

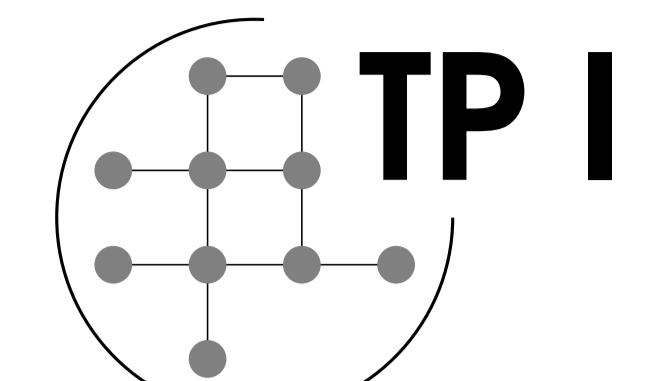


# Variational Cluster Approach to antiferromagnetic and superconducting phases in hole- and electron-doped cuprates

M. Aichhorn<sup>1</sup>, E. Arrigoni<sup>2</sup>, M. Potthoff<sup>1</sup>, and W. Hanke<sup>1</sup>

<sup>1</sup>Institut für Theoretische Physik und Astrophysik, Universität Würzburg, Germany

<sup>2</sup>Institut für Theoretische Physik - Computational Physics, Technische Universität Graz, Austria



Lehrstuhl für  
Theoretische Physik I  
Universität Würzburg

## Statement of the problem - Model

The high-temperature superconductors (HTSC) and other strongly correlated electron systems, such as heavy-fermion and a variety of transition-metal oxide systems, exhibit many ordered phases, which appear to compete and sometimes coexist.

- Antiferromagnetic (AF) order
- Superconducting (SC) order
- Coexisting AF and SC orders
- Charge- and Spin-“stripe” phases

Description on a microscopic level:

Single-band Hubbard model:

$$H = \sum_{ij\sigma} t_{ij} c_{i\sigma}^\dagger c_{j\sigma} + U \sum_i n_{i\uparrow} n_{i\downarrow} - \mu \sum_i (n_{i\uparrow} + n_{i\downarrow})$$

• Hopping integrals:

– n.n. hopping  $t = 1$  (unit of energy)

