Introduction to Theoretical Physics

Problem set 4

Task 10

The magnetic field of a pole of a bar magnet can be assumed to be approximately radially symmetric (around the pole) of the form

$$\mathbf{B} = \alpha \frac{\mathbf{r} - \mathbf{r}_0}{|\mathbf{r} - \mathbf{r}_0|^3} \tag{1}$$

around the position $\mathbf{r}_0$ of the pole with a constant α.

The pole now moves with a constant velocity v along the axis of a coil with N turns. The coil has the shape of a circular ring with radius a. Calculate the induced voltage V_I in the coil in dependence of v and the distance x between the center of the coil and the pole. (Hint: assume $x \gg a$).

Task 11 (Inhomogeneous Maxwell equation - Lorentz and Coulomb gauge)

Consider in space a given current density

$$\mathbf{J}(\mathbf{r}, t) = Re[\mathbf{J}_0 \ e^{i(\mathbf{k}\cdot\mathbf{r} - \omega \ t)}] \tag{2}$$

($\omega \neq 0$, $\omega \neq c|\mathbf{k}|$, $|\mathbf{k}| > 0$, $\mathbf{J}_0$ real). Give a possible (particular) solution to

a) the charge density (continuity equation!)

b) the vector and scalar potentials in Lorentz gauge ($\mathbf{A}_L, \phi_L$)

c) the $\mathbf{E}$ and $\mathbf{B}$ fields

d) the scalar and vector potentials in Coulomb gauge ($\mathbf{A}_C, \phi_C$)
(Hint: First calculate ϕ_C, then determine χ so that $\phi_C = \phi_L - \frac{\partial \chi}{\partial t}$. Then use χ in order to calculate $\mathbf{A}_C$.)

Hint: Look for solutions for ρ and for the potentials of the form $\mathbf{J}$: $Re[\text{const } e^{i(\mathbf{k}\cdot\mathbf{r} - \omega \ t + \phi)}]$ ($\phi = 0$ or $\pi/2$). Use the fact that $\nabla \ Re \cdots = Re \ \nabla \cdots = Re \ i\mathbf{k} \ \cdots$ (and similarly with $\partial/\partial t$). Then calculate $\mathbf{B}$ and $\mathbf{E}$ from the potentials (in one of the gauges).