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ON EXTRACTING WEAK QUARK COUPLINGS FROM BOTTOM AND TOP DECAYS

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We list and discuss the various types of systematic uncertainties one faces when extracting quark mixing angles from the decays of bottom and top hadrons. Evaluating various suggested methods we conclude that a study of the lepton energy spectrum in B and T decays provides a reasonable way of obtaining the ratio of mixing angles; yet analysis of just the endpoint region will not lead to a reliable determination. Such results should be backed up by other studies based on very energetic kaons and on the mode $B^- \to \tau^- \bar{\nu}_\tau$. A dedicated effort should be made to search for flavour changing neutral currents in channels like $B, T \to \tau^+\tau^- X$.

1. Goals

We have quite substantial evidence that the vector mesons of the Υ family are made up of a fifth quark which carries electric charge of one third unit; this quark is usually referred to as bottom (or beauty) quark. Hadrons carrying a net bottom quantum number appear to be produced with a mass $\frac{1}{2}M(\Upsilon'') < M(B) < \frac{1}{2}M(\Upsilon''')$ and one has begun to study their weak decays [1]. The goals in such an analysis can be summarized as follows.

(i) Within the Standard Model the bottom quarks open up a new doublet of weak isospin with the hypothetical top quarks being their partners. The Kobayashi-Maskawa (KM) quark mixing angles [2] represent basic and crucial parameters of any theory of the weak forces (although they can of course not be explained within the standard model). The KM angles involving b quarks can best and most directly be determined in bottom decays.

(ii) The successes of the simple standard model have so far been impressive, but also somewhat unreasonable. Therefore the range of its validity has to be scrutinized very carefully, for example by searching for genuine flavour changing neutral currents in the bottom sector. From existing CESR data one can already conclude that such neutral currents do not mediate a very sizeable fraction of B decays [3] (most topless models are thus eliminated); yet these results do not close the chapter on such couplings since they can be generated on a smaller level by the exchange of Higgs states or family gauge bosons.

2. Problems

The operators relevant for weak decays are basically determined by short distance dynamics and therefore calculable in a constituent picture with some confidence. We can list four effective operators for charged currents [4]

$$\mathcal{L}_{\mathrm{eff}}(\Delta B=1)=\frac{G_F}{\sqrt{2}}\left\{\frac{c_+}{2}\left[O_+^{\mathrm{bc}}+\rho O_+^{\mathrm{bu}}\right]+\frac{c_-}{2}\left[O_-^{\mathrm{bc}}+\rho O_-^{\mathrm{bu}}\right]\right\},$$

$$\rho=\frac{U(\mathrm{b}\to\mathrm{u})}{U(\mathrm{b}\to\mathrm{c})},$$

where $U(\mathrm{i}\to\mathrm{j})$ stands for the mixing angle between quark i and j; O_+ $[O_-]$ are Fierz [anti] symmetric four-fermion operators and the coefficients $c_\pm$ are calculable in perturbation theory. We ignore penguin contributions since their coefficients in the Wilson expansion are rather small and the matrix element enhancement which makes them so important in strange decays is not significant in bottom decays [5].

Big uncertainties arise when one endeavours to compute matrix elements of these operators in order to describe on-shell processes like weak decays. The matrix elements will in general contain long distance effects which are notoriously hard even to estimate. This complication affects both the *normalization* of transition rates and the *shape* of distributions. The problem of normalization, which is at the heart of the "charm puzzle", will not be discussed here in great detail since we expect the spectator ansatz [4] to give a very reasonable description of bottom decays. Therefore we will concentrate on possible effects of *hadronic long distance dynamics* on the *shape* of distributions from which one hopes to extract the weak parameter ρ.

3. KM angles in semi-leptonic B decays

It has been stated many times before that an analysis of semi-leptonic B decays offers the best and most direct handle on the KM angles $U(\mathrm{b}\to\mathrm{c})$ and $U(\mathrm{b}\to\mathrm{u})$. This statement is based on a number of reasons.

(a) It appears so far that only the non-leptonic decays of strange and charmed hadrons defy a definitive theoretical description whereas semi-leptonic decays seem to fit into the existing theoretical picture.

(b) The charged lepton and its accompanying neutrino will carry off a large fraction of the available energy; therefore less energy is left over for the hadrons and thus fewer hadronic channels with lower multiplicities will contribute making hadronic complexities less awesome.

