

Atomic physics and Quantum mechanics accompanying notes

Univ.-Prof. Dr. Enrico Arrigoni TU Graz - Austria

Version from: October 12, 2026

Copyright notice:

This material can only be downloaded and used by students of the course MAS.090UF “Atom Physics - Quantum Mechanics” By Prof. Enrico Arrigoni for their personal use. Other

persons can request permission by e-mail: arrigoni@tugraz.at

Content of this lecture

Content of this lecture

Mainly follows the book by

S. M. Blinder

“Introduction to Quantum Mechanics in Chemistry, Materials Science, and Biology”

Contents

1	Introduction: atoms and electromagnetic waves	2
1.1	Atoms and their constituents	2
1.2	Electromagnetic waves	4
2	Failures of classical physics	5
2.1	Blackbody radiation	5
2.2	Photoelectric effect	8
2.3	Line spectra	9
3	Wave and Particle duality	11
3.1	Double-slit experiment	11
3.2	Light carries momentum: Compton scattering	14
3.3	Matter (Electrons) as waves	15

Suggested literature

Suggested literature

1. S. M. Blinder, **Introduction to Quantum Mechanics in Chemistry, Materials Science, and Biology** (Elsevier, 2004)
the lecture is essentially based on this book

2. L. van Dommelen **Fundamental Quantum Mechanics for Engineers**
3. J. E. House, **Fundamentals of Quantum Chemistry**
some mathematical aspects are treated in more detail here
4. P. W. Atkins, **Physical Chemistry** Chap. 2
also a good book, many details and examples, many physical aspects discussed.
5. P. A. Tipler and R. A. Llewellyn, **Moderne Physik**
simpler treatment
6. J.J. Sakurai, **Modern Quantum Mechanics**
High level book
7. D. Ferry **Quantum Mechanics: An Introduction for Device Physicists and Electrical Engineers**
More advanced, special topics of interest for material physicists. Device physics, transport theory.
8. Applets <http://www.quantum-physics.polytechnique.fr/en/index.html>

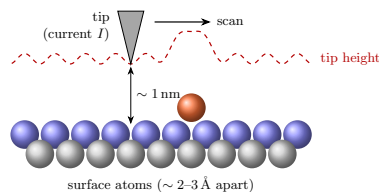
1 Introduction: atoms and electromagnetic waves

Quantum mechanics was born at the beginning of the 20th century from the study of the interaction between *matter* and *electromagnetic radiation*. In this introductory chapter we recall the basic facts about these two ingredients: the constituents of atoms and electromagnetic waves. (See also Blinder, Chap. 1.1–1.2.)

1.1 Atoms and their constituents

Atoms

- Matter consists of tiny particles: the *atoms* (Greek *atomos* = “indivisible”), an idea going back to Leucippus and Democritus (5th century BC).
- For a long time: only **indirect evidence** (chemistry, kinetic theory of gases, Brownian motion).
- Today it is possible to actually *see individual atoms*, for example with a scanning tunneling microscope (STM):



Size of an atom: $\sim 10^{-10} \text{ m} = 1 \text{ Å}$.

But: the *atom* is not the *ultimate particle of matter*.

The idea that matter is made of atoms is more than 2000 years old, but for a long time the evidence for it was only indirect. Today individual atoms can even be imaged: in a scanning tunneling microscope a very sharp tip is moved across a surface at a distance of about 1 nm, and the electric current between tip and surface makes the tip follow the contour of the single

atoms, as sketched in the figure. Atoms turn out to have a size of about 10^{-10} m. However, as we will see now, they are not the ultimate constituents of matter.

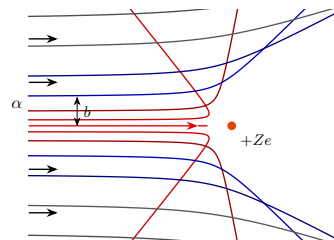
Subatomic particles

- 1896, H. Becquerel: *radioactive decay*.
Atoms emit α , β and γ radiation
(α : helium nuclei, β : electrons, γ : high-energy electromagnetic radiation)
- 1897, J. J. Thomson: the *electron*, a universal constituent of all atoms, with negative charge $-e$
 $e = 1.602 \times 10^{-19}$ C, $m_e = 9.109 \times 10^{-31}$ kg
- Atoms are electrically neutral $\Rightarrow$ they must also contain a *positive charge*.
How is it distributed?

Radioactivity showed that atoms can emit particles, and Thomson identified the electron as a constituent common to all atoms. Since atoms are neutral, the negative charge of the electrons must be compensated by a positive charge. How this positive charge is distributed in the atom was found out by Rutherford's experiment.

Rutherford's scattering experiment (1911)

- α particles from a radioactive source are shot onto a thin *gold foil*
- most of them pass (almost) undeflected
- a few are strongly deflected, some even backwards!



$\Rightarrow$ the positive charge and almost all the mass are concentrated in a *small but massive nucleus* at the center.

Nuclear model: a neutral atom of *atomic number* Z consists of a nucleus of charge $+Ze$, containing almost the entire mass, surrounded by Z *electrons* of charge $-e$.
Simplest atom: *hydrogen*, $Z = 1$: one electron and one proton (charge $+e$).

If the positive charge were spread over the whole atom, all α particles would be deflected only slightly. The few strong deflections show that the positive charge is concentrated in a very small region, the nucleus: only the α particles which happen to come very close to it are strongly repelled, as shown by the trajectories in the figure, while most of them pass far away from any nucleus.

