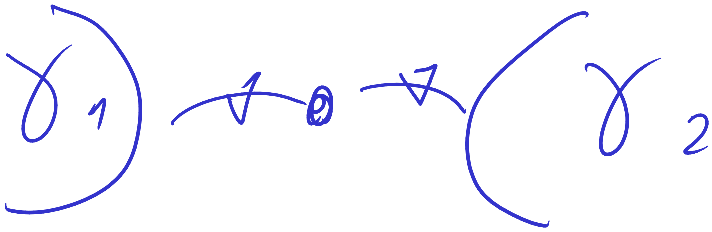
Summary:

1 FERMIONIC SITE WITH DISSIPATION



$$\Gamma_1 = \sqrt{\gamma_1}\, a \qquad \Gamma_2 = \sqrt{\gamma_2}\, a^\dagger$$

$$H = \varepsilon\, a^\dagger a$$

$$\hat{n} = a^\dagger a \qquad n = \langle \hat{n} \rangle$$

$$\frac{dn}{dt} =$$

1 FROM H

$$i[H, n] = 0$$

② FROM γ_1

$$\gamma_1\left(a^\dagger \hat{n} a - \frac{1}{2}\{a^\dagger a, n\}\right)$$

$$\hat{n} = a^\dagger a = -a a^\dagger + 1$$

$$\hat{n}^2 = \hat{n}$$

$$= \gamma_1\left(-\hat{n}^2 + \hat{n} - \hat{n}^2\right) = -\gamma_1 \hat{n}$$

③ FROM γ_2

$$\gamma_2\left(a \hat{n} a^\dagger - \frac{1}{2}\{a a^\dagger, \hat{n}\}\right)$$

$$= \gamma_2\left((a a^\dagger)^2 - a a^\dagger(-a a^\dagger + 1)\right)$$

$$(a a^\dagger)^2 = a a^\dagger = 1 - \hat{n}$$

$$= \gamma_2\left(1 - \hat{n}\right)$$

$$\Rightarrow \quad \frac{dm}{dt} = -\gamma_1 m + \gamma_2 (1-m)$$

STEADY STATE (SOL. OF INHOMOG.)

$$m_0 = \frac{\gamma_2}{\gamma_1 + \gamma_2}$$

HOMOG. PART

$$m_H = a\, e^{-(\gamma_1 + \gamma_2) t}$$

GENERAL SOL.

$$m(t) = a\, e^{-(\gamma_1 + \gamma_2) t} + m_0$$

CORR. FUNCTION

$$h(\tau, t) \equiv Tr\, a^{+}(t+\tau)\, a(t)\, \rho_0$$

$$= Tr\, a^{+} \underbrace{e^{-iH\tau}\, a\, \rho_0(t)\, e^{iH\tau}}$$

$$A_{st}(\tau) = Tr_E$$

in. value $A_{st}(0) = a\, \rho_s(t)$

we take steady state $t \to \infty$

$$A_s(\tau) \equiv A_{s\infty}(\tau)$$

$$A_s(0) = a\, \rho_s \leftarrow \text{STEADY state}$$

$$h(0, t \to \infty) = \frac{\gamma_2}{\gamma_1 + \gamma_2} = n_0$$

$$\underset{\sim}{\bar{\mathcal{L}}}_H a^+ = i[H, a^+] = i\varepsilon a^+$$

$$\underset{\sim}{\bar{\mathcal{L}}}_1 a^+ = \gamma_1\Big(-\underbrace{a^+ a^+}_{=0} a - \frac{1}{2}\{a^+ a, a^+\}\Big)$$

$$= -\frac{\gamma_1}{2} a^+ a a^+ = -\frac{\gamma_1}{2} a^+$$

SIMILARLY

$$\bar{\mathcal{L}}_2 a^+ = \gamma_2\Big(-a a^+ a^+ - \frac{1}{2}\{a a^+, a^+\}\Big)$$

$$= -\frac{\gamma_2}{2} a^+ a a^+ = -\frac{\gamma_2}{2} a^+$$

$$\frac{d h(\tau)}{d\tau} = \Big(i\varepsilon - \frac{\gamma_1}{2} - \frac{\gamma_2}{2}\Big) h(\tau) \qquad \tau > 0$$

$$\Rightarrow h(\tau) = n_0\, e^{i\varepsilon\tau}\, e^{-\frac{1}{2}\underbrace{(\gamma_1+\gamma_2)}_{=2\gamma}\tau}$$

FOR $\boxed{\tau < 0}$

$$h^*(\tau, t) = \mathrm{Tr}\, a^+(t)\, a(\underbrace{t+\tau}_{t'})\, \varrho_v$$

$$= \mathrm{Tr}\, a^+(t'-\tau)\, a(t')\, \varrho_v$$

$$= h(\tau', t') \quad \text{in steady state } t' \to \infty \qquad \tau' = -\tau > 0$$

$$= m_0 e^{i\varepsilon\tau'} e^{-\gamma\tau'}$$

$$\tau = -\tau' < 0$$

$$\Rightarrow h(t+\tau, t) = m_0 e^{i\varepsilon\tau} e^{\gamma\tau}$$

SUMMARY

$$h(\tau) = h(t+\tau, t)\Big|_{t\to\infty} = m_0 e^{i\varepsilon\tau} e^{-\gamma|\tau|}$$

SIMILARLY

$$\bar{h}(\tau, t) = \mathrm{Tr}\, a(t+\tau) a^{+}(t) \rho_s$$

$$\bar{h}(\tau) \overset{t\to\infty}{=} \underbrace{(1-m_0)}_{\frac{\gamma_1}{\gamma_1+\gamma_2}} e^{-i\varepsilon\tau} e^{-\gamma|\tau|}$$

$$G_R(\tau) \equiv -i\,\underset{t\to\infty}{\Theta(\tau)}\,\langle\{a(\tau+t), a^\dagger(t)\}\rangle$$

$$= -i\,\Theta(\tau)\left(\bar{h}(\tau) + h(-\tau)\right)$$

$$= -i\,\Theta(\tau)\, e^{-i\varepsilon\tau}\, e^{-\gamma|\tau|}$$

$$G_<(\tau) \equiv i\,h(-\tau) =$$

$$= i\, e^{-i\varepsilon\tau}\, e^{-\gamma|\tau|}\, n_0$$

FOURIER TRANSFORMS

$$G_R(\omega) = \int_{-\infty}^{+\infty} G_R(\tau)\, e^{i\omega\tau}\, d\tau$$

$$= -i\int_0^{\infty} e^{i(\omega-\varepsilon+i\gamma)\tau}\, d\tau$$

$$= \frac{1}{\omega-\varepsilon+i\gamma}$$

$$G_<(\omega) = i n_0 \int_{-\infty}^{+\infty} d\tau \; e^{i(\omega - \varepsilon + i\gamma \, \mathrm{sgn}\, \tau)\tau} d\tau$$

$$= i n_0 \left(\int_0^{\infty} d\tau \; e^{i(\omega - \varepsilon + i\gamma)\tau} + c.c. \right)$$

$$= n_0 \left(- \frac{1}{\omega - \varepsilon + i\gamma} - c.c. \right)$$

$$= n_0 \frac{2i\gamma}{(\omega - \varepsilon)^2 + \gamma^2}$$