(c) More specifically it has been suggested that semi-leptonic D decays can basically be described in terms of two channels:

$$(D \to \ell^+ \nu_\ell X) \simeq (D \to \ell^+ \nu_\ell K) + (D \to \ell^+ \nu_\ell K^*) \tag{3.1}$$

and data on B decay multiplicities indicate the analogous pattern to hold for B decays [3]:

$$(B \to \ell^- \bar{\nu}_\ell X) \simeq (B \to \ell^- \bar{\nu}_\ell D) + (B \to \ell^- \bar{\nu}_\ell D^*) . \tag{3.2}$$

Yet, more theoretical finesse has to be employed when extracting $U(b \to c)$ and $U(b \to u)$ from decay rates because the spectator ansatz (like the other approaches) is not derived from the theory. It merely represents a recipe considered to be plausible. In that scheme one describes $B \to \ell^- \bar{\nu}_\ell +$ charm [$B \to \ell^- \bar{\nu}_\ell +$ no charm] by $b \to \ell^- \bar{\nu}_\ell c$ [$b \to \ell^- \bar{\nu}_\ell u$]. It is a trivial exercise to calculate the spectrum of the charged lepton (e or μ): $d\Gamma/dE_\ell(b \to \ell^- \bar{\nu}_\ell c[u])$; the distributions are shown in fig. 1 as they appear in the b quark rest system for the parameters $M_b = 4.8$ GeV, $m_c = 1.3$–1.8 GeV and $m_u = 0.25$ GeV. (We will come back to the question of the "proper choice" of these parameters.) Fig. 1 exemplifies the usual statement that the endpoint region of the charged lepton spectrum is the best place to search for $b \to u$ transitions.

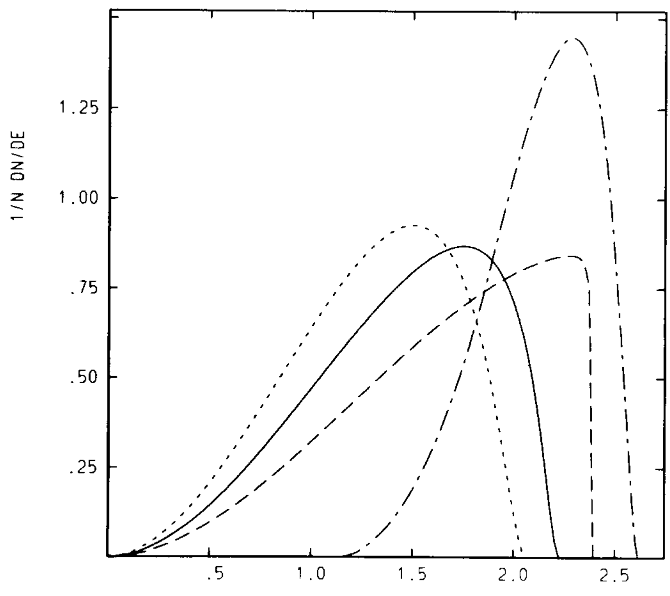

Fig. 1. Energy spectra of primary leptons: $b \to \ell^- \bar{\nu}_\ell q$ with $m_q = 0.25$ GeV (– – –), 1.3 GeV (—), 1.8 GeV (------), $B^- \to \ell^- \bar{\nu}_\ell +$ glue with $m_B = 5.255$ GeV, m (glue) averaged over 0.28 GeV to 1.5 GeV and E (glue) cut off at 1.5 GeV (· – · –). These and subsequent curves are normalized to unit area unless stated otherwise.

One appeals then to duality ideas to argue that this simple quark-lepton spectrum should be a good approximation to the "real" spectrum where hadrons are involved.

The crux however is that the ratio $|\rho| = |U(\mathrm{b} \to \mathrm{u})|/|U(\mathrm{b} \to \mathrm{c})|$ which we want to extract from the data is presumably quite small:

$$|\rho| \ll 1 .$$

This sets the degree of reliability required for our theoretical tools. There are basically three types of uncertainties which we will discuss in turn:

(i) the degree to which hadronic resonances in the final state affect the lepton spectra in the $\mathrm{b} \to \mathrm{c}$ and $\mathrm{b} \to \mathrm{u}$ transitions. Even if it turns out that the process $\mathrm{B} \to \ell^+ \bar{\nu}_\ell + \text{charm}$ is almost saturated by the two channels $\mathrm{B} \to \ell^- \bar{\nu}_\ell \mathrm{D}$ and $\mathrm{B} \to \ell^- \bar{\nu}_\ell \mathrm{D}^*$ this does not imply by any means that $\mathrm{B} \to \ell^- \bar{\nu}_\ell + \text{(no charm)}$ is saturated by $\mathrm{B} \to \ell^- \bar{\nu}_\ell \pi$ and $\mathrm{B} \to \ell^- \bar{\nu}_\ell \rho$. The mode $\mathrm{B} \to \ell^- \bar{\nu}_\ell \pi$ should actually be a very atypical one considering the size of the available phase-space. Other modes $\mathrm{B} \to \ell^- \bar{\nu}_\ell \mathrm{X}$ with $\mathrm{X} = \omega, \delta, \mathrm{S}^*, \mathrm{A}_i$, multipions etc. should be very prominent among this class of semi-leptonic B decays and this could affect the endpoint spectrum of the charged leptons;

(ii) the significance of the effect of weak annihilation, presumably not important on the overall scale for B decays, on the endpoint region and thus on the value of ρ derived there;

(iii) the proper values for the various quark masses and the appropriate Lorentz boosts to apply.