The atomic nucleus

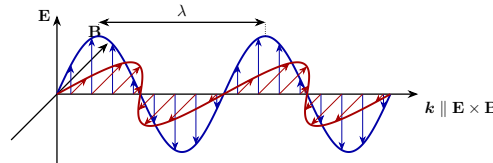
- 1932, J. Chadwick: the *neutron* (no charge, mass $\approx$ proton mass)
 - a nucleus with atomic number Z and mass number A consists of Z *protons* and $A - Z$ *neutrons*
 - *nuclear diameters*: a few 10^{-15} m,
an atom ($\sim 10^{-10}$ m) is $\sim 10^5$ *times larger*
- $\Rightarrow$ for atomic physics, the nucleus behaves (almost) like a *point charge* $+Ze$ with a large mass ($m_p \approx 1836 m_e$)
- protons and neutrons themselves have a structure (quarks, Gell-Mann, 1960s) $\rightarrow$ “**Standard Model**” of particle physics. Not needed in atomic physics.

Since the nucleus is about 10^5 times smaller than the atom but carries almost all of its mass, for the purposes of atomic physics it can be treated as a point charge $+Ze$. Its internal structure plays no role in this lecture.

1.2 Electromagnetic waves

Electromagnetic waves

- 1864, J. C. Maxwell: *unification of electricity, magnetism and optics*.
Maxwell's equations predict *electromagnetic waves* propagating with the speed of light: *light is an electromagnetic wave*.
- 1887, H. Hertz: first production and detection of electromagnetic (radio) waves, emitted by *accelerated electric charges*.



Electric field $\mathbf{E}$ and magnetic field $\mathbf{B}$ are perpendicular, oscillate in phase, and propagate in the direction of $\mathbf{E} \times \mathbf{B}$.

According to Maxwell's theory, light is an electromagnetic wave, consisting of oscillating electric and magnetic fields, as shown in the figure. Such waves are produced by accelerated charges, as Hertz demonstrated with radio waves.

Wavelength, frequency, speed of light

- *wavelength* λ : distance between successive maxima of the field
- *frequency* ν : number of oscillations per second at a fixed point (unit: $\text{Hz} = \text{s}^{-1}$)
- *period* $\tau = 1/\nu$: duration of one oscillation

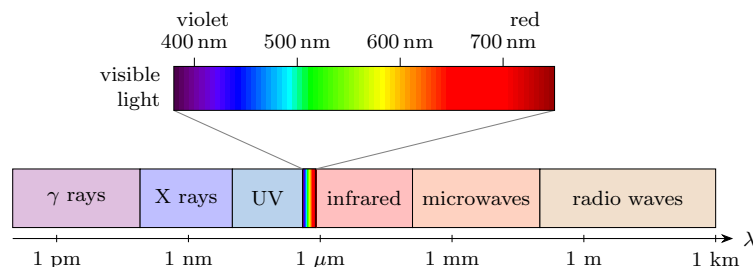
In one period the wave moves by one wavelength: $\lambda = c\tau$, i. e.

$$\boxed{\lambda \nu = c} \quad (1.1)$$

$c = 2.9979 \times 10^8 \text{ m/s}$: *speed of light*, the same for all electromagnetic waves in vacuum.

Often used: **angular frequency** $\omega = 2\pi\nu$ and **wave number** $k = |\mathbf{k}| = 2\pi/\lambda$, so that $\omega = ck$.

The electromagnetic spectrum



- Electromagnetic radiation exists in an *immense range of wavelengths*: γ rays, X rays, ultraviolet (UV), visible light, infrared (IR), microwaves, radio waves.
- *Visible light* is only a tiny window: $\lambda \approx 400 \text{ nm}$ (*violet*) – 700 nm (*red*).

All these kinds of radiation are electromagnetic waves of the same nature; they only differ in their wavelength, and visible light occupies only a very small part of the spectrum.

2 Failures of classical physics

Three failures of classical physics

At the end of the 19th century, classical mechanics, Maxwell's electrodynamics and thermodynamics seemed to explain all known phenomena.

However, three phenomena involving *electromagnetic radiation* could *not* be explained by classical physics:

1. *Blackbody radiation* → M. Planck (1900)
2. *Photoelectric effect* → A. Einstein (1905)
3. *Line spectra* of atoms → N. Bohr (1913)

A coherent formulation of *quantum mechanics* was developed only in 1925–26 (Heisenberg, Schrödinger, Dirac, ...).

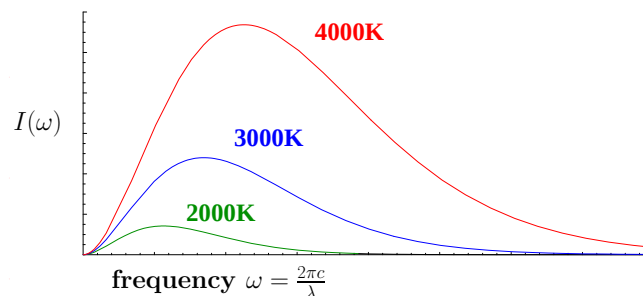
In this chapter we discuss these three experimental facts and the new, “quantum” ideas which were introduced to explain them. (See also Blinder, Chap. 1.3–1.6.)

2.1 Blackbody radiation

Thermal radiation

At high temperatures matter (for example metals) emits a continuum radiation spectrum.

