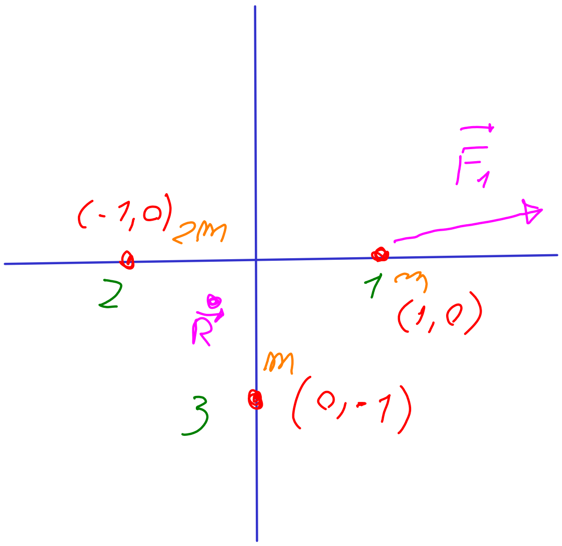
K2



$$\vec F_{ij} = \alpha(\vec r_j - \vec r_i) = \alpha\, |\vec r_j - \vec r_i|\, \vec e_{ji}$$

$$\Rightarrow U_{ij} = -\frac{\alpha}{2} |\vec r_{ij}|^2 \qquad \left(\vec F_{ij} = -\vec\nabla_{r_j} U_{ij} \right)$$

$$\vec F_1 = \alpha m^2 \underbrace{(\vec r_1 - \vec r_3)}_{=\binom{1}{1}} + 2\alpha m^2 \underbrace{(\vec r_1 - \vec r_2)}_{=\binom{2}{0}} = \alpha m^2 \begin{pmatrix} 1 \\ 1 \end{pmatrix} + \begin{pmatrix} 4 \\ 0 \end{pmatrix}$$

$$= \alpha m^2 \begin{pmatrix} 5 \\ 1 \end{pmatrix}$$

$$|r_{13}| = |r_{23}| = \sqrt{2} \qquad r_{12} = 2$$

$$\Rightarrow U = U_{12} + U_{13} + U_{23} = -\frac{\alpha}{2} m^2 (2 \cdot 4 + 1 \cdot 2 + 2 \cdot 2)$$

$$= -7 \alpha m^2$$

$$\vec R = \frac{\binom{1}{0} + 2\binom{-1}{0} + \binom{0}{-1}}{4} = -\frac{1}{4}\begin{pmatrix} 1 \\ 1 \end{pmatrix}$$

$$\vec v_1 = u \begin{pmatrix} 0 \\ 1 \end{pmatrix} \qquad \vec V_R = \frac{u}{4} \begin{pmatrix} 0 \\ 1 \end{pmatrix}$$

$$T = \frac{1}{2} m u^2 \qquad T_R = \frac{1}{2}\, 4m \frac{u^2}{16} = \frac{m u^2}{8}$$

$$T_r = T - T_R = m u^2 \left(\frac{4}{8} - \frac{1}{8} \right) = \frac{3}{8} m u^2$$

K3

(a) $x^2+y^2=R$ $\qquad f=2$

(b)
$$x = R\cos\varphi$$
$$y = R\sin\varphi$$
$$z = z$$

(c)
$$\dot{x} = -R\dot{\varphi}\sin\varphi$$
$$\dot{y} = R\dot{\varphi}\cos\varphi$$
$$\dot{z} = \dot{z}$$

(d)
$$T = \frac{1}{2}m\left(R^2\dot{\varphi}^2+\dot{z}^2\right)$$
$$U = -mgx = -mg.R\cos\varphi$$
$$L = T-U$$