Consider (i): To calculate all the exclusive transitions like $\mathrm{B} \to \ell^- \bar{\nu}_\ell \rho, \mathrm{B} \to \ell^- \bar{\nu}_\ell \mathrm{A}_1$, etc. one has to know the appropriate form factors of the various resonances including their momentum dependence. Being currently impossible, we adopt an alternative procedure for predicting *inclusive* rates and estimating their sensitivity to resonance formation. We calculate the lepton spectrum first in the spectator ansatz for some quark mass parameters. This computation ignores all resonance effects. Then we notice that six prominent resonances can be found in that kinematical regime, namely the $\eta, \rho/\omega, \delta, \mathrm{A}_1, \mathrm{A}_2, \mathrm{A}_3/\rho'$. The invariant mass of the hadronic system is viewed as coming from the c or u quark originating in b quark decay and the spectator $\bar{\mathrm{q}}$. One does not know the momentum distribution of the antiquark $\bar{\mathrm{q}}$ inside B (or D) mesons. But we consider it highly unlikely that the light quarks or gluons carry a significant fraction of energy since the mass of the B meson is viewed as coming mainly from a large b quark mass. We actually employ a parton picture with the B meson consisting of the b quark and the spectator $\bar{\mathrm{q}}$. Using $M_\mathrm{B} = 5.255$ GeV, $E_\mathrm{b} = M_\mathrm{b} = 4.8$ GeV and $m_{\bar{\mathrm{q}}} = 0$ or 0.25 GeV it then involves a trivial algebraic operation to determine the invariant hadronic mass. We then take the differential rate $\mathrm{d}^2\Gamma/\mathrm{d}E_\mathrm{lept}\,\mathrm{d}E_\mathrm{c,u}$ of the spectator process and weight it with six Breit-Wigner curves of equal strength at the appropriate resonance mass. Due to the concept of duality we retain the same normalization for the integrated distribution thus obtaining a change in *shape*. We then repeat the same procedure but leave out one of the resonances as a further sensitivity test. It turns out that the largest variation is obtained when leaving out either the ρ/ω or the A_3, amounting to a change of roughly 10–15% only, a very welcome result, shown in fig. 2. The distribution depends only very little on $m_{\bar{q}}$.

The analogous procedure is adopted for the transition $B \to \ell^- \bar{\nu}_\ell +$ charm with $B \to \ell^- \bar{\nu}_\ell D$ and $B \to \bar{\nu}_\ell \ell^- D^*$ largely saturating the inclusive rate. The endpoint spectrum depends on the relative size of the two rates and the D and D* form factors. The freedom in the choice of the charm quark mass introduces the main uncertainty into our ansatz. In fig. 3 we have put $m_b = 4.8$ GeV and $m_{\bar{q}} = 0.25$ GeV while varying m_c between 1.3 and 1.7 GeV. Resonance effects are clearly significant, but not in the endpoint region; we will comment on that later.

In fig. 4 we have put $m_b = 5.0$ GeV while varying m_c between 1.4 and 1.7 GeV to test further the sensitivity of the model on the input parameters. The spectator and the b quark are then almost at rest to each other inside the B meson. For $m_c = 1.4$ GeV there is a large difference between the curves including and excluding resonance formation. However a more detailed analysis of the kinematical regimes involved for that set of parameters reveals that the application of the model might be doubtful there since one deals with basically just one very narrow state, the D.

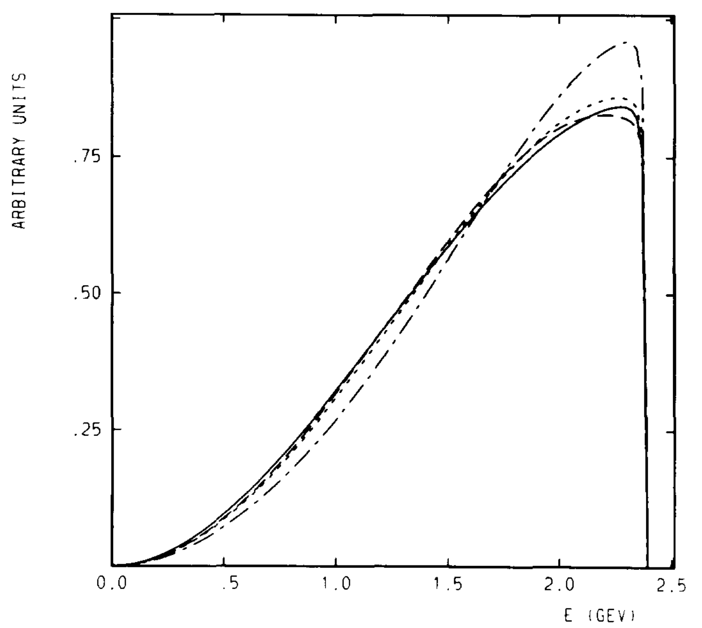

Fig. 2. Energy spectra of primary leptons in $b \to \ell^- \bar{\nu}_\ell u$ with $m_u = 0.25$ GeV and no resonances (—), six resonances (------), five resonances with ρ/ω (– – –) or A_3/ρ' (· – · – ·) left out.

