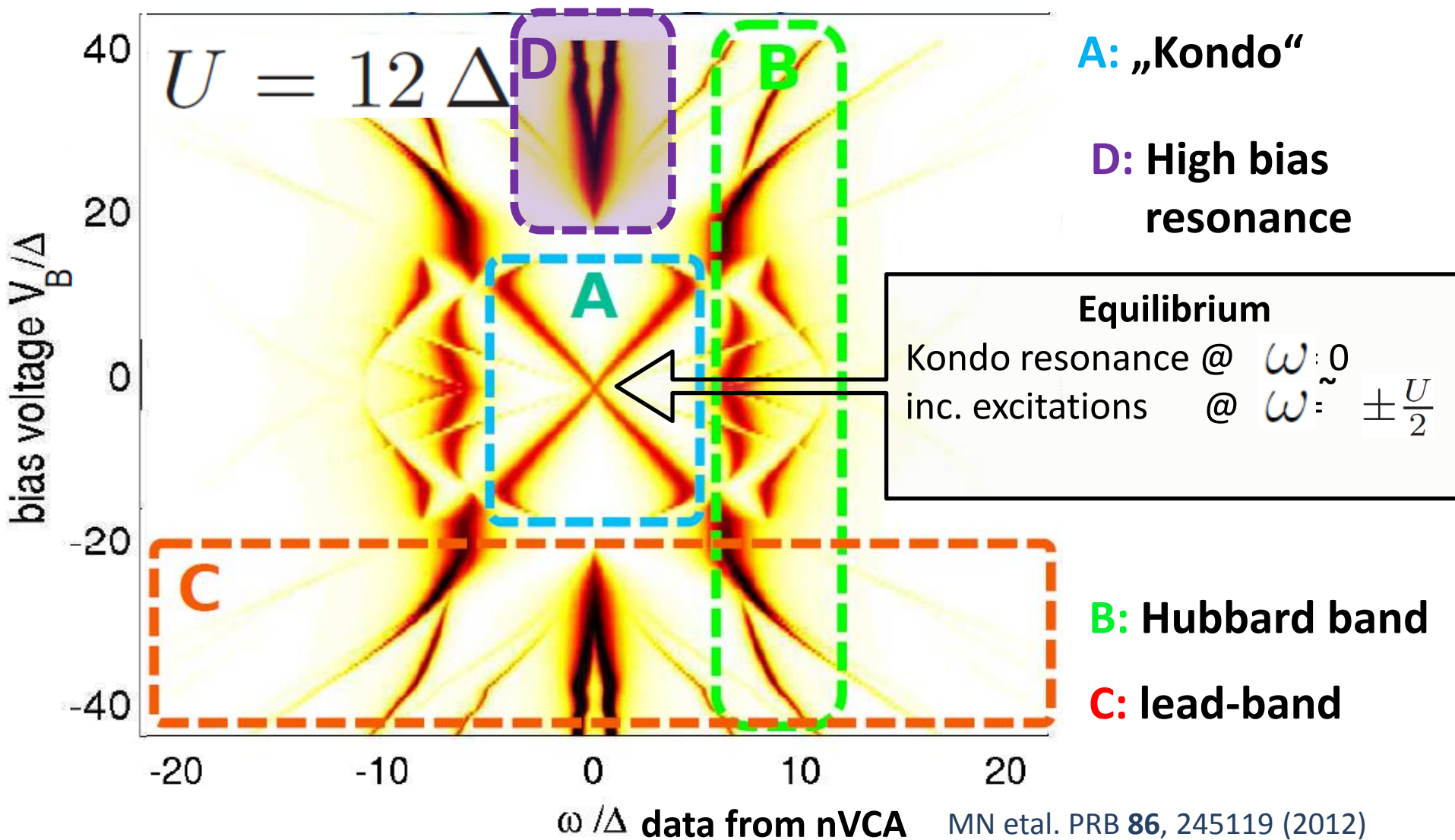


structured lead DOS can promote electronic correlations also at high bias voltage



