1.2

(1)

$$\dot{x}_1 = v_1 = \frac{1}{m}(p_1 - qA_1)$$
$$\dot{p}_1 = 0 \longrightarrow p_1 = \text{CONST} = p_{10}$$
$$v_1 = \frac{1}{m}\left(p_{10} - \frac{qE_0}{2\alpha}\exp(-2\alpha t)\right)$$

(2)

$$\dot{x}_2 = v_2 = \frac{p_2}{m}$$
$$\dot{p}_2 = qE_0\exp(-2\alpha t) \longrightarrow p_2 = p_{20} - \frac{qE_0}{2\alpha}\exp(-2\alpha t)$$
$$v_2 = \frac{p_{20}}{m} - \frac{qE_0}{2m\alpha}\exp(-2\alpha t)$$

$p_1 \neq p_2$ NOT GAUGE INV.

$v_1 = v_2$ GAUGE INV.

$$a = \dot{v} = \frac{qE_0}{m}\exp(-2\alpha t) = \frac{qE}{m}$$

$$\left(x = x_0 + \frac{p_{20}}{m}t + \frac{qE_0}{m(2\alpha)^2}\exp(-2\alpha t)\right)$$ NOT REQUESTED

7.5

d) In cylindrical coordinates

$$x^2+y^2=r^2$$

$$L_z = -i\frac{\partial}{\partial \varphi}$$

$$-\hat{P}^2 = +\nabla^2 = \frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r} + \frac{1}{r^2}\frac{\partial^2}{\partial \varphi^2} + \frac{\partial^2}{\partial z^2}$$

$$\frac{\partial^2}{\partial \varphi^2} = -L_z^2 \qquad\qquad \frac{\partial^2}{\partial z^2} = -P_z^2$$

P_z have eigenvalues K_z $\quad (\hbar = 1)$

L_z " " m

$\Longrightarrow$

$$-P^2 = \underbrace{\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}}_{=\nabla_r^2} - \frac{1}{r^2} m^2 - K_z^2$$

see expression above

$$H = \frac{1}{2m}\left[\overbrace{-\nabla_r^2 + \frac{1}{r^2}m^2 + k_z^2}^{p^2} + \left(\frac{qB}{2}\right)^2 r^2 - qBm\right]$$

Schrödinger equation

$$H\psi = E\psi$$

(1) $k_z = m = 0$

$$\left(-\nabla_r^2 + A^2 r^2\right)\psi = \overbrace{2mE}^{\lambda}\psi$$

Ansatz $\psi = e^{-\beta r^2}$

$$\underbrace{\frac{1}{r}\frac{\partial}{\partial r} r \frac{\partial}{\partial r}}_{\nabla_r^2}\psi = \frac{1}{r}\frac{\partial}{\partial r} r\left(-2\beta r e^{-\beta r^2}\right)$$

$$-2\beta \frac{1}{r} \frac{\partial}{\partial r} r^2 e^{-\beta r^2} =$$

$$= -2\beta \frac{1}{r}\left(2r - 2\beta r^3\right) e^{-\beta r^2}$$

$$= -4\beta\left(1 - \beta r^2\right) e^{-\beta r^2}$$

$\Rightarrow$ Schröd. eq.

$$\left(4\beta\left(1-\beta r^2\right) + A^2 r^2 - \lambda\right) e^{-\beta r^2} = 0$$

$$\Rightarrow \beta^2 = \frac{A^2}{4} \qquad \lambda = 4\beta = 2A$$

$$2mE = 2A = 2\left(\frac{qB}{2}\right)$$

$$E = \frac{1}{2} \frac{qB}{m} = \frac{1}{2} W_c$$

↑ cyclotron frequency

Corresponds to the energy of the lowest Landau level (see script)