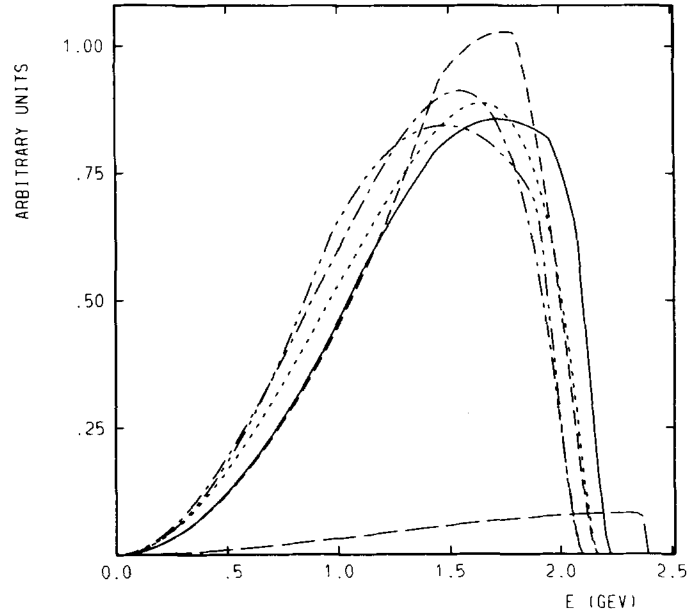

Fig. 3. Energy spectra of primary leptons in $b \to \ell^- \bar{\nu}_\ell c$ with $E_b = m_b = 4.8$ GeV, spectator mass $m_u = 0.25$ GeV and $m_c = 1.3$ GeV and D + D* (—), $m_c = 1.5$ GeV and no resonances (------), $m_c = 1.5$ GeV and D + D* (– – –), $m_c = 1.7$ GeV and no resonances (– · – ·), $m_c = 1.7$ GeV and D + D* (– · · –). For comparison: the curve describing $b \to \ell^- \nu_\ell u$, $m_u = 0.25$ GeV normalized to an area of 0.1 units.

Consider (ii). There are good reasons to expect weak annihilation (WA) to play a relatively minor role overall in B decays. Yet WA (in the *s* channel) can contribute to semi-leptonic B decays. In that process the hadrons have to be generated from gluons; those are expected to carry little hadronic energy – thus only light hadrons like η, ω, $\pi\pi$ would be produced. This in turn implies that the lepton spectrum is hard. Therefore WA will contribute to $B^- \to \ell^- \bar{\nu}_\ell X$ mainly in the endpoint region and will thus obtain a higher significance for extracting ρ from there. In order to make this statement quantitative we treated WA as

$$\begin{array}{l} B^- \to W^- + \text{gluons} \\ \qquad\quad\;\; \llcorner\!\!\to \ell^- \bar{\nu}_\ell \qquad \text{gluons} \to \text{hadrons} \end{array}$$

with the "gluon mass" distributed evenly between 0.28 GeV ($\pi\pi$ threshold) and 1.5 GeV. The resulting lepton spectrum is included in fig. 1.

Consider (iii). We have discussed corrections of a basically dynamical nature in (i) and (ii) which affect the shape of the spectrum. The position of its endpoint reflects kinematical constraints which depend sensitively on the choice for the quark masses. This fact is actually the raison d'être for this approach to the KM angles. With our incomplete understanding of QCD we have however to allow for considerable uncertainties in the quark masses: $m_b = 4.8$–5.2 GeV, $m_c = 1.3$–1.8 GeV and $m_u = 0$–0.3 GeV. As shown in fig. 3 varying m_c between 1.3 and 1.7 GeV shifts the endpoint considerably thus jeopardizing identification of the few leptons in the endpoint spectrum of $b \to \ell^- \bar{\nu}_\ell u$. But fig. 3 also shows that as long as one can measure the lepton spectrum (almost) in the rest system of the decaying b quark the rapid fall off of the spectrum which is hardly affected by resonance effects still allows to separate $b \to c$ from $b \to u$ initiated transitions.

In general the b quark will not be at rest inside the meson and the B meson will have some lab momentum. Even a very small Lorentz boost will smear out the rapid drop of the spectrum thus making identification of b transitions extremely difficult. In this context it is highly desirable to determine the mass of B mesons either from exclusive decays like $B \to D^*\pi$, $\psi\bar{K}^{(*)}$, $\Lambda_c\bar{N}$ or from inclusive studies [6].