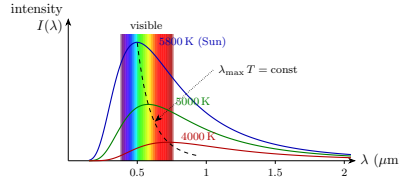
- A piece of metal in a flame becomes “red hot”, at higher temperatures “white hot”.
- The “color” they emit is pretty much the same at a given temperature, independent of the particular substance
(Wedgwood, 1782: different materials become red hot at the same temperature).
- With increasing temperature the color shifts towards shorter wavelengths (higher frequencies).



Energy intensity $I(\omega)$ versus **frequency** ($\omega = 2\pi\nu = \frac{2\pi c}{\lambda}$) of blackbody radiation at different temperatures:

- The energy intensity $I(\omega)$ vanishes at small and large ω , there is a maximum in between.
- The frequency ω_{max} of the maximum (“color”) of the distribution obeys the law (Wien’s law) $\omega_{max} = \text{const. } T$

Wien’s displacement law



Plotted versus the *wavelength*, the maximum is at λ_{max} with

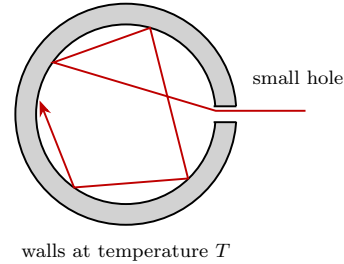
$$\lambda_{max} T = 2.898 \times 10^{-3} \text{ m K} \quad (2.1)$$

- room temperature ($T \approx 300 \text{ K}$): $\lambda_{max} \approx 10 \mu\text{m}$, in the *infrared*
- surface of the Sun ($T \approx 5800 \text{ K}$): $\lambda_{max} \approx 500 \text{ nm}$, in the *middle of the visible range*

Wien's law tells us that the color of a hot body is determined by its temperature: at room temperature the radiation is in the infrared and therefore invisible, while the Sun, at about 5800 K, emits most strongly in the visible range. Note that the maximum of the distribution plotted versus λ and the one of the distribution versus ω (previous figure) do not correspond to the same wavelength; in both cases, however, the maximum shifts proportionally to T . In the following we use $I(\omega)$.

The blackbody

An idealized description is the so-called black-body model, which describes a *perfect absorber and emitter of radiation at all wavelengths*. Specifically, radiation is absorbed and has enough time to thermalize before being emitted (in contrast to “white” or a “specular” body).



A blackbody can be realized by a cavity with a small hole, whose walls are kept at temperature T (see the figure): radiation entering the hole is reflected many times and eventually absorbed, and the radiation coming out of the hole is in thermal equilibrium with the walls.

Electromagnetic waves as oscillators

One single electromagnetic wave is characterised by a *wavevector* $\mathbf{k}$ which indicates the propagation direction and is related to the frequency and wavelength by $|\mathbf{k}| = \frac{2\pi}{\lambda} = \frac{\omega}{c}$.

In a blackbody, electromagnetic waves of all wavevectors $\mathbf{k}$ are present and distributed in equilibrium.

One can consider an electromagnetic wave with wavevector $\mathbf{k}$ as an independent oscillator (“mode”).

For a given frequency ω ($= 2\pi\nu$), there are many oscillators $\mathbf{k}$ having that frequency. Since $\omega = c |\mathbf{k}|$ the number (density) $n(\omega)$ of oscillators with frequency ω is proportional to the surface of a sphere with radius ω/c , i. e.

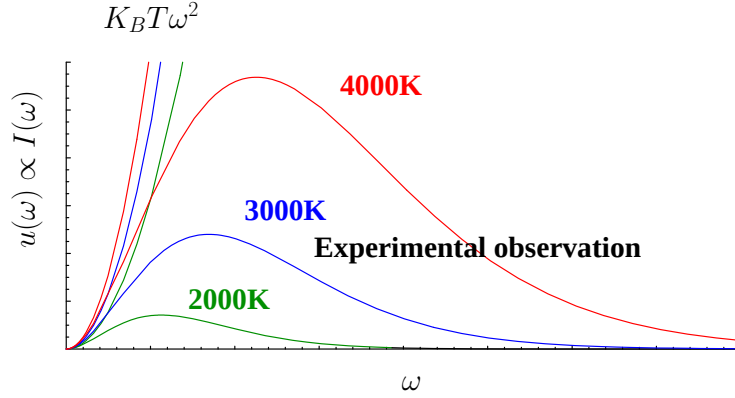
$$n(\omega) \propto \omega^2 \quad (2.2)$$

The *energy equipartition law of statistical physics* tells us that at temperature T *each mode is excited (on average) to the same energy $K_B T$* . Therefore, at temperature T the energy density $u(\omega, T)$ of all oscillators with a certain frequency ω would be given by

$$u(\omega, T) \propto K_B T \omega^2 \quad (2.3)$$

(Rayleigh–Jeans law).

The modes with frequency between ω and $\omega + d\omega$ correspond to wavevectors in a thin spherical shell of radius ω/c in $\mathbf{k}$ space, whose surface grows as ω^2 . Classically, in thermal equilibrium each of these oscillators has on average the energy $K_B T$ (K_B is Boltzmann's constant), independently of its frequency. Together, this gives the Rayleigh–Jeans law above.



Since $I(\omega) \propto u(\omega)$, this indeed agrees with experiments at small ω , but **not at large ω** . At large ω , $u(\omega, T)$ **must decrease again and go to zero**, otherwise the total energy

$$U = \int_0^\infty u(\omega, T) d\omega \quad (2.4)$$

would diverge (“*ultraviolet catastrophe*”)!