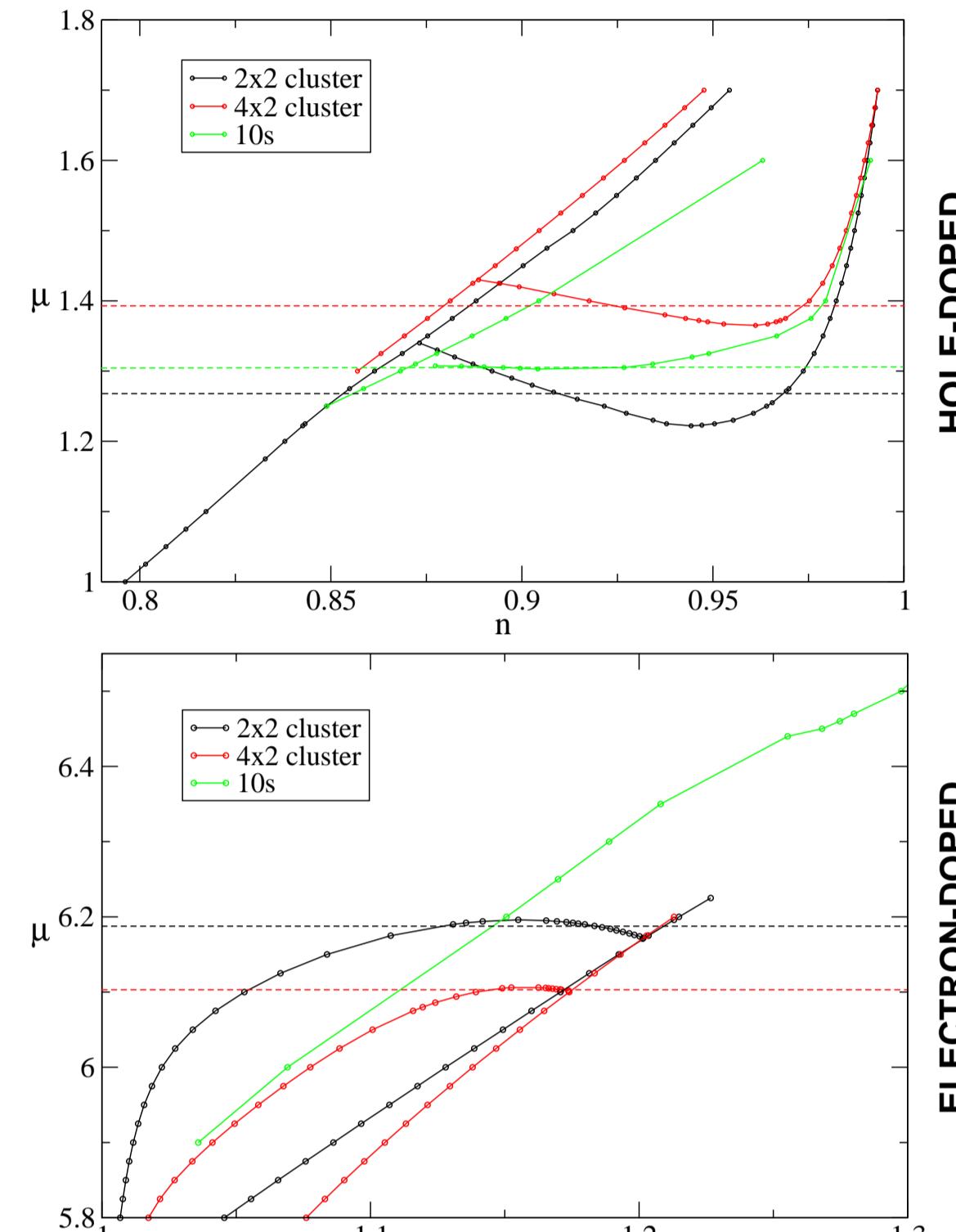
– n.n.n. hopping  $t' = -0.3t$

• Hubbard interaction  $U = 8t$

• Chemical potential  $\mu$ : Filling control

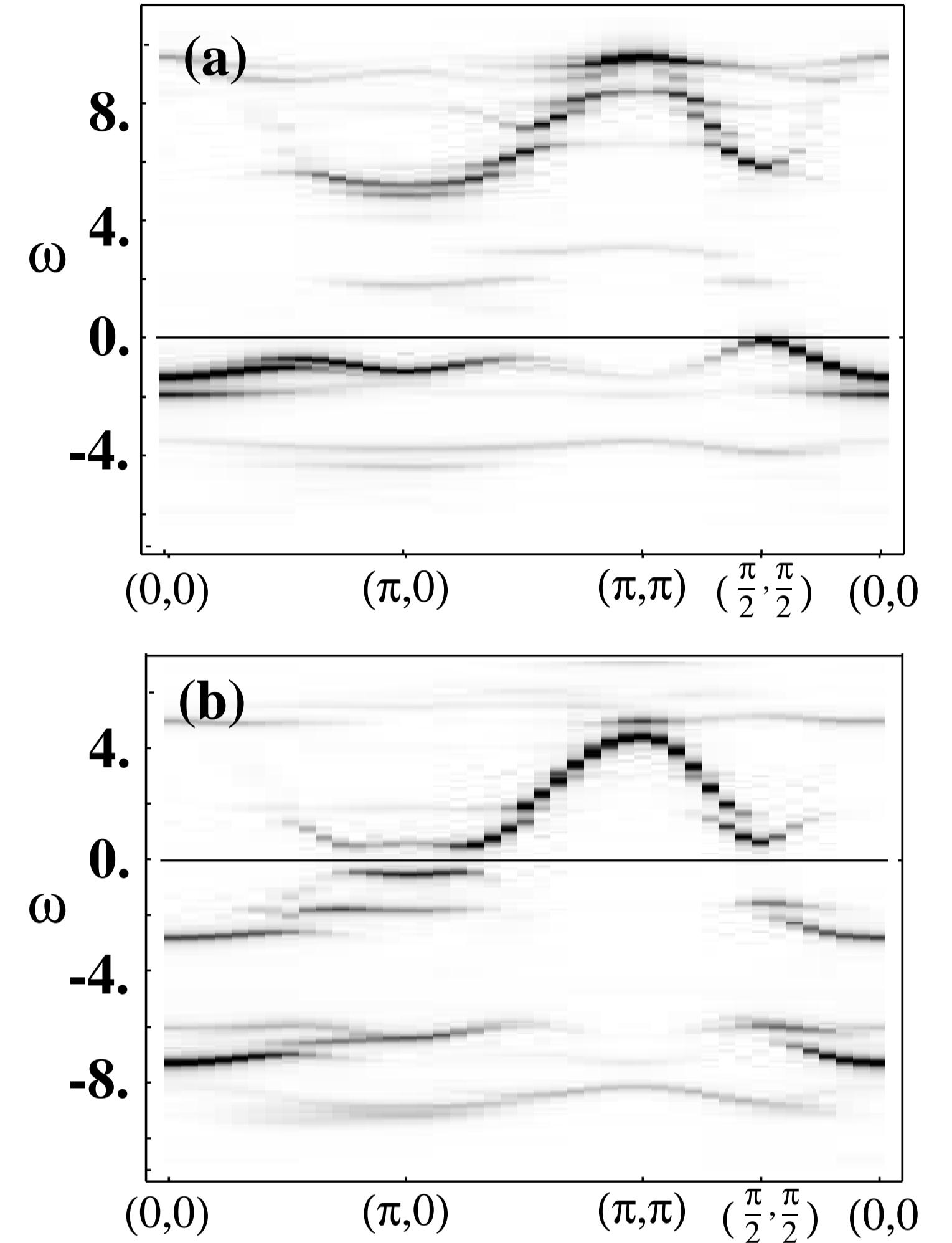
## Competing AF and SC orders

Small reference systems show tendency to phase separation.



Chemical potential  $\mu$  as function of the electron density  $n$ .  
