7.3

$\vec{q} = q\hat{e}_x \qquad \vec{u}_{k1} = \vec{e}_y$

(i) $\quad |n\rangle = \frac{1}{\sqrt{n!}} \left(a^\dagger_{q1}\right)^n |0\rangle$

$$\vec{E}_k = -4\pi \vec{\Pi}_k = 4\pi \sum_s \hat{p}_{ks} \vec{u}_{ks}$$

$$\hat{p}_{ks} = -i \underbrace{\sqrt{\frac{\hbar k}{8\pi}}}_{\gamma_k} \left(a_{ks} - a^\dagger_{-k,-s}\right)$$

$$\vec{E}(\vec{r}) = \frac{1}{\sqrt{\Omega}} \sum_k e^{i\vec{k}\cdot\vec{r}} \vec{E}_k$$

$$\langle n | \vec{E}(\vec{r}) | n \rangle = 0$$

since $\vec{E}$ is linear in $a, a^\dagger$, which changes $n \to n \pm 1$

$$\vec{E}(\vec{r}=0)|n\rangle = \frac{4\pi}{\sqrt{\Omega}} \sum_{k,s}^{\text{half}} \vec{u}_{ks}(-i\gamma_k)\left(a_{ks} - a^\dagger_{-k-s} + a_{-k-s} - a^\dagger_{ks}\right) \cdot |n\rangle$$

+ this means for example only $K_x > 0$ or only $s = 1$

$$= -\frac{i\,\gamma_q}{\sqrt{\Omega}}\, 4\pi\, \vec{\mu}_{q1} \left(-\sqrt{n+1}\,|n+1\rangle + \sqrt{n}\,|n-1\rangle \right)$$

+ terms in which photons with $k, s \neq (q, 1)$ are created.

$$\langle n+1 | \vec{E}(0) | n \rangle = \frac{i\,\gamma_q}{\sqrt{\Omega}}\, \vec{\mu}_{q1} \sqrt{n+1}\; 4\pi$$

$$|\psi\rangle = \left(|n\rangle + i\alpha\, |n+1\rangle \right) \frac{1}{\sqrt{2}}$$

$$\langle \psi | \vec{E}(0) | \psi \rangle = \frac{1}{2} \left(i\alpha\, \langle n | \vec{E}(0) | n+1 \rangle + \text{c.c.} \right)$$

$$= -\frac{2\pi\,\gamma_q\,\alpha}{\sqrt{\Omega}}\, \vec{\mu}_{q1} \sqrt{n+1} + \text{c.c.}$$

$$= -\frac{4\pi\,\gamma_q\, \vec{\mu}_{q1}}{\sqrt{\Omega}} \sqrt{n+1}\; \mathrm{Re}\,\alpha$$

$$= -\sqrt{2\pi q}\; \frac{\vec{\mu}_{q1}}{\sqrt{\Omega}} \sqrt{n+1}\; \mathrm{Re}\,\alpha \qquad \vec{\mu}_{q1} = \vec{e}_x$$

† There might be a factor 2 incorrect somewhere

$$B(\vec{r}) = \vec{\nabla}\times\vec{A} = \frac{1}{\sqrt{\Omega}}\vec{\nabla}\sum_{k} e^{i\vec{k}\cdot\vec{r}} A_k$$

$$= \frac{1}{\sqrt{\Omega}}\sum_{k} i\vec{k}\times\vec{A}_k\, e^{i\vec{k}\cdot\vec{r}}$$

$$= \frac{i}{\sqrt{\Omega}}\sum_{k,s}^{r=0} \vec{k}\times\vec{u}_{ks}\left(\hat{q}_{ks} + \hat{q}_{-k,-s}\right)$$

$$\hat{q}_{ks} + \hat{q}_{-k-s} = \underbrace{\beta_k}_{\sqrt{\frac{2\pi\hbar}{k}}}\left(a_{ks} + a^{+}_{ks} + a_{-k-s} + a^{+}_{-k-s}\right)$$

ONLY RELEVANT TERM

$k = q \quad s = 1$

$$\Rightarrow \vec{B}(\vec{0})|m\rangle = \frac{i}{\sqrt{\Omega}}\beta_q \underbrace{\vec{q}\times\vec{u}_{q,1}}_{q\hat{e}_y}\left(a_{q1} + a^{+}_{q1}\right)|m\rangle$$

$$= \frac{i\,\beta_q\, q\,\hat{e}_y}{\sqrt{\Omega}}\sqrt{m+1}\;|m+1\rangle$$

$$\langle\psi|B(0)|\psi\rangle = \frac{i}{\sqrt{2}}\left(i\alpha\frac{\beta_q q}{\sqrt{\Omega}}\hat{e}_y\sqrt{m+1} - c\,c\right)$$

$$= -\sqrt{\frac{2}{\Omega}}\beta_q q\, e_y\sqrt{m+1}\;\mathrm{Re}\,\alpha$$

$$\beta_q q = \sqrt{2\pi\hbar q}$$

$$= -\sqrt{\frac{4\pi\hbar q}{\Omega}}\, e_y\sqrt{m+1}\;\mathrm{Re}\,\alpha$$