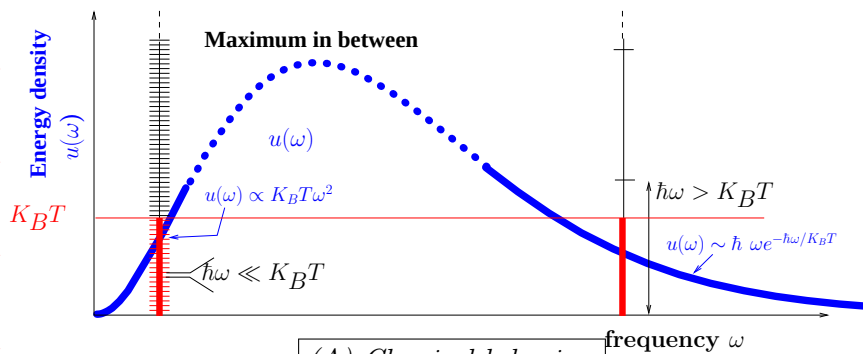
The classical theory thus predicts that the energy of the radiation in a cavity is infinite, most of it being at very high frequencies, which is clearly absurd.

Planck’s hypothesis: The “oscillators” (electromagnetic waves) cannot have a continuum of energies. *Their energies come in “packets” (quanta) of size $h \nu = \hbar \omega$.*

$h \approx 6.6 \times 10^{-34} \text{ Js}$ ($\hbar = \frac{h}{2\pi}$): Planck’s constant.

At small frequencies, as long as $K_B T \gg \hbar \omega$, this effect is irrelevant. The effect will start to be important at $K_B T \sim \hbar \omega$: here $u(\omega, T)$ will start to decrease. And in fact, Wien’s empiric observation is that $u(\omega, T)$ displays a maximum at $\hbar \omega \propto K_B T$. Eventually, for $K_B T \ll \hbar \omega$ the oscillators are not excited at all, their energy is vanishingly small. A more elaborate theoretical treatment gives the correct functional form.

Average energy of “oscillators”



(A) Classical behavior:

Average energy of oscillator $\langle E \rangle = K_B T$.

$\Rightarrow$ Distribution $u(\omega) \propto K_B T \omega^2$ at all frequencies!

(B) Quantum behavior: Energy quantisation

Small ω : Like classical case: oscillator is excited up to $\langle E \rangle \approx K_B T$. $\Rightarrow u(\omega) \propto K_B T \omega^2$.

Large ω : first excited state ($E = 1 \cdot \hbar \omega$) is occupied with probability $e^{-\hbar \omega / K_B T}$ (Boltzmann Factor): $\Rightarrow \langle E \rangle \approx \hbar \omega e^{-\hbar \omega / K_B T} \Rightarrow u(\omega) \propto \omega^2 \langle E \rangle \sim \hbar \omega^3 e^{-\hbar \omega / K_B T}$

In summary: classically every oscillator has the energy $K_B T$, so that $u(\omega) \propto \omega^2$ grows without limit. With Planck’s hypothesis, oscillators with $\hbar \omega \ll K_B T$ still behave classically,

while those with $\hbar\omega \gg K_B T$ are only rarely excited (Boltzmann factor $e^{-\hbar\omega/K_B T}$), so that $u(\omega)$ decreases exponentially at large ω . In between, $u(\omega)$ has a maximum at $\hbar\omega \propto K_B T$, which is Wien's law.

2.2 Photoelectric effect

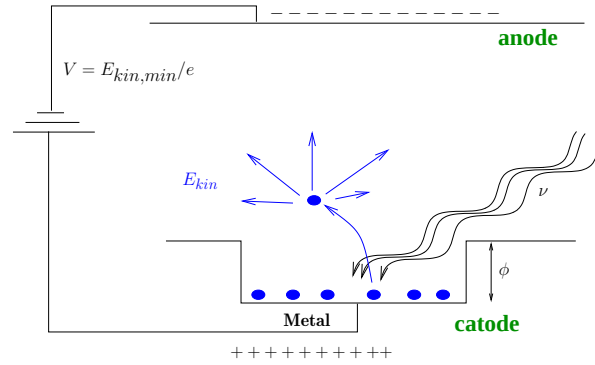
Electrons in a metal are confined by an *energy barrier* (work function) ϕ .

One way to extract them is to shine light onto a metallic plate.

Light transfers an energy E_{light} to an electron.

The rest of the energy $E_{light} - \phi$ goes into the kinetic energy of the electron $E_{kin} = \frac{1}{2} m v^2$.

By measuring E_{kin} , one can get E_{light} .



In the experiment (see the figure) light shines on a metal plate in vacuum, and the emitted electrons are collected by a second electrode, which gives an electric current. Their kinetic energy can be measured, for example, from the voltage which is needed to stop them.

Photoelectric effect

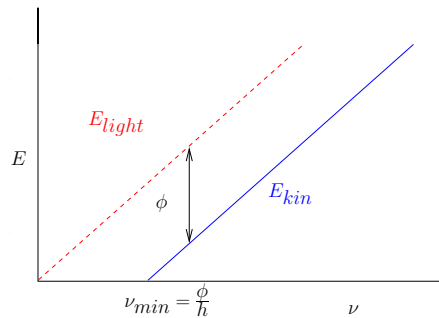
Classically, we would expect the total energy transferred to an electron $E_{light} = \phi + E_{kin}$ to be *proportional to the radiation intensity*. The experimental results give a different picture:

while the *current* (i. e. the number of electrons per second expelled from the metal) is *proportional to the radiation intensity*,

E_{light} is *proportional*

to the frequency of light:

$$E_{light} = h \nu \quad (2.5)$$



Each electron thus receives the energy $h\nu$, so that its kinetic energy is $E_{kin} = h\nu - \phi$, a straight line as a function of ν (see the figure). Below the frequency $\nu_{min} = \phi/h$ no electrons are emitted at all, however intense the light is. This is explained if light consists of quanta of energy $h\nu$, each of which is absorbed by a single electron (Einstein, 1905): a higher intensity means more quanta, and thus more electrons, but not more energy per electron.

