4.1

$$U(\rho) = \frac{\alpha}{\rho^2} \qquad \ell = \mu\, b\, v$$

$$E = \frac{1}{2}\mu v^2 \qquad \mu = \frac{m_1 m_2}{m_1+m_2}$$

$$\tilde{F}(\rho) = \left(\frac{\mu\rho^2}{\ell}\right)^2 \frac{2}{\mu}\left(E - U - \frac{\ell^2}{2\mu\rho^2}\right) = \underbrace{\frac{2E\mu}{\ell^2}}_{=A^2}\rho^4 - \rho^2\underbrace{\left(1 + \frac{2\alpha\mu}{\ell^2}\right)}_{=A^2B^2}$$

$$= A^2\rho^2(\rho^2 - B^2)$$

$$A^2 = \frac{\mu v^2 \mu}{\mu^2 b^2 v^2} = \frac{1}{b^2} \qquad A^2B^2 = 1 + \frac{2\alpha\mu}{\mu^2 b^2 v^2} = 1 + \frac{\alpha}{b^2 E}$$

UMKEHRPUNKT ρ_1 : $E = U_{EFF} \Rightarrow \tilde{F}(\rho_1) = 0 \Rightarrow \rho_1 = B$

$$\Delta\varphi = 2\int_B^\infty \frac{d\rho}{\sqrt{A^2\rho^2(\rho^2-B^2)}} = \frac{2}{A}\int_B^\infty \frac{d\rho}{\rho\sqrt{\rho^2-B^2}} \qquad \begin{aligned}\rho &= Bx\\ d\rho &= B\,dx\end{aligned}$$

$$= \frac{2}{AB}\int_1^\infty \frac{dx}{x\sqrt{x^2-1}} = \frac{\pi}{AB} = \frac{\pi}{\sqrt{1+\frac{\alpha}{b^2E}}}$$

$$\tilde{\Theta} = \pi - \Delta\varphi = \pi\left(1 - \frac{1}{\sqrt{1+\frac{\alpha}{b^2E}}}\right) \qquad \left(1+\frac{\alpha}{b^2E}\right)^{-\frac{1}{2}} \approx 1 - \frac{1}{2}\frac{\alpha}{b^2E}$$

FÜR b GROß

$$= \pi\left(1 - \left(1 - \frac{\alpha}{2b^2E}\right)\right) = \frac{\pi}{2}\frac{\alpha}{b^2E} \quad \Rightarrow \quad g = \sqrt{\frac{\alpha}{E}}$$

$$\Rightarrow b^2 = \frac{\pi}{2}\frac{g^2}{\tilde{\Theta}}$$

$$\frac{d\sigma}{d\Omega} = \frac{1}{2\sin\tilde{\Theta}}\frac{db^2}{d\tilde{\Theta}} = \frac{\pi}{4}\frac{g^2}{\tilde{\Theta}^3}$$

$$\sin\tilde{\Theta} \approx \tilde{\Theta}$$

4.2

SKRIPT: $\tan\Theta_L = \dfrac{\sin\tilde{\Theta}}{\cos\tilde{\Theta} + K}$ KLEINE $\tilde{\Theta}$ $\Rightarrow$ $\Theta_L = \dfrac{\tilde{\Theta}}{1+K}$

$K = \dfrac{m_1}{m_2}$

$$b^2 = \frac{\pi}{2} \frac{q^2}{\tilde{\Theta}} = \frac{\pi}{2} \frac{q^2}{\Theta_L} \frac{1}{1+K}$$

$$\Rightarrow \frac{d\sigma}{d\Omega_L} = \frac{\pi}{4} \frac{q^2}{(1+K)\,\Theta_L^3}$$

$\dfrac{dN}{dt}(\Delta\Omega) = \text{FLUSS} \cdot \dfrac{d\sigma}{d\Omega_L} \Delta\Omega$: TEILCHEN PRO ZEIT, DIE IN $\Delta\Omega$ GESTREUT WERDEN

$\text{FLUSS} = \rho \cdot v$ $\Rightarrow$

$$\left.\frac{dN}{dt}\right|_{\text{zw. }\Theta_1, \Theta_2} = \rho v \int \frac{d\sigma}{d\Omega} \, d\Omega =$$

$\int d\varphi$

$$\rho v \, 2\pi \frac{\pi}{4} \frac{q^2}{1+K} \int_{\Theta_1}^{\Theta_2} \frac{\sin\Theta}{\Theta^3} \, d\Theta = \frac{\pi^2}{2} \rho v \frac{q^2}{1+K} \frac{1}{2}\left(\frac{1}{\Theta_1} - \frac{1}{\Theta_2}\right)$$

4.3

$$x^2+y^2+(z-vt)^2-R^2=0$$

$$x = R\sin\theta\cos\varphi$$
$$y = R\sin\theta\sin\varphi$$
$$z = R\cos\theta + vt$$

$$\dot{x} = R\dot{\theta}\cos\theta\cos\varphi - R\dot{\varphi}\sin\theta\sin\varphi$$
$$\dot{y} = R\dot{\theta}\cos\theta\sin\varphi + R\dot{\varphi}\sin\theta\cos\varphi$$
$$\dot{z} = -R\dot{\theta}\sin\theta + v$$

$$T = \frac{1}{2}m\left(\dot{x}^2+\dot{y}^2+\dot{z}^2\right) =$$

$$= \frac{1}{2}m\left(R^2\dot{\theta}^2 + R^2\dot{\varphi}^2\sin^2\theta + v^2 - 2vR\dot{\theta}\sin\theta\right)$$

$$U = mgz = mg\left(R\cos\theta + vt\right)$$

$$L = T - U$$

$$\frac{d}{dt}\frac{\partial \mathcal{L}}{\partial \dot{\Theta}} = m\frac{d}{dt}\left(R^2\dot{\Theta} - v R \sin\Theta\right) = m\left(R^2\ddot{\Theta} - \cancel{v R\dot{\Theta}\cos\Theta}\right)$$

$$= \frac{\partial \mathcal{L}}{\partial \Theta} = m\left(R^2\dot{\varphi}^2\cos\Theta\sin\Theta - \cancel{vR\dot{\Theta}\cos\Theta}\right) + mgR\sin\Theta$$

$$\frac{\partial \mathcal{L}}{\partial \varphi} = 0 \quad \Rightarrow \quad \varphi \text{ ZYKLISCH}$$

$$\frac{\partial \mathcal{L}}{\partial \dot{\varphi}} = m R^2 \sin^2\Theta\,\dot{\varphi} = P$$

$$\Rightarrow \quad \dot{\varphi} = \frac{P}{mR^2\sin^2\Theta}$$

$$R^2\ddot{\Theta} = R^2\dot{\varphi}^2\cos\Theta\sin\Theta + gR\sin\Theta$$

$$= \frac{P^2\cos\Theta\,\cancel{\sin\Theta}}{m^2R^2\sin^{\cancel{4}\,3}\Theta} + gR\sin\Theta$$

$$= \frac{P^2\cos\Theta}{m^2R^2\sin^3\Theta} + gR\sin\Theta$$

$$\frac{\sin^4\Theta}{\cos\Theta} = -\frac{P^2}{m^2R^3g}$$