MONTE CARLO ANALYSIS OF GAUGE INVARIANT TWO-POINT FUNCTIONS IN AN SU(2) HIGGS MODEL ☆

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New order parameters involving ratios of gauge invariant correlation functions for distinguishing different phases – confined/screened or free charges – in systems with lattice gauge fields coupled to matter fields have been proposed by Fredenhagen and Marcu and by Bricmont and Fröhlich. Our Monte Carlo analysis of those quantities for an SU(2) gauge field coupled to scalar matter fields in the fundamental representation supports the theoretically expected behaviour. Also a new mass parameter determining the exponential decay of two-point functions is observed.

In euclidean gauge theories on the lattice the potential $V(R)$ between two external static charges can be derived from the large-T behaviour of the expectation value

$$W(T, R) = \langle \mathrm{tr}\, \mathrm{U}\big(\underset{R}{\square} T \big) \rangle . \qquad (1)$$

Here U(□) denotes the Wegner–Wilson loop [1] of path-ordered products of gauge group valued link variables U_ℓ on a rectangular contour with length R in spatial and length T in (euclidean) temporal direction. If the potential

$$V(R) = -\lim_{T\to\infty} \frac{1}{T} \ln W(T, R) \qquad (2)$$

behaves like σR for large R (area law), then one has confinement of the charges associated with the gauge group, but if $V(R)$ tends to a constant $V(\infty) < \infty$ (perimeter law), then there are no long-range forces between the charges and they may be free. Thus, for pure gauge theories the qualitatively different behaviour of $W(T, R)$ for large T and R makes it possible to use $W(T, R)$ as an order parameter which distinguishes between the two phases of confined and free charges, respectively.

If the gauge fields are coupled to matter fields, then the Wegner–Wilson loop is no longer useful as an order parameter, because it also has perimeter-behaviour in the confinement/screening phase [2,3]. The physical reason is that now dynamical particle–antiparticle pairs of the matter fields can be created from the ground state and the charges of these pairs can shield the external charges, if the matter fields belong to the fundamental representation. The construction of new order parameters testing for the existence or non-existence of free charges in the presence of matter fields [4–7] is severely restricted by the fact that there is no spontaneous gauge symmetry breaking in gauge-invariant lattice field theories [8]. On the other hand, there is an intimate relationship between confinement and Higgs mechanisms [9,10,3], if the scalar matter fields are in the fundamental representation of the gauge group, a property which makes it possible to analyse Higgs mechanism and confinement by means of the same order parameter.

In systems with local gauge invariance there is

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no long-range order and gauge invariant two-point functions decay exponentially with the distance [11]. But if the mass parameters which determine this decay are the same for two different correlation functions in one phase, and different in another, then the ratio of such functions may provide a suitable order parameter. The same holds if the power law corrections to the exponential decay are different for different phases. Such ideas are behind the proposals of Fredenhagen and Marcu [6] and of Bricmont and Fröhlich [5] for order parameters which test for confined/screened charges or free ones in lattice gauge theories coupled to matter.

Fredenhagen and Marcu proposed the following limit

$$\rho^{\infty}_{\mathrm{FM}} = \lim_{R\to\infty} \rho_{\mathrm{FM}}(R, T = \tfrac{1}{2}R),$$

$$\rho_{\mathrm{FM}}(R, T) = G_R(R, T)/W(2T, R)^{1/2} \qquad (3)$$

of the ratio of the gauge-invariant two-point function

$$G_R(R, T) = \left\langle \Phi^{+}_{x}\, \mathrm{U}\left(T \overset{R}{\underset{x\qquad y}{\sqcap}} \right) \Phi_y \right\rangle \qquad (4)$$

and the square root of the expectation value $W(2T, R)$ of the Wegner–Wilson loop. Here Φ^{+}_{x} and Φ_y denote the matter fields – we consider only scalar ones – carrying a gauge charge and anticharge, respectively. $\mathrm{U}(\sqcap)$ denotes the path-ordered product of link variables along a rectangular path from x to y as indicated. Pictorially we can write

$$\rho_{\mathrm{FM}}(R, T) = \left\langle T \overset{R}{\underset{x\qquad x}{\sqcap}} \right\rangle \Big/ \left\langle 2T \overset{R}{\square} \right\rangle^{1/2}. \qquad (5)$$

In the same way as $W(T, R)$ measures the response of the gauge fields to an external charge–anticharge pair separated by a distance R, the function $G_R(T, R)$ measures the response of the coupled system of gauge and matter fields to such an external pair [6], now with a dynamical charge and anticharge "sitting on top" of their corresponding external partners at the spatial points x and y.