Dotted lines  $\equiv$  critical  $\mu$ . Maxwell construction for phase separation.

## Spectral functions



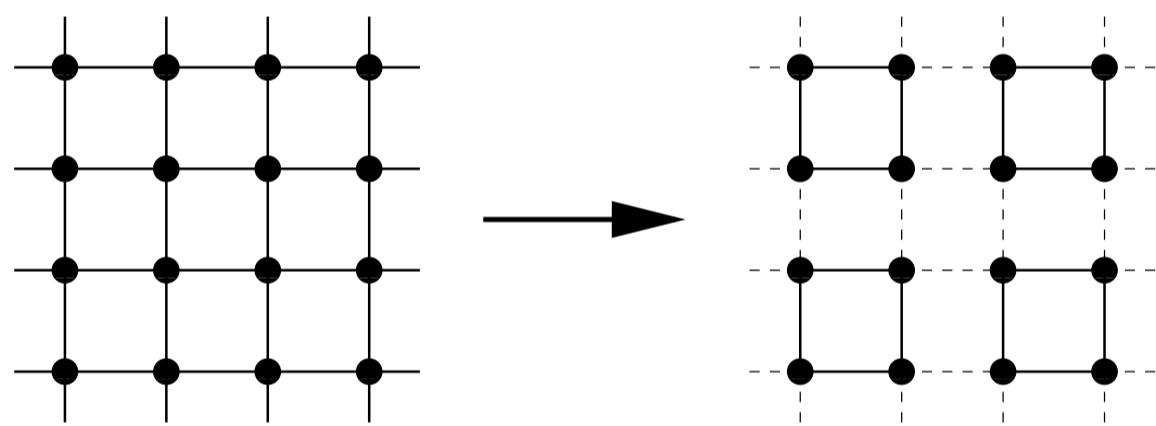
Spectral functions just before the transition to a non-magnetic state:

Upper panel: Hole doping,  $x \approx 0.03$

Lower panel: Electron doping,  $x \approx 0.13$

## Method - Variational Cluster Approach

First step: Tiling of the lattice into clusters of finite size:



Finite clusters  $\equiv$  Reference systems for SFA

The main property of the SFA/VCA method is the Grand potential:

$$\Omega(\Delta) = \Omega'(\Delta) + T \sum_{\omega_n, Q} \text{tr} \ln \frac{-1}{G_Q^{(0)}(i\omega_n)^{-1} - \Sigma(\Delta, i\omega_n)} - LT \sum_{\omega_n} \text{tr} \ln (-G'(\Delta, i\omega_n)).$$

Exact quantities (reference system):  $\Omega'(\Delta)$ ,  $\Sigma(\Delta, i\omega_n)$ , and  $G'(\Delta, i\omega_n)$

Variational parameters  $\Delta$ :

Staggered magnetic field,  $d$ -wave pairing field, onsite potential  $\Delta\mu'$

Variational principle:

$$\text{Physical solution } G \Leftrightarrow \frac{\partial \Omega}{\partial \Delta_i} = 0$$

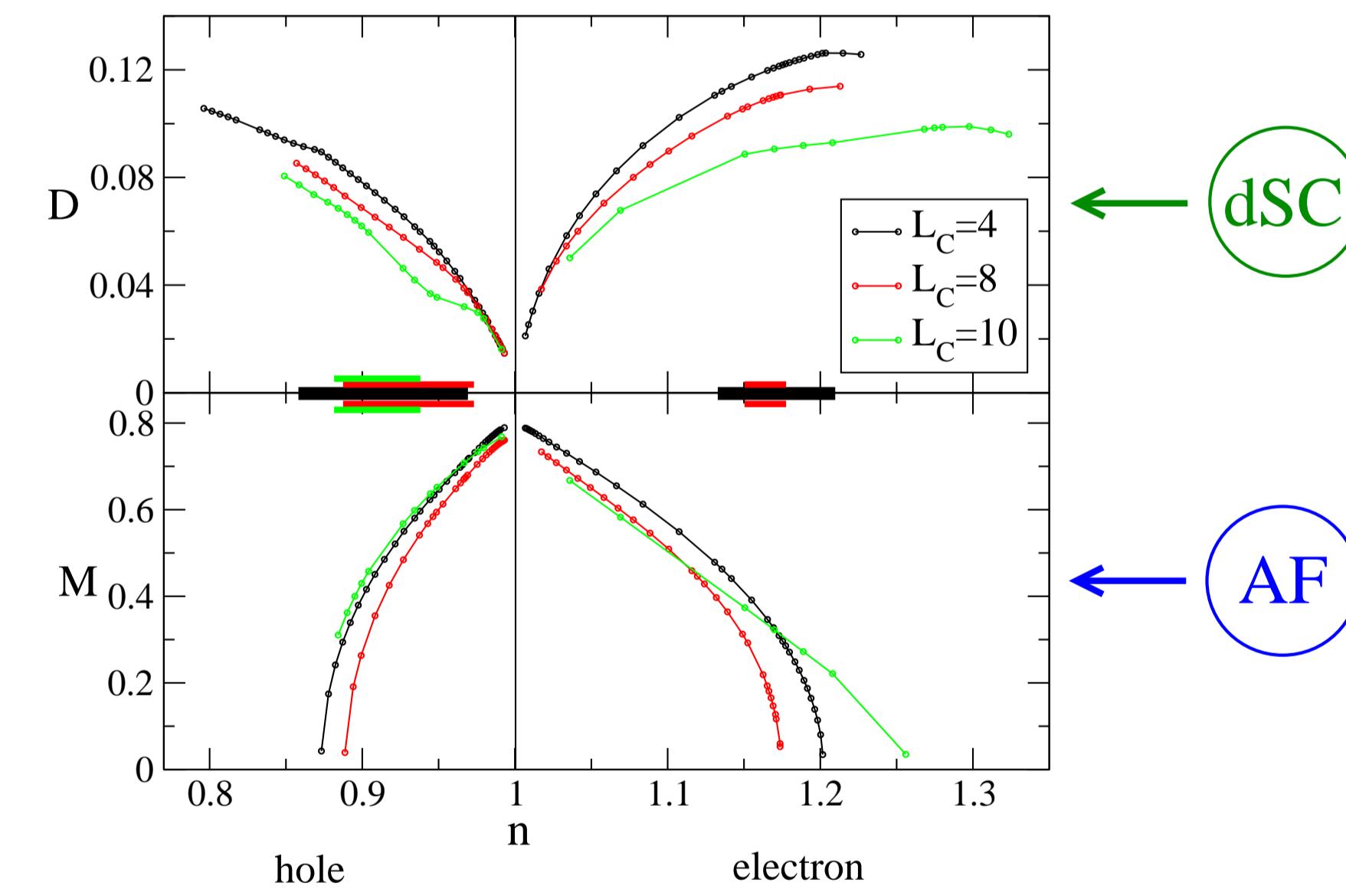
## Order parameters

Antiferromagnetic order parameter:

$$M = -\frac{1}{\pi} \text{Im} \frac{1}{L} \sum_{\mathbf{k}} \int d\omega \sum_{\sigma} (-1)^{\sigma} G_{\sigma}(\mathbf{k}, \mathbf{k} + \mathbf{Q}, \omega), \quad \mathbf{Q} = (\pi, \pi)$$

Superconducting order parameter:

$$D = -\frac{1}{\pi} \text{Im} \frac{1}{L} \sum_{\mathbf{k}} \int d\omega g(\mathbf{k}) F(\mathbf{k}, \mathbf{k}, \omega), \quad g(\mathbf{k}) = \cos(k_x) - \cos(k_y)$$



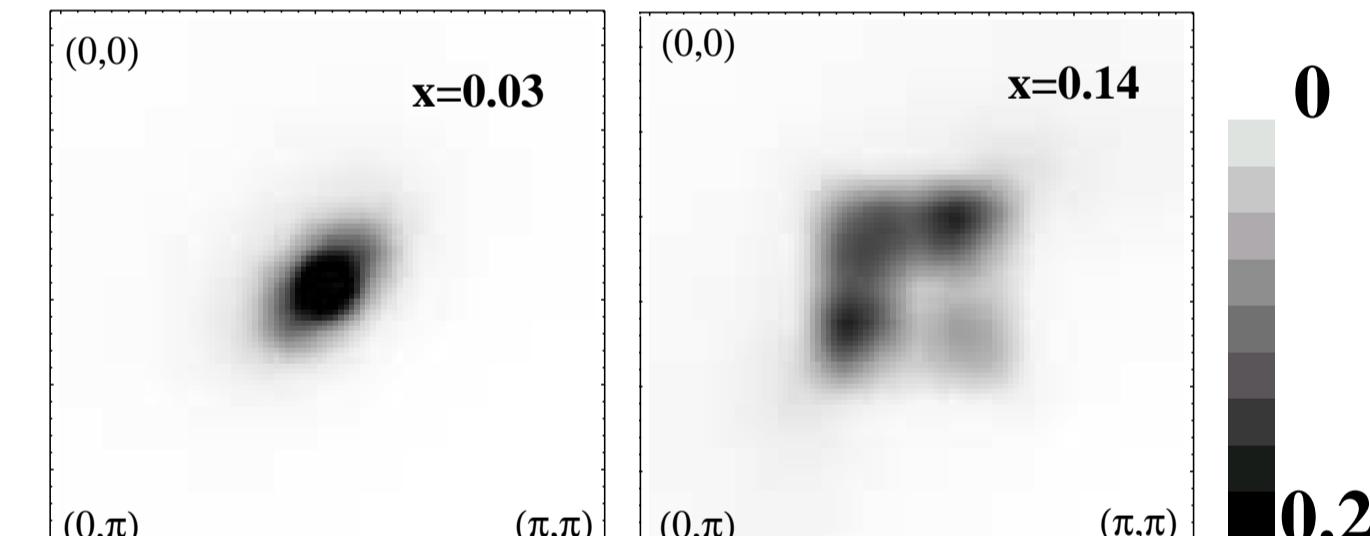
Bars mark the regions of Phase Separation.

Order parameters show coexisting SC+AF phases!