Summary: Planck's energy quantum

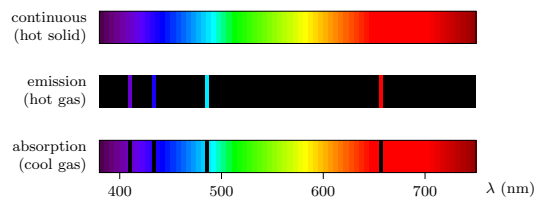
Blackbody radiation and the photoelectric effect are explained by the idea (Planck 1900, Einstein 1905) that light carries energy only in “quanta” of size

$$E = h\nu \quad (2.6)$$

This means that light is *not* a *continuous* object: rather, its constituents are *discrete*: *the photons*.

2.3 Line spectra

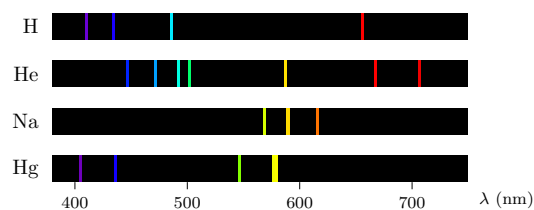
Continuous and line spectra



- *Continuous spectrum*: emitted by an incandescent solid or a gas at high pressure (e. g. blackbody radiation).
- *Emission spectrum*: a *gas at low pressure*, excited by heat or by collisions with electrons, emits light only at a series of *sharp wavelengths*.
- *Absorption spectrum*: light from a continuous source passing through a cooler gas: *dark lines at the same wavelengths*.

To obtain a spectrum, the light is decomposed into its wavelengths by a prism or a grating. For a given gas, the emission and the absorption lines appear at the same wavelengths (see the figure).

Line spectra of atoms



- The emission *line spectra* are characteristic of the element (“fingerprint”).
- *Consistent with classical electrodynamics*: moving charges within the atom can emit and absorb radiation.
- *Completely mysterious*: why only *discrete frequencies*, rather than a continuum?

Bohr's explanation (1913)

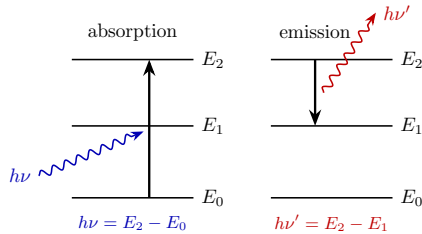
Bohr postulated that the *energy levels of atoms* take only a *discrete set of values* E_n , rather

- *Emission*: in a transition from a higher level E_m to a lower level E_n a photon is emitted, with

$$h\nu = E_m - E_n \quad (2.7)$$

than a continuum as in classical mechanics.

- *Absorption*: a photon of the same energy is annihilated, the atom is excited from E_n to E_m .



⇒ Absorption and emission occur at the *same*

set of frequencies.

Bohr's idea connects the discrete lines with the quanta of light: if light is emitted and absorbed only in quanta $h\nu$, discrete frequencies mean that the atom itself can only have discrete energies. The frequencies of the lines are then differences of energy levels, divided by h .

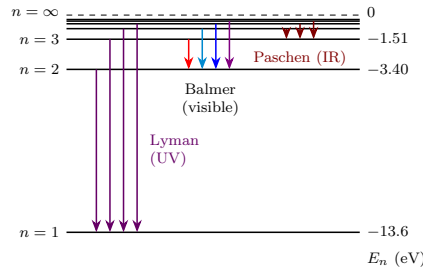
The spectrum of hydrogen: Rydberg's formula

Rydberg (1890): all lines of *atomic hydrogen* are fitted by the empirical formula

$$\frac{1}{\lambda} = \mathcal{R} \left(\frac{1}{n_1^2} - \frac{1}{n_2^2} \right) \quad (2.8)$$

$$n_1 = 1, 2, 3, \dots, \quad n_2 > n_1.$$

$\mathcal{R} = 109\,677\,\text{cm}^{-1}$: *Rydberg constant*.



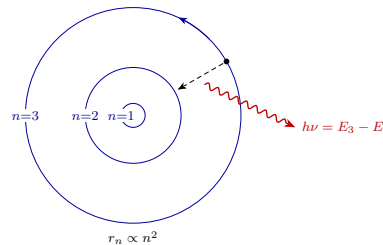
$$\Rightarrow \text{Comparing with (2.7) with } \nu = c/\lambda: E_n = -\frac{hc\mathcal{R}}{n^2} \approx -\frac{13.6\,\text{eV}}{n^2}$$

- *No analogously simple formula* has been found for any atom other than hydrogen.

The figure shows the energy levels of hydrogen obtained in this way, together with the series of lines: transitions ending in $n_1 = 1$ give lines in the ultraviolet, those ending in $n_1 = 2$ the visible lines, and those ending in $n_1 = 3$ lines in the infrared.