In the case of Z_2 gauge fields coupled to Z_2 matter fields in dimensions $D \geqslant 3$ Fredenhagen and Marcu proved that the order parameter $\rho^{\infty}_{\mathrm{FM}}$ has a value $\neq 0$ in the confinement/screening phase and vanishes in the phase with free charges. They argued that this behaviour should hold for more general gauge groups, the essential argument being the following: The charge–anticharge "string" state

$$|\Psi_R\rangle = \Phi^{+}_{x}\, \mathrm{U}\left(T \overset{R}{\underset{x\qquad y}{\sqcap}} \right) \Phi_y |0\rangle \qquad (6)$$

is gauge invariant. If one moves one of the charges to infinity, then either the remaining charge becomes free and the resulting free-charge state is orthogonal to the ground state $|0\rangle$ which has charge 0, or the charges remain confined or screened (i.e. the confining flux tube becomes fragmented) and the state $|\Psi_R\rangle$ stays in the vacuum sector so that $\langle 0|\Psi_R\rangle \neq 0$. The denominator $W(2T, R)^{1/2}$ in the function $\rho_{\mathrm{FM}}(R, T)$ results from a regularization of the norm $\langle \Psi_R|\Psi_R\rangle^{1/2}$.

In addition Fredenhagen and Marcu proved analytically for the Z_2 Higgs model that in a region of the screening/confinement phase their order parameter $\rho^{\infty}_{\mathrm{FM}}$ coincides with that of Bricmont and Fröhlich [5], which is defined by

$$\rho^{\infty}_{\mathrm{BF}} = \lim_{T\to\infty} \rho_{\mathrm{BF}},$$

$$\rho_{\mathrm{BF}} = G^2_T(T, R=0)/G_T(2T, R=0), \qquad (7)$$

where

$$G_T(T, R) = \left\langle \Phi^{+}_{x}\, U\left(T \left[\begin{smallmatrix} R & \\ & y \\ & x \\ R & \end{smallmatrix} \right. \right) \Phi_y \right\rangle. \qquad (8)$$

Notice that in an euclidean field theory we have $G_T(T, R) = G_R(T, R)$. Conceptually there is a difference however, because we can – in the temporal gauge, where $U_{x,\nu=0} = 1$ – translate the field variables along the euclidean time axis by means of the transfer matrix $\exp(-Ha)$, where a is the lattice constant. Thus we have

$$G_R(R, T) = \langle 0|\Phi^{+}_{x} \exp(-Hna)\, \mathrm{U}\left(x \overset{R}{—} y\right)$$

$$\times \exp(Hna)\, \Phi_y|0\rangle,$$

$$x = (0, x),\ y = (0, y),\ T = na, \qquad (9a)$$

$$G_T(T,\ R) = \langle 0 \,|\, \Phi_x^+ \mathrm{U}\Big(\underset{x}{|} R \Big) \times \exp(-Hna)\, \mathrm{U}^{-1}\Big(\underset{x}{|} R \Big) \Phi_x \,|\, 0 \rangle . \tag{9b}$$

Eq. (9b) suggests that the mass parameter

$$\mu = -\lim_{T \to \infty} \frac{1}{T} \ln G_T(T,\ R), \quad R \text{ fixed} \tag{10}$$

can be interpreted as the lowest energy of fields screening an external charge. In the language of the constituent picture μ may be seen as the energy of a dynamical charge bound by an external charge. The R-independence of μ follows from the fact that in eq. (9b) the same intermediate states contribute for all R. This interpretation means that 2μ is equal to the value of the screened potential in eq. (2) for $R \to \infty$,

$$\mu = \tfrac{1}{2} V(R \to \infty). \tag{11}$$

This relationship between the decays of the two-point function and of the Wilson loop makes it plausible that the ratio (5) of two exponentially decaying functions may be non-zero.