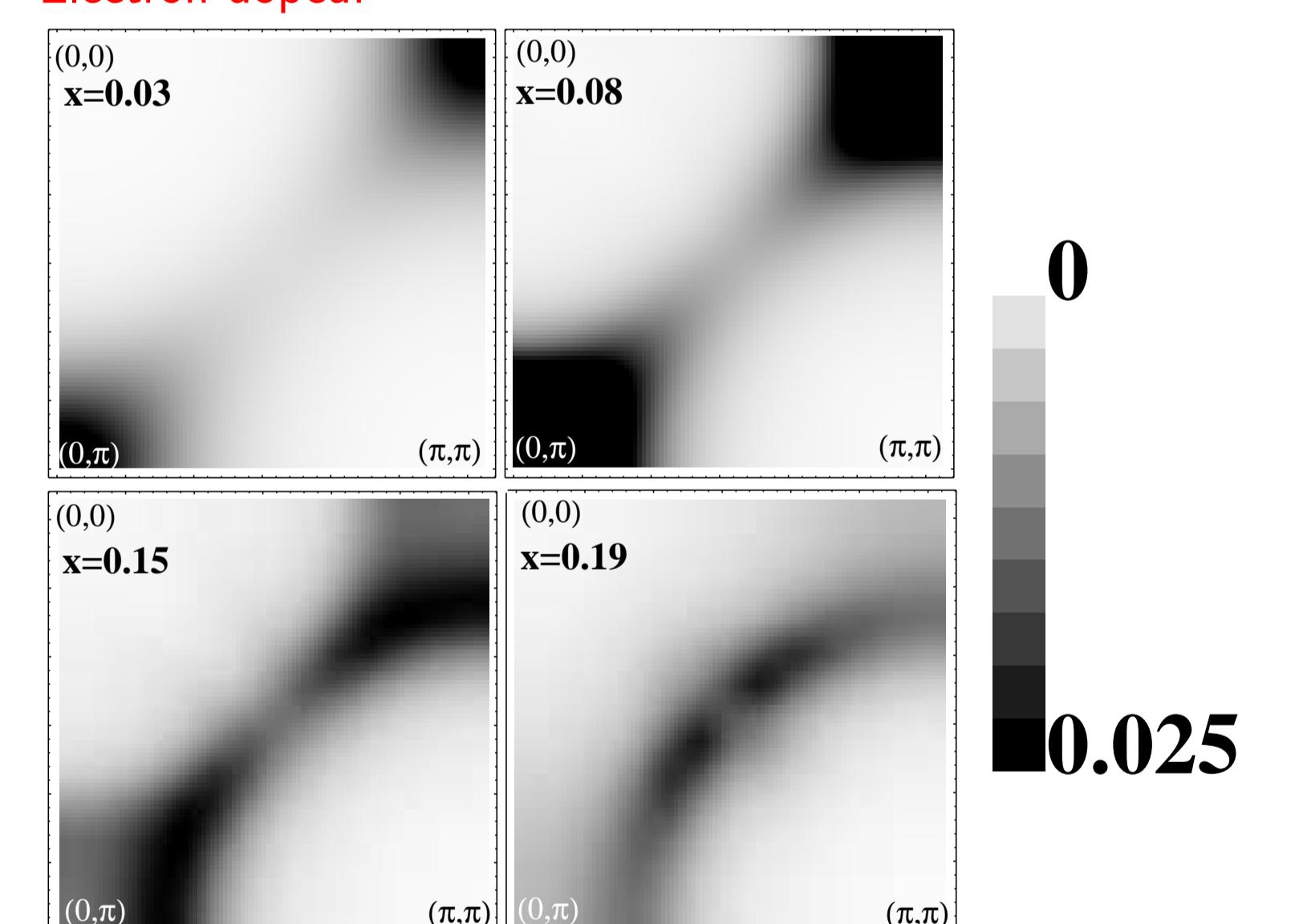
AF regime is enhanced on the electron-doped side (as in experiments).

## Fermi surfaces

Hole doped:



Electron doped:



Hole doped:

Hole pockets around  $(\pi/2, \pi/2) \rightarrow$  FS around  $(\pi, \pi)$ .