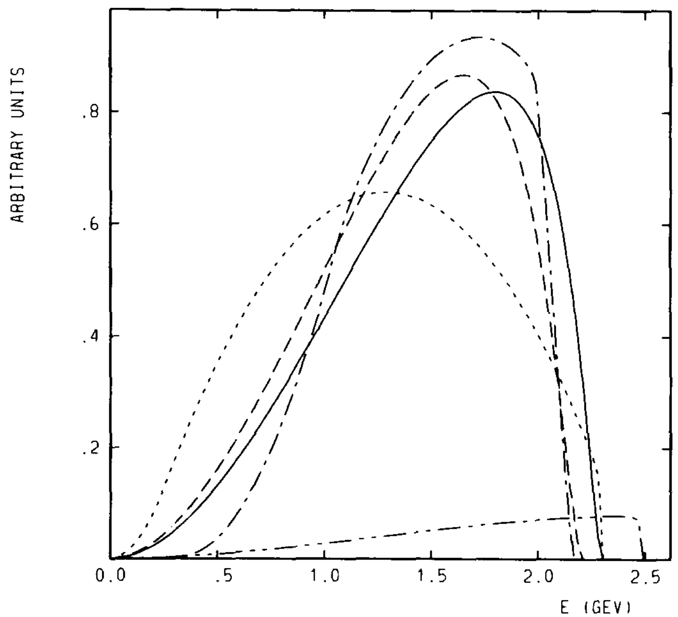

Fig. 4. Energy spectra of primary leptons in $b \to \ell^- \bar{\nu}_\ell c$ with $E_b = m_b = 5.0$ GeV and $m_c = 1.4$ GeV and no resonances (—), $m_c = 1.4$ GeV and D + D* (------), $m_c = 1.7$ GeV and no resonances (–––), $m_c = 1.7$ GeV and D + D* (·–·–). For comparison: the curve describing $b \to \ell^- \nu_\ell u$ normalized to an area of 0.1 units.

To summarize the results presented so far:

(a) We conclude from our model calculations that resonance formation affects $b \to \ell^- \bar{\nu}_\ell u$ transitions very little ($\sim$ 10–15%).

(b) It can on the other hand have a considerable impact on $b \to \ell^- \bar{\nu}_\ell c$ modes.

(c) The endpoint spectrum depends of course very sensitively on the quark masses m_b and m_c and on the momentum of the b quark, quantities that are a priori not well known. Yet if the b quarks possess little lab momentum then these ambiguities will not curtail the ability to identify $b \to u$ modes due to the then very sharp drop in the lepton spectrum for $b \to c$ transitions. This sharp drop is hardly affected by resonance formation.

(d) If however the b quarks have non-negligible lab momenta the lepton spectrum will fall off more gently; uncertainties in the proper choice of m_c and resonance effects will become more crucial thus making an analysis of KM angles much more difficult. In that case one has to measure the energy spectrum over as wide a range as possible to get a better handle on m_c and on the *impact of resonances* before extrapolating into the endpoint region.

(e) A separate analysis of B^0 and B^- semi-leptonic decays would be desirable in order to evaluate the impact of weak annihilation.

4. Other handles on KM angles in B-decays

With all these uncertainties and problems one will want to obtain ρ using a second, independent method as back-up. We list 3 options:

(a) secondary leptons from charm decay,

$$\begin{array}{l} B \to D + X \\ \quad\;\; \hookrightarrow \ell^+ + \tilde{X}, \end{array}$$

(b) the kaon yield,

(c) the exclusive mode $B^- \to \tau^- \bar{\nu}_\tau$.

Consider (a). This does not represent an attractive method since it contains large systematic uncertainties: the secondary leptons from charm decays are

(α) presumably too soft to be identified in measurements near B threshold and

(β) do *not* offer an unambiguous handle on the size of the charm contribution. For the semi-leptonic branching ratios of the different charm hadrons seem to vary strongly and D* production with its subsequent decay preferentially into D^0 has to be included.

Secondary leptons do however present a background if the analysis of primary leptons is extended to low energies.

Consider (b). There are many sources of kaons:

(α) primary kaons:

$$\begin{array}{l} \mathrm{b} \to \mathrm{c\bar{c}s} \\ \qquad\quad \rightarrow \mathrm{K} \end{array}$$

$$\begin{array}{l} \mathrm{b} \to \mathrm{c\bar{u}s} \qquad \text{(Cabibbo suppressed)} \\ \qquad\quad \rightarrow \mathrm{K} \end{array}$$

$$\begin{array}{l} \mathrm{b} \to \mathrm{u\bar{c}s} \\ \qquad\quad \rightarrow \mathrm{K}, \end{array}$$

(β) secondary kaons from charmed hadrons

$$\begin{array}{l} \mathrm{b} \to \overset{(-)}{\mathrm{c}} + \mathrm{X} \\ \qquad\ \ \rightarrow \mathrm{K}\ , \end{array}$$

(γ) secondary kaons from τ leptons

$$\begin{array}{l} \mathrm{b} \to \tau^- + \mathrm{X} \\ \qquad\ \ \rightarrow \mathrm{K}, \end{array}$$

(δ) tertiary kaons due to $\mathrm{s\bar{s}}$ excitation from the vacuum.