(e) z ist zyklisch $\Rightarrow$ (a) $\dfrac{\partial L}{\partial \dot{z}} = m\dot{z} = \text{const.}$ $\quad\left(\begin{matrix}\text{oder}\\ m\ddot{z}=0\end{matrix}\right)$

$$\frac{d}{dt}\frac{\partial L}{\partial \dot{\varphi}} = mR^2\ddot{\varphi} = \frac{\partial L}{\partial \varphi} = -mgR\sin\varphi \quad (b)$$

(f) (a) $z = a + bt$

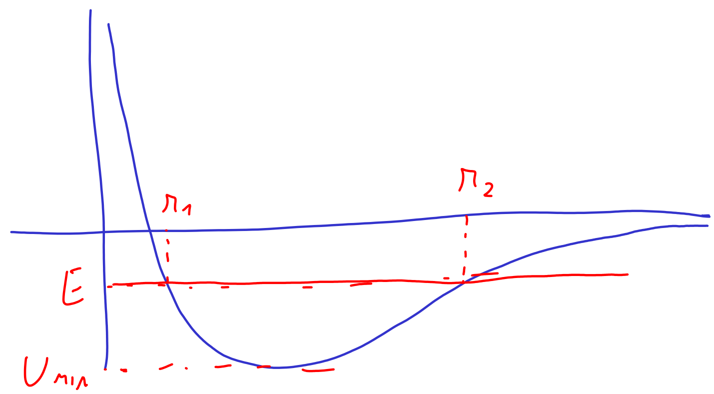
(b) $\cancel{m}R^2\ddot{\varphi} = -\cancel{m}gR\varphi \qquad \ddot{\varphi} = -\frac{g}{R}\varphi$

$$\varphi = A\cos\omega t + B\sin\omega t \qquad \omega = \sqrt{\frac{g}{R}}$$

K1

(a) $U_{eff} = \frac{\ell^2}{2mr^2} - \frac{\alpha^2}{r^2+c^2}$

(b)



(c) $U_{min} \le E < 0$ $\qquad \beta^2 = \frac{\ell^2}{2m}$ $\qquad z = r^2$

$$U_{eff} = \frac{\beta^2}{z} - \frac{\alpha^2}{z+c^2}$$

$$U'_{eff} = -\frac{\beta^2}{z^2} + \frac{\alpha^2}{(z+c^2)^2} = 0 \quad = \quad \frac{-(\beta(z+c^2))^2 + (\alpha z)^2}{\text{NENNER}} = 0 .$$

$\alpha z = \beta(z+c^2)$ (ODER GL. 2° ORDNUNG) $\qquad U_{min} = \frac{\beta^2}{z_{min}} - \frac{\alpha^2}{z_{min}+c^2}$

$$z_{min} = \frac{\beta c^2}{\alpha - \beta}$$

(d) Wenn $\beta > \alpha$ Weil in diesem Fall
und die Kurve oberhalb der E=0 Achse bleibt $\lim_{r\to\infty} U_{eff} = 0^+$ $\Rightarrow$ $\ell = \sqrt{2m}\,\alpha$

(e) $E = U_{eff}(r_i)$ $\qquad \frac{\beta^2}{r^2} - \frac{\alpha^2}{r^2+c^2} - E = 0$

$$\frac{\beta^2 r^2 + \beta^2 c^2 - \alpha^2 r^2 - E r^2 (r^2+c^2)}{r^2(r^2+c^2)}$$

$r^2 = z$

$E < 0 \Rightarrow |E| = -E$

$$-(-\beta + \alpha + |E|c^2)z + \beta c^2 + |E| z^2 = 0$$

$$b = \alpha - \beta + |E|c^2$$

$$z_{1,2} = \frac{b \mp \sqrt{b^2 - 4\beta c^2 |E|}}{2|E|} \qquad r_{1,2} = \sqrt{z_{1,2}}$$

(f) $T = 2\int_{r_1}^{r_2} \frac{dr}{\sqrt{F(r)}} = 2\int_{r_1}^{r_2} \frac{dr}{\sqrt{\frac{2}{m}(E - U_{eff}(r))}}$