Introduction to Theoretical Physics

Problem set 3

Task 8

Assume a cylinder of length L and radius R with a given charge density $qn(\rho)$. The charge density only depends on the distance ρ to the cylinder axis.
The cylinder (as well as the charge density) rotates with the angular velocity ω.
(a) Give an integral expression for the **B**-field for a point P located on the cylinder axis with a given distance z_0 to the center of the cylinder.
(b) Calculate **B** for a homogenous charge density $n(\rho) = n_0$: consider only the case where $L \ll z_0$ and $R \ll z_0$.

Task 9

Along a cylindrical conductor of infinite length with radius R flows the current I. The current density is parallel to the cylinder axis and it's absolute value $J(\rho)$ only depends on the distance ρ to this axis.

(a) Give an expression for the **B**-field both on the inside and on the outside of the cylinder using Stoke's theorem.
(Hint: the absolute value of **B** only depends on the distance ρ to the cylinder axis. The **B** field lines are concentric circles around the cylinder axis. Write down the circulation of **B** alongside one of these circles. What is the relation between the circulation of the magnetic field **B** and the current I?)

From now assume a current density $J(\rho) = \alpha\rho$ with a constant α:

(b) Determine α and calculate **B** both on the inside and on the outside of the cylinder.

(c) Calculate the vector potential **A** both on the inside and on the outside of the cylinder (in Coulomb gauge) using Poisson's equation.
(Hint: from Poisson's equation you can assume **A** to be parallel to the cylinder axis. Because of symmetries it's absolute value therefore only depends on the distance ρ to the cylinder axis. Next solve Poisson's equation in cylindrical coordinates. **A** must not have any singularities at $\mathbf{r} = 0$. **A** is both continuous and differentiable on the surface of the cylinder.)