Bohr's planetary model

- Inspired by Rutherford's nuclear atom, Bohr suggested a *planetary model*: the electron moves around the proton on one of a set of *allowed circular orbits*.
- Each orbit corresponds to an energy E_n ; light is emitted only in a jump between two orbits.
- Bohr's model reproduces Rydberg's formula and $\mathcal{R}$ (see the section on Bohr's atom).



A deeper understanding of the discrete energy levels came only with quantum mechanics (1925–26), but Bohr's model was an important stepping stone. We will treat the hydrogen atom in detail later.

3 Wave and Particle duality

Waves and particles

- Quantum mechanics describes the *behavior of matter on the atomic scale*.
- It is the most successful quantitative theory in physics, tested in many fields: chemistry, elementary-particle physics, solid-state electronics, molecular biology, cosmology, ...
- *Transistors, lasers, computers, nuclear reactors* are offspring of quantum theory.
- However, the *quantum world differs dramatically from the world of everyday experience*.

A *paradox* at the origin of quantum theory concerned the *nature of light*: light usually behaves as a *wave*, but occasionally it shows a *particle-like* aspect:

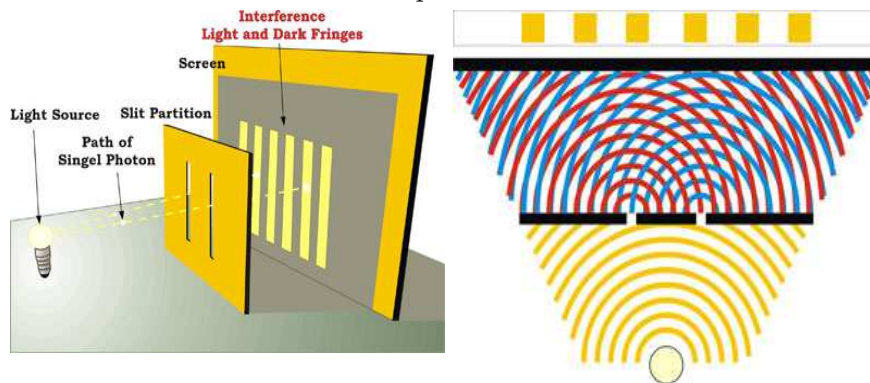
wave-particle duality.

Quantum mechanics is the theory which describes matter on the atomic scale. Its predictions have been confirmed in very different fields, and much of modern technology relies on it. At the same time, the quantum world behaves very differently from what we know from everyday life. In this chapter we see how this became apparent for light, which depending on the experiment behaves as a wave or as a stream of particles, and that the same is true for electrons.

3.1 Double-slit experiment

Double-slit experiment

This experiment (T. Young, 1801) was done in order to distinguish whether light behaves as particle or as a wave.

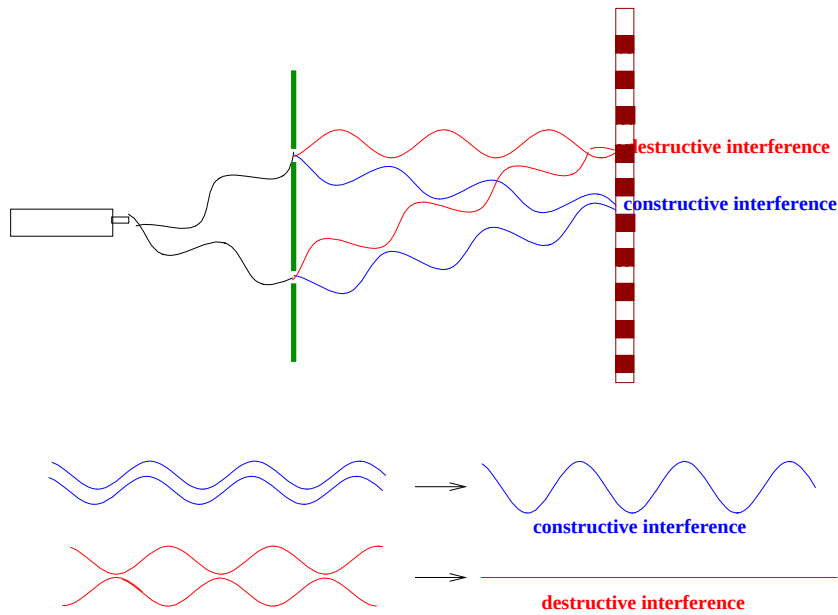


Double

slit: interference pattern

Monochromatic, coherent light is first sent through a single slit. If light consisted of classical particles, behind two slits we would just expect two bright bands, the images of the two slits. Instead, one observes a sequence of bright and dark fringes: an interference pattern, which is typical for waves.

Double-slit experiment



The observation of

an interference pattern proves the wave nature of light!

The waves coming from the two slits travel different distances to a given point on the screen. Where they arrive in phase they add up (constructive interference, bright fringe); where they arrive with opposite phase they cancel (destructive interference, dark fringe).

