6.5*

$$C^+_{1\uparrow} C^+_{1\downarrow} |0\rangle = |G_2\rangle$$

$$\hat{V} |G_2\rangle = U |S_2\rangle \qquad (\text{from } 6.2)$$

similarly $\hat{V} |S_2\rangle = \hat{V} C^+_{2\uparrow} C^+_{2\downarrow} |0\rangle = U |G_2\rangle +$ terms outside of the grad

MATRIX ELEMENTS OF $\hat{H}_I = \hat{H} + \hat{V}$ IN THIS SUBSPACE

$$\begin{array}{c} \langle G_2| \\ \langle S_2| \end{array} \begin{pmatrix} 0 & U \\ U & 2\alpha \end{pmatrix}$$

$$E_\pm = \alpha \pm \sqrt{\alpha^2 + U^2}$$

$$E_- \simeq \alpha - \alpha\sqrt{1 + \frac{U^2}{\alpha^2}} \simeq -\frac{1}{2}\frac{U^2}{\alpha} + o(U^4)$$

OK

Eigenvector $\begin{pmatrix} -2\alpha - E_- & U \\ U & -E_- \end{pmatrix} \begin{pmatrix} x \\ y \end{pmatrix} = Ux - E_- y = 0$

$$x = 1 \qquad y = \frac{U}{E_-} \simeq -\frac{U}{2\alpha}$$

$$\Rightarrow |\bar{G}\rangle = |G_2\rangle + \frac{U}{E_-} |S_2\rangle$$

First order correction to the ground state

$$\delta |\bar{G}\rangle \simeq -\frac{\langle S_2 | \hat{V} | G_2 \rangle}{E_{S_2} - E_{G_2}} |S_2\rangle \simeq -\frac{U}{2\alpha} |S_2\rangle$$ V

There are 4 more states $C^+_{1\sigma_1} C^+_{0\sigma_2} |0\rangle$ but their order U^0 energy is $-\alpha$ so the correction can be $-\alpha + o(U)$ and cannot be lower than E_- if U is sufficiently small