Tertiary kaons are of course uninteresting; the information carried by the secondary kaons is highly diluted since the finally observed kaons are the product of a long and complex fragmentation process. Furthermore one has to measure *both* charged and neutral kaons since D^0 and D^+ seem to possess greatly differing branching ratios into K^- and $\overline{\mathrm{K}}^0$. Finally one has to keep in mind the problem of "missing" K mesons in D decays, namely $\mathrm{BR}(\mathrm{D}^+ \to \mathrm{K} + \mathrm{X}) < \mathrm{BR}(\mathrm{D}^0 \to \mathrm{K} + \mathrm{X}) < 1 - \tan^2\theta_\mathrm{c}$! [7].

The *primary* kaons hold out some promise of being useful in extracting ρ. As shown in fig. 5 the energy spectrum of primary kaons is considerably harder in $\mathrm{b} \to \mathrm{u\bar{c}s}$ than in $\mathrm{b} \to \mathrm{c\bar{c}s}$ or that of secondary etc. kaons. For primary kaons probe the mass scale inherent in the hadronic recoil system in a way similar to what leptons do in semi-leptonic B decays. The mode $\mathrm{b} \to \mathrm{c\bar{u}s}$ generates an almost identical spectrum but since it is Cabibbo suppressed it should not lead to an overwhelming background. Of course the precise form of the spectrum depends also on the quark fragmentation.

Consider (c). The decay rate for $\mathrm{B}^- \to \tau^- \bar{\nu}_\tau$ is given by [8]

$$\Gamma(\mathrm{B}^- \to \tau^- \bar{\nu}_\tau) = \frac{|U(\mathrm{b} \to \mathrm{u})|^2}{8\pi} G_\mathrm{F}^2 |f_\mathrm{B}|^2 M_\mathrm{B} m_\tau \left(1 - \frac{m_\tau^2}{M_\mathrm{B}^2}\right)^2 . \tag{4.1}$$

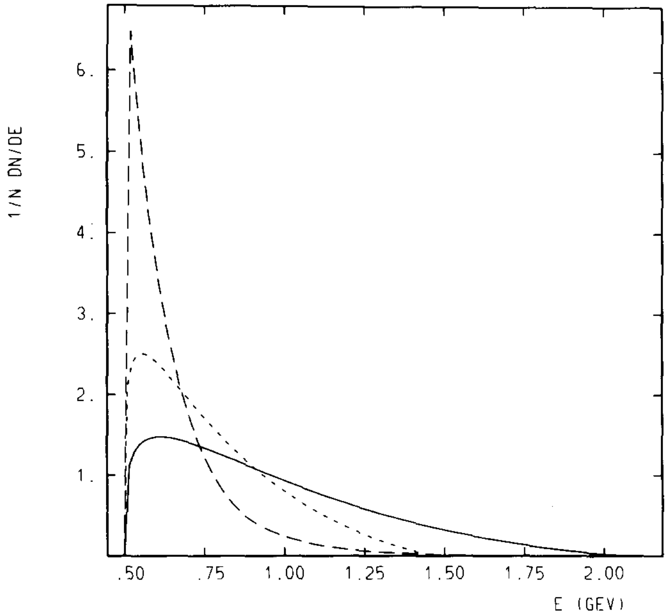

Fig. 5. Energy spectra of kaons: $b \to u\bar{c}s \to$ primary kaon (—), $b \to c\bar{c}s \to$ primary kaon (------) and $b \to c\bar{u}d \to$ secondary kaon (– – –) with $m_c = 1.5$ GeV, $m_s = 0.5$ GeV and $m_u = m_d = 0.25$ GeV. $D(z) =$ const. was used to describe charm quark fragmentation; strange quark fragmentation was treated à la Field-Feynman.

Before one can extract $|U(b \to u)|$ one has of course to know $|f_B|$. We do not possess a method to compute $|f_B|$ where reliability has been clearly established. We are however impressed by the successes of the SVZ approach and believe their estimate to be quite a reasonable one [9]:

$$|f_B| \lesssim 241 \text{ MeV}. \tag{4.2}$$

We are aware that if WA is significant in *inclusive* charm decays then the value of f_D as obtained in the SVZ ansatz is much too small. But as discussed elsewhere in more detail [10] the SVZ approach offers a handle on the decay constant in the *exclusive two-body* decay only; the corresponding quantity for inclusive transitions could be quite different. Thus we think that observation of $B^- \to \tau^- \bar{\nu}_\tau$ would provide us with important information on $|U(b \to u)|$, in particular if one has also observed $F^+ \to \tau^+ \nu_\tau$ [9].