Amplitude and intensity of a wave

The energy density of an electromagnetic field

$$\rho = \frac{1}{2} \left(\epsilon_0 E^2 + \frac{B^2}{\mu_0} \right) \quad (3.1)$$

depends *quadratically* on the fields. For a wave $B = E/c$, so that both terms are equal. With the *amplitude* $\Psi(\mathbf{r}, t) = \sqrt{\epsilon_0} E(\mathbf{r}, t) = B(\mathbf{r}, t)/\sqrt{\mu_0}$ the intensity is

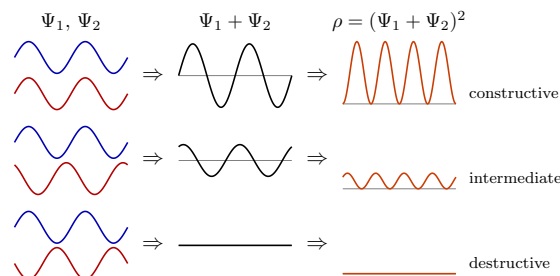
$$\rho(\mathbf{r}, t) = [\Psi(\mathbf{r}, t)]^2 \quad (3.2)$$

Later, Ψ will have *complex values*; then

$$\rho(\mathbf{r}, t) = |\Psi(\mathbf{r}, t)|^2 = \Psi(\mathbf{r}, t)^* \Psi(\mathbf{r}, t) \quad (3.3)$$

In quantum mechanics, Ψ is called the *wavefunction*.

Interference



With both slits open the *amplitudes add*:

and the intensity is

$$\Psi = \Psi_1 + \Psi_2 \quad (3.4) \quad \rho = \Psi^2 = \Psi_1^2 + \Psi_2^2 + 2\Psi_1\Psi_2 \quad (3.5)$$

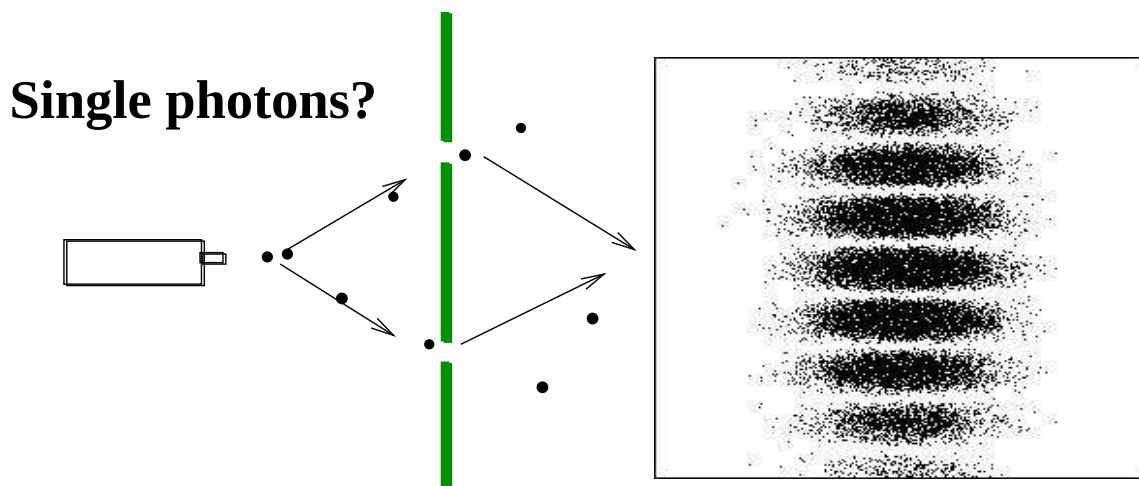
The intensity is *not* the sum of the single-slit intensities, $\rho \neq \rho_1 + \rho_2$: the *cross term* $2\Psi_1\Psi_2$ is responsible for the *constructive* (bright bands) and *destructive* (dark bands) *interference*.

To describe interference we need the amplitude of the wave. The energy carried by an electromagnetic wave is quadratic in the fields, and it is convenient to define an amplitude Ψ such that the intensity is simply its square. The amplitude can be positive or negative. When two waves overlap, it is their amplitudes, and not their intensities, which add. Where Ψ_1 and Ψ_2 have the same sign, the intensity is larger than the sum of the two intensities; where they have opposite sign, they partly or completely cancel, as shown in the figure. The same structure, an amplitude whose square gives an intensity, will reappear in quantum mechanics.

Light: waves or particles?

- Interference experiments (Young, Fresnel, early 19th century) seemed to have *settled that light is a wave* (contrary to Newton's view).
- But about a century later, *blackbody radiation* and the *photoelectric effect* could not be explained by the wave theory (Sec. 2.1).
- Modification of the double-slit experiment: *dim the light source* by several orders of magnitude.
- One begins to see *localized flashes* (scintillations) at random positions on the screen: *single photons*.

Light: particles or waves ?



When the light is dimmed, the interference pattern does not simply become fainter. At very low intensity the photons arrive one by one, each of them producing a single flash at some point of the screen. After many photons have arrived, the familiar interference pattern appears, as shown in the figure.

Wave-particle duality

- Each photon arrives at a random position: the *behavior of an individual photon is unpredictable*.
- The *statistical behavior* of many photons, however, reproduces the interference pattern of intense light: it is *predictable*.
- Each photon, although detected like a particle, carries a “knowledge” of the whole wavelike pattern: in some sense, a *single photon must go through both slits*.

⇒ **Wave–particle duality**: under appropriate circumstances light behaves as a wave or as a particle.

The probability to detect a photon at a point $\mathbf{r}$ is proportional to the intensity $\rho(\mathbf{r})$ of the wave.

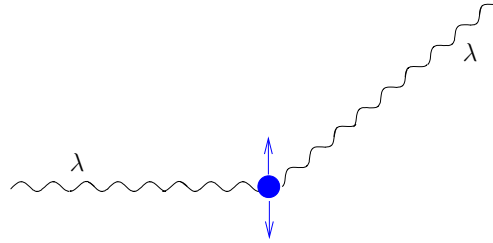
We thus cannot predict where a single photon will be detected, but we can predict the probability of detecting it at a given point, which follows the intensity of the wave. Since the interference pattern appears only when both slits are open, each photon is in some way influenced by both slits, even though it is always detected at a single point.