Electron doped:

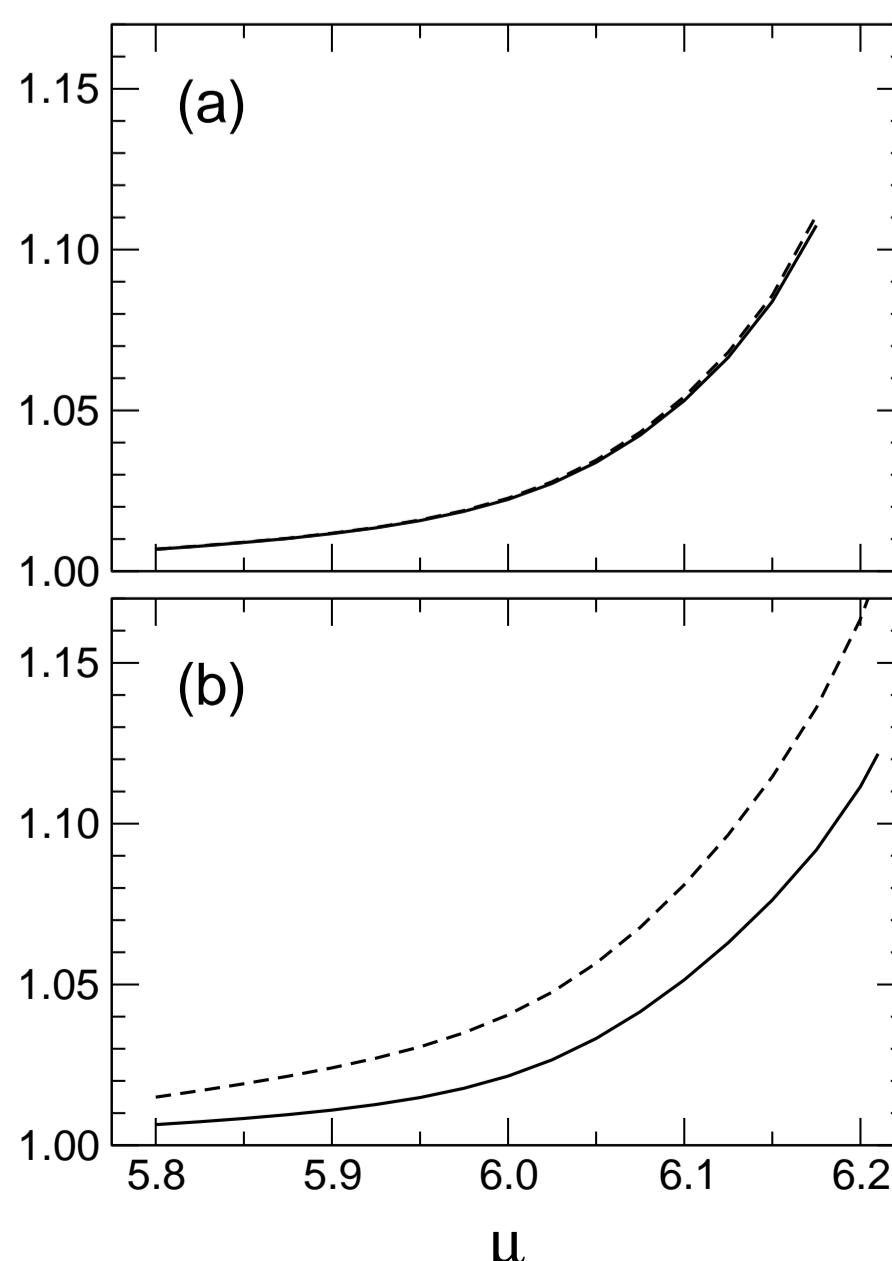
Pockets around  $(\pi, 0) \rightarrow$  FS around  $(\pi, \pi)$ .

