Introduction to theoretical physics

Sheet 2

Exercise 5:
Consider the following vector fields:
$\mathbf{v} = \mathbf{a}(\mathbf{r} \cdot \mathbf{a})$
$\mathbf{u} = (0, 0, (x^2 + y^2)z)$,
a sphere S with radius R centered on the origin of the coordinates, and a cyilinder C with length L and radius R, also centered on the origin.
$\mathbf{a}$ is perpendicular to the axis of the cylinder.
Determine the flux

a) of $\mathbf{v}$ through the surface ∂S of the sphere S.

b) of $\mathbf{u}$ through the surface ∂S von der sphere S.

c) of $\mathbf{v}$ through the surface ∂C of the cylinder C.

verify in all these cases Gauss' law.
[Hint: use the results of exercises 2-g and 2-h]

Exercise 6

Determine the $\mathbf{E}$-field of an infinitely long, charged cylinder with radius R and charge density
$\rho(r) = a/r$ (for $r \leq R$)
(with r the distance to the axis of the cylinder and a a constant) for a point P outside and inside of the cylinder.
Hint: use the symmetries of the field as well as Gauss' law for a cylindrical Gauss surface.
Determine the voltage between the axis of the cylinder and a point P outside of the cylinder.

Exercise 7

A point charge q is located in front of two metallic half planes, which cut each other at an angle of 90^o and have the potential $\Phi = 0$.
Using the method of the mirror charge (here you need 3 mirror charges) determine the potential $\Phi(\mathbf{r})$ for an arbitrary point $\mathbf{r}$ within the sector delimited by the two planes.
Determine the surface charge density at the surface of the metal.
Write down the potential in cartesian coordinates.