3.2 Light carries momentum: Compton scattering

Classical view (Thomson scattering)

Scattering of high-energy radiation (x-rays, gamma-rays) from an electron.

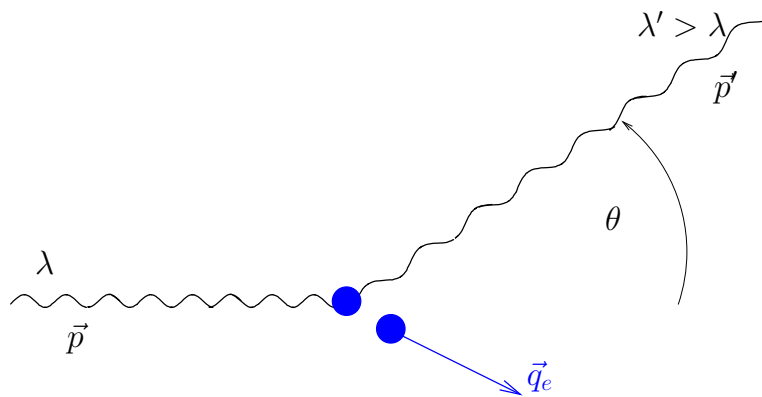
The electron oscillates at the frequency of the incident wave. The electron acts as an oscillating dipole and emits radiation at the same frequency. **Wavelength of the scattered radiation remains unchanged !**



At high intensities, the electron is also pushed by the radiation and the Doppler effect must be taken into account:

light is emitted in a broader frequency range.

Experimental result (Compton scattering, 1923)



$$\vec{p} = \vec{p}' + \vec{q}_e \quad (3.6)$$

$$\lambda' - \lambda = \frac{h}{m_e c} (1 - \cos \theta) \quad (3.7)$$

Classically, the scattered radiation should have the same wavelength as the incident one. Compton found instead that the scattered x-rays have a larger wavelength λ' , and that the

shift depends only on the scattering angle θ . This is exactly what one obtains by treating the scattering as a collision between two particles, a photon and an electron, in which energy and momentum are conserved: part of the momentum of the photon is transferred to the electron.

This result can be understood if one assumes that the particles constituting *electromagnetic waves (photons)* have a momentum

$$|\mathbf{p}| = \frac{h}{\lambda} \quad (3.8)$$

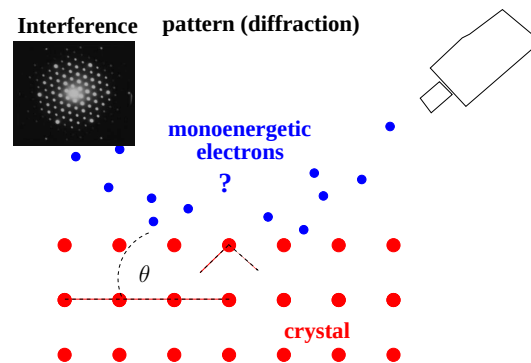
(in the direction of propagation) and due to the kinematics part of the momentum is transferred to the electron. This is consistent with Planck's energy formula for photons and with relativity: for a particle with zero rest mass, moving with velocity c , $E = pc$, so that

$$E = pc = h\nu \quad \Rightarrow \quad p = E/c = h\nu/c = h/\lambda \quad (3.9)$$

Light thus carries not only energy $h\nu$ but also momentum, and in a collision photons behave like particles. The value of the momentum, h/λ , is the one expected from relativity for a particle without rest mass, which always moves with the speed of light.

3.3 Matter (Electrons) as waves

“Double slit” experiment with crystals



For x -rays the natural “slit” system consists of an arrangement of atoms in a crystalline structure the distance between atoms is of the order of the wavelength of x -rays. One can do the same thing for electrons.

Surprisingly, an *interference pattern* was observed for electrons as well. This had been predicted by de Broglie (1924), who suggested that matter (electrons) might also behave as waves.

These “matter waves” have a wavelength (cfr. (3.8))

$$\lambda = \frac{h}{p} \quad (3.10)$$

In a crystal the atoms are arranged regularly, at distances comparable to the wavelength of x -rays, so that the crystal acts like a system of many slits, and the scattered x -rays show an interference pattern. When the same is done with a beam of electrons, an interference pattern is observed as well: electrons behave as waves, with the wavelength given by de Broglie's relation, the same relation between momentum and wavelength which holds for photons.

Electrons as waves

- The double-slit experiment can also be done with *electrons* instead of light (technically difficult): an *electron gun* produces a beam of electrons with a selected velocity, focused and guided by electric and magnetic fields.

- Everything that happens for photons has its *analog for electrons*: electrons also exhibit *diffraction and interference*.
- 1927, Davisson and Germer: *diffraction of monoenergetic electrons by a metal crystal*, much like the diffraction of x-rays, with the wavelength given by de Broglie's formula (3.10).
- Example: electrons accelerated by 54 V have $\lambda = h/p \approx 0.17 \text{ nm}$, comparable to the distance between atoms.

$\Rightarrow$ *Wave-particle duality holds for matter as well.*

For electrons, too, everything that was found for photons repeats itself: they are detected as particles at single points, but their distribution shows interference. The wavelength of electrons with typical energies of some tens of eV is of the order of the distance between atoms, which is why crystals are suited to observe their diffraction.