5. $\Delta B \neq 0$ neutral currents

The question of flavour changing neutral currents is one of the most intriguing problems in flavour dynamics. In each new system it has to be studied anew in the most careful fashion. We already know that bottom changing neutral currents à la $B \to e^+e^-X$ do not generate a major part of B decays. This rules out almost all topless models. Yet we have to inquire whether there are any genuine flavour changing neutral currents. B decays allow looking for it in a somewhat novel way, namely via a mode involving τ leptons:

$$B \to \tau^+\tau^- X. \tag{5.1}$$

This is potentially a very powerful candidate if such neutral currents are mediated by Higgs bosons. Their coupling to fermions is typically given by the fermion masses; thus the strength of such neutral currents in $B \to \tau^+\tau^-X$ is $(m_b m_\tau)^2/(m_s m_\mu)^2 \sim$ few $\times\, 10^5$ times stronger than in $K_L \to \mu^+\mu^-$ for example. It poses of course quite a challenge to identify a mode like $B \to \tau^+\tau^-K$ or $B \to \tau^+\tau^-K^*$.

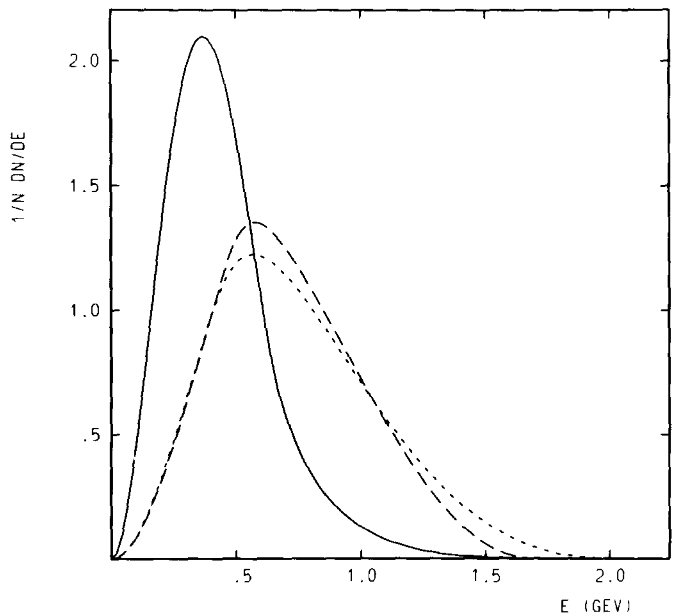

Fig. 6. Energy spectra of secondary leptons:

$$b \to \tau^+ \underset{\hookrightarrow \ell^-}{\tau^-} s \text{ via Higgs exchange } (---).$$

For comparison:

$$b \to \underset{\hookrightarrow \ell^-}{\tau^-} \nu_\tau c \;(------), \qquad b \to \underset{\hookrightarrow \ell^+}{\bar{c}} \bar{u} d \;(—).$$

$D(z) =$ constant was assumed for charm quark fragmentation.

Assuming $M(\text{Higgs}) = M_{\mathrm{H}} \gg M_{\mathrm{b}}$ one can calculate $\mathrm{b} \to \tau^+\tau^-\mathrm{s}$ in terms of a local four-fermion interaction and a normalization proportional to $1/M_{\mathrm{H}}^2$. In fig. 6 we show the energy spectrum for the Higgs mediated process

$$\begin{array}{l} \mathrm{b} \to \tau^+ \tau^- \mathrm{s} \\ \qquad\quad\; \llcorner\!\!\to \ell^- \bar{\nu}_\ell \nu_\tau , \end{array} \tag{5.2}$$

with the area arbitrarily normalized to unity. We can see that the lepton spectra for the two reactions

$$\begin{array}{l} \mathrm{b} \to \tau^+ \tau^- \mathrm{s} \\ \qquad\quad\; \llcorner\!\!\to \ell^- \end{array} , \qquad \begin{array}{l} \mathrm{b} \to \tau^- \bar{\nu}_\tau \mathrm{c} , \\ \qquad \llcorner\!\!\to \ell^- \end{array}$$

look very similar: unless one can use tagged events it might be very difficult to distinguish the two decay modes.

6. KM angles in top decays and $\Delta T \neq 0$ neutral currents

6.1. KM ANGLES

In top decays there are three types of charged current reactions namely

(a) $\mathrm{t} \to \mathrm{b}\ell^+\nu_\ell$,

(b) $\mathrm{t} \to \mathrm{s}\ell^+\nu_\ell$,

(c) $\mathrm{t} \to \mathrm{d}\ell^+\nu_\ell$,

and it is highly desirable to determine them separately. The dynamical situation here is even cleaner than in bottom decays since the energies involved are so much larger. Resonance effects etc. should therefore not play a significant role.

Again it is a careful analysis of the charged lepton spectrum which should provide us with crucial information on the mixing angles. By measuring the spectrum of primary leptons in $\mathrm{T} \to \ell^+\nu_\ell \mathrm{X}$ one can distinguish whether X contains bottom, i.e. heavy hadrons, or not. As a bonus even the secondary leptons from bottom decays,

$$\begin{array}{l} \mathrm{T} \to \mathrm{B} + \mathrm{X} \\ \qquad\;\; \llcorner\!\!\to \ell^- + \tilde{\mathrm{X}} , \end{array}$$

can be harnessed in such an analysis: (i) they should be sufficiently energetic to be identifiable and (ii), as already mentioned, we anticipate the semi-leptonic branching fractions of bottom states to be very close to each other. The spectra of primary and secondary leptons are shown in fig. 7.