## Thermodynamic consistent particle density

Variational parameter  $\Delta\mu'$  ensures thermodynamic consistent density

$$-\frac{\partial \Omega}{\partial \mu} = \sum_{i\sigma} \int f(\omega) A_{i\sigma}(\omega) d\omega$$

with optimization of  $\Delta\mu'$ :



without optimization of  $\Delta\mu'$ :

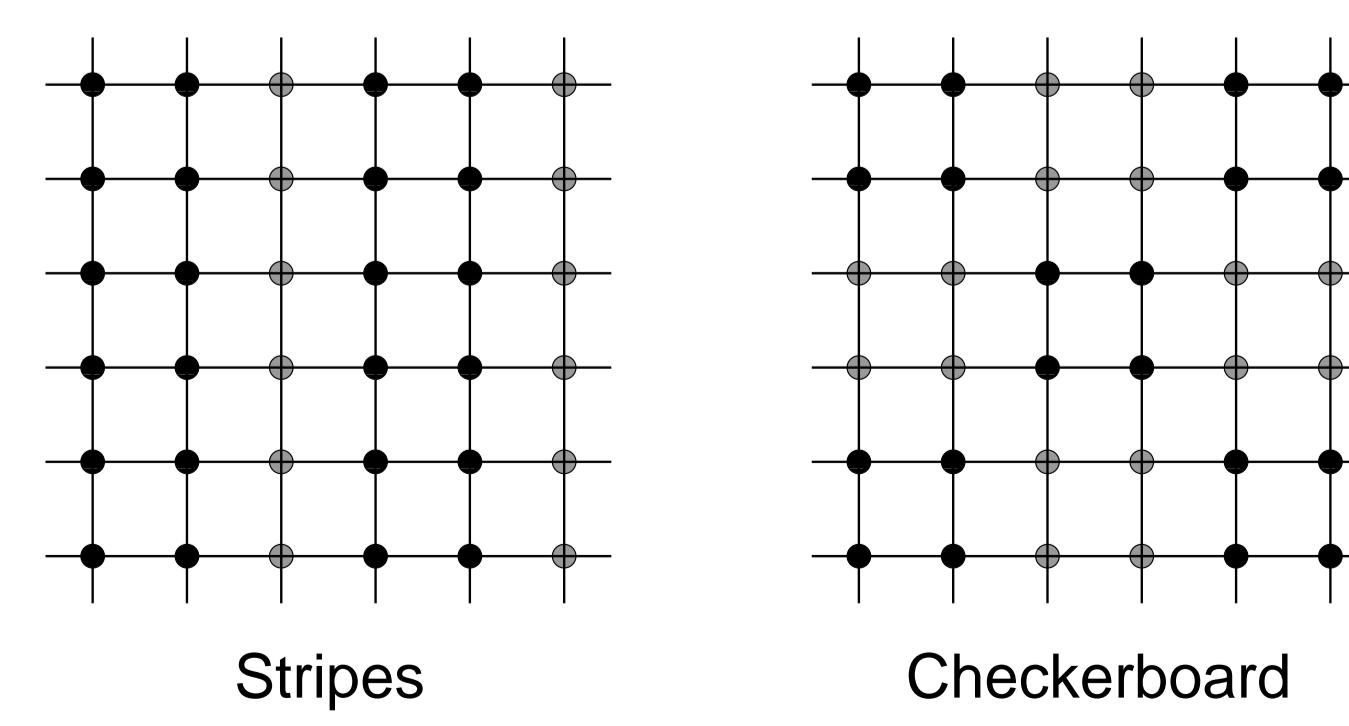
## Is it phase separation?

Small reference systems (2x2, 4x2) show Phase Separation.

BUT: Phase separation energy decreases with cluster size!  
Longer-range Coulomb interactions neglected!

Alternative scenario: Microscopic inhomogeneities!

Possible microscopic order patterns:



Can be simulated by additional variational parameter (to be done...)

## Acknowledgments

This work was supported by the Doctoral Scholarship Program of the Austrian Academy of Sciences, by the DFG Forschergruppe 538, and by the KONWIHR supercomputing network in Bavaria.

## References

1. M. Aichhorn and E. Arrigoni, *Europhys. Lett.* **71**, 117–123 (2005).
2. M. Aichhorn, E. Arrigoni, M. Potthoff, and W. Hanke, *cond-mat/0511460* (preprint).
3. M. Potthoff, M. Aichhorn, and C. Dahnken, *Phys. Rev. Lett.* **91**, 206402 (2003).