Introduction to Theoretical Physics

Problem Set 5

Task 12
Consider the following potentials

$$\mathbf{A} = \mathbf{u}\ f(k\ |\mathbf{r}| + \omega t) \qquad \Phi = 0\ , \tag{1}$$

where $\mathbf{u}$ denotes a constant vector with $|\mathbf{u}| = 1$ and where $f(x)$ (with $x \equiv k\ r + \omega\ t$) is an arbitrary, yet sufficiently differentiable function.

(a) Calculate the $\mathbf{E}$ and $\mathbf{B}$ field.

(b) Calculate the Poynting vector $\mathbf{S}$

(c) Calculate the flux of the Poynting vector (energy current flux) through the surface of a given sphere K with radius R centered around the origin.

(d) Calculate the energy density of the electromagnetic field.

Hint: Use (and verify) that for $\mathbf{A}$:

$$\nabla \to k\ \frac{\mathbf{r}}{r}\ \frac{d}{dx} \qquad\qquad \frac{\partial}{\partial t} \to \omega\ \frac{d}{dx} \tag{2}$$

Task 13
Assume a transversal electromagnetic wave in vacuum being

a) linearly polarized,

$$\mathbf{E} = f\ \mathbf{e}_x \sin(kz - \omega t),$$

b) elliptically polarized,

$$\mathbf{E} = \alpha\ \cos(kz - \omega t)\mathbf{e}_x + \beta\ \sin(kz - \omega t)\mathbf{e}_y,$$

(α, β, f are real-valued constants) propagating in the z-direction.
Rewrite the fields in the complex notation.
Calculate

1. the $\mathbf{B}$ field $\mathbf{B}(\mathbf{r}, t)$,
2. the mean value of the Poynting vector $\overline{\mathbf{S}(\mathbf{r})}$ with respect to time.

Hint: $\mu_0\ \epsilon_0\ c^2 = 1$.