A study of very energetic kaons offers in principle a handle on the $\mathrm{t} \to \mathrm{s}$ transition; a large background is provided, however, by the reaction $\mathrm{t} \to \mathrm{bc\bar{s}}$ and a

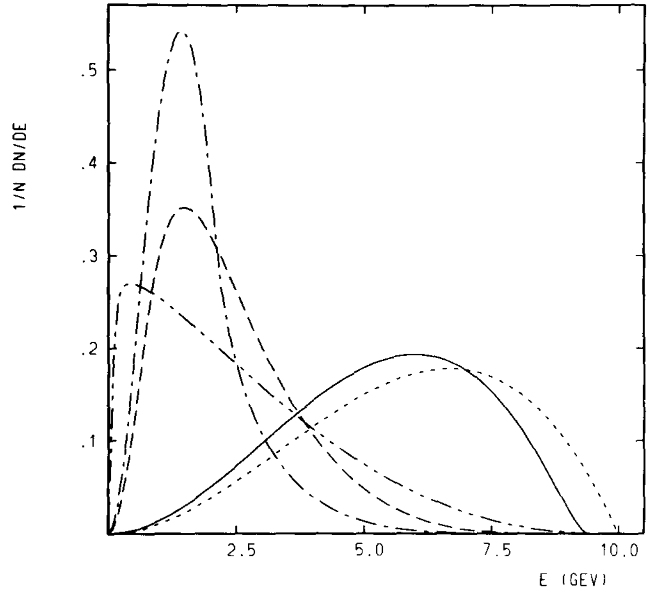

Fig. 7. Energy spectra of leptons in top decays:

$t \to \ell^+ \nu_\ell s$ (------), $\quad t \to \ell^+ \nu_\ell b$ (—),

$t \to b\bar{d}u$, bottom fragmentation $D_b(z) = (1 - 1/z - 0.1/(1-z))^{-2}$ [23] (- - -),
$\quad \llcorner\!\to \ell^- \bar{\nu}_\ell u$

$t \to b\bar{s}c$, bottom fragmentation $D_b(z) = \text{constant}$ (· — · —),
$\quad \llcorner\!\to \ell^- \bar{\nu}_\ell c$

$t \to \tau^+ \tau^- u$ via Higgs exchange (· · — · ·).
$\quad \llcorner\!\to \ell^- \bar{\nu}_\ell \nu_\tau$

$E_t = m_t = 20$ GeV, $m_b = 4.8$ GeV, $m_c = 1.5$ GeV, $m_s = 0.5$ GeV and $m_u = m_d = 0.25$ GeV were used.

final evaluation of the various methods can be made only when the actual top mass is known.

6.2. $\Delta T \neq 0$ NEUTRAL CURRENTS

The mode $T \to \tau^+\tau^- X$ appears as a promising candidate and should be searched for. Relative to $B \to \tau^+\tau^- X$ one gains another factor of $(m_t/m_\ell)^2 \gtrsim 16$ in coupling strength if Higgs bosons are involved. Identification of $T \to \tau^+\tau^- X$ should be simplified by jet characteristics: such energetic τ leptons will lead to one-prong "jets" with a large amount of missing energy.

6.3. COMPLETENESS RELATIONS OF KM ANGLES

With three generations of quarks there are only three independent KM angles and one phase. Thus completeness relations had to exist between the various mixing angles; e.g.

$$|U(\mathrm{b}\to\mathrm{u})|^2+|U(\mathrm{b}\to\mathrm{c})|^2+|U(\mathrm{b}\to\mathrm{t})|^2=1\,. \tag{6.1}$$

In principle one could check such relations. If the left-hand side of (6.1) did not saturate the unitarity bound one would conclude that there must be more generations of quarks; if it exceeded it, universality of the weak forces would have to be abandoned. Since however mixing angles are expected to be small on general grounds we consider it to be unlikely that measurements in the bottom and top sector beset with their inherent uncertainties will force us into that direction.

7. Summary

Very severe requirements have to be placed on the quality of the theoretical tools when one wants to extract small, but crucial quantities like quark mixing angles. After discussing a number of uncertainties and caveats we concluded that an analysis of semi-leptonic decays does indeed offer a reasonable method for obtaining quark mixing angles. In view of the complications one should make a strong effort to obtain an independent handle on the KM angles from a study of primary, i.e. very hard kaons and of the exclusive mode $\mathrm{B}^-\to\tau^-\bar{\nu}_\tau$.

We have also stressed that the crucial issue of flavour changing neutral currents merits a dedicated search for modes like $\mathrm{B},\mathrm{T}\to\tau^+\tau^-\mathrm{X}$.

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