TUG

Non Equilibrium Spectral Function

itp^{cp}

Prof. Wolfgang von der Linden



Prof. Enrico Arrigoni



Non equilibrium group at



Prof. Hans Gerd Evertz



Martin Ganahl



Markus Aichhorn



Benjamin Kollmitzer

Christoph Heil



Michael Knap



Anna Fulterer



Max Sorantin



Delia Fugger



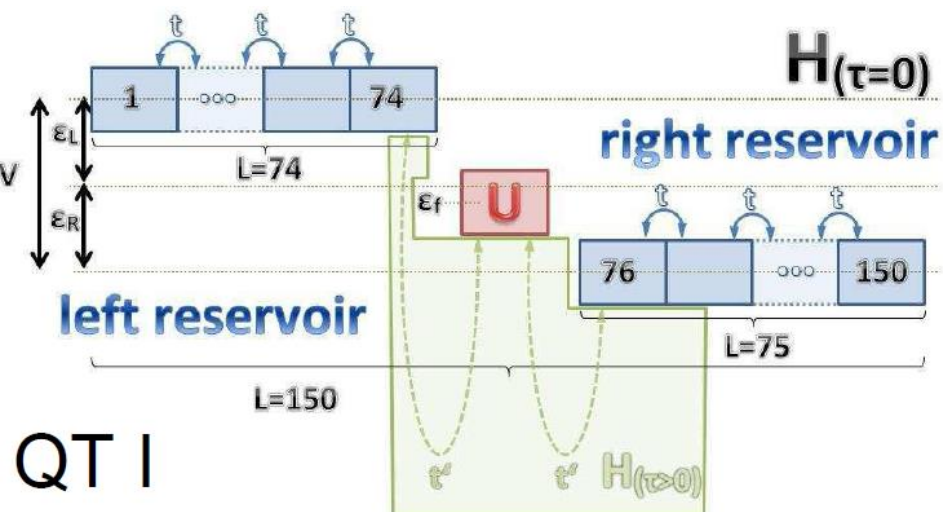
Antonius Dorda



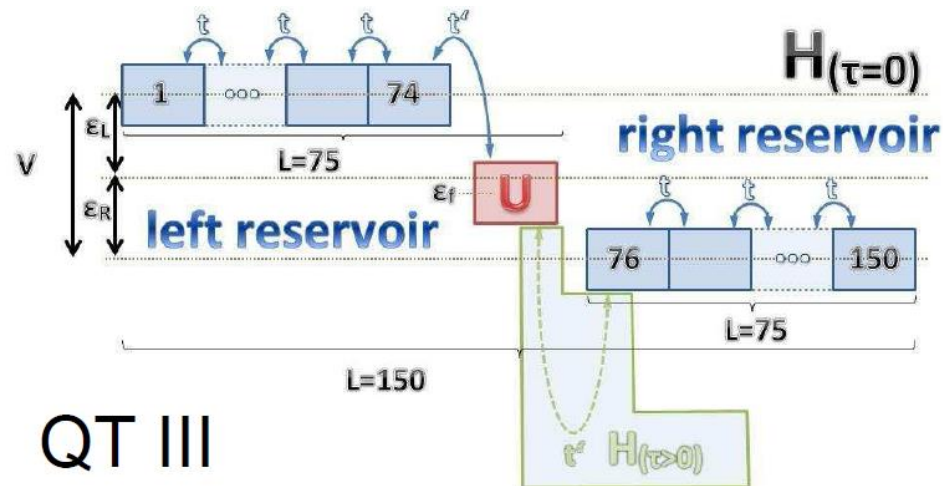
Martin Nuss



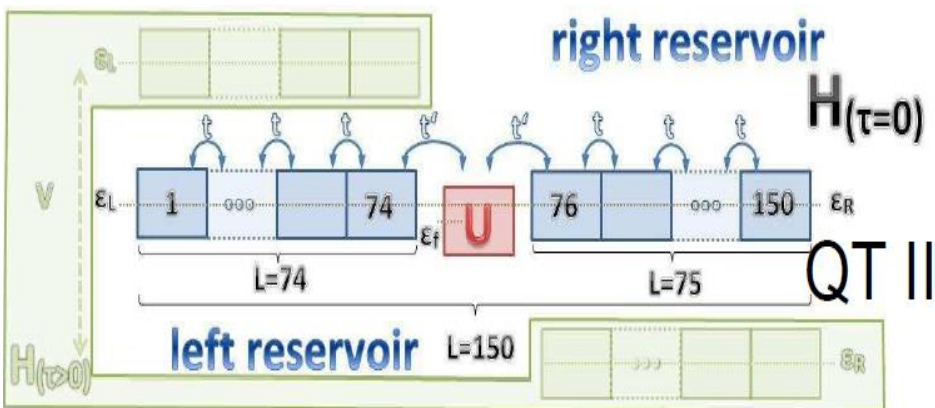
3 Quneches



QT I



QT III



QT II

comparison of short time quench
dynamics
+ quasi-steady-state

Time evolution of Current