An illustrative example – not to be taken literally, however – for such a behaviour is Debye–Hückel screening [12]: Let $V_0(r)$ be a potential associated with an external charge $q = 1$ which behaves like $1/r$ in the neighbourhood of the origin $r = 0$. If such a test charge is brought into a plasma containing an equal number of positive and negative charges, then the potential $V_0(r)$ in general will be screened, say by a factor $\exp(-\mu r)$ [in the Debye–Hückel case $\mu = (8\pi q^2 n/kT)^{1/2}$, where n is the density of positive charges in the plasma]. The potential difference induced by a cloud of opposite charges in the neighbourhood of q is $V_{\mathrm{cl}}(r) = V_0(r)\,[\exp(-\mu r) - 1]$, yielding $V_{\mathrm{cl}}(0) = -\mu$, $V_{\mathrm{cl}}(\infty) = 0$, or, if we renormalize $V_{\mathrm{cl}}(r)$ such that $V_{\mathrm{cl}}(0) = 0$, then $V_{\mathrm{cl}}(\infty) = \mu$. Thus, if we bring a second test charge from infinity to $r = 0$, then its energy gain associated with the potential V_{cl} is μ. If we now, heuristically, identify the constant $V(\infty)$ obtained from eq. (2) for the external charges in the screening region with $2V_{\mathrm{cl}}(\infty) = 2\mu$, then the lowest mass appearing in the two-point functions $G_R(R,\ T)$ should be $\mu = V(\infty)/2$.

In the limit $R \to \infty$, $T = R/2$ clustering properties can lead to a cancellation of the contributions of those pieces of the paths in the ratio (5) which are parallel to the $T = 0$ hyperplane and we may have, approximately and symbolically,

$$\left\langle T \underset{x \qquad x}{\overset{R}{\sqcap}} \right\rangle \Big/ \left\langle 2T \overset{R}{\square} \right\rangle^{1/2} \approx \left\langle T \underset{x}{\overset{x}{|}} \right\rangle^{2} \Big/ \left\langle 2T \underset{x}{\overset{x}{\vdots}} \right\rangle , \tag{12}$$

which illustrates why the two order parameters $\rho_{\mathrm{BF}}^{\infty}$ and $\rho_{\mathrm{FM}}^{\infty}$ may coincide in the screening/confinement phase.

Is it not clear, however, even in the Z_2 Higgs model, whether the two order parameters also coincide in the free charge phase, because their intuitive meaning is not the same [5,6]. Whereas $\rho_{\mathrm{FM}}^{\infty}$ measures the overlap of the gauge-invariant charge–anticharge state with the ground state, if one of the charges is sent to infinity, the two-point function $G_T(T,\ R = 0)$ represents the bound state ("meson") of a dynamical and an external charge, both at the same spatial point x.

We have investigated the different correlation functions mentioned above and the properties of the associated order parameters $\rho_{\mathrm{BF}}^{\infty}$ and $\rho_{\mathrm{FM}}^{\infty}$ by a Monte Carlo analysis of an SU(2) lattice Higgs model in four dimensions, with the scalar field in the fundamental representation [13]. One of the advantages of this model is that the properties of the order parameters $\rho_{\mathrm{BF}}^{\infty}$ and $\rho_{\mathrm{FM}}^{\infty}$ are not yet known analytically for an SU(2) group. Thus one obtains a genuinely new test beyond the results concerning the groups Z_2 [5,6] and U(1) [14]. The main disadvantage is that the model has only one phase [10,3], the screening/confinement phase, which, however, is partially separated into different screening (Higgs) and confinement regions, very similar to the single phase of a fluid, which contains vapour and liquid regions.

The model is defined by the following action [13,15]:

$$S = -\tfrac{1}{4}\beta \sum_{p} \mathrm{tr}\big(\mathrm{U}_p + \mathrm{U}_p^+\big) - \kappa \sum_{x\nu} \big(\Phi_x^+ \mathrm{U}_{x\nu} \Phi_{x+\nu} + \text{h.c.}\big) + \lambda \sum_{x} \big(\Phi_x^+ \Phi_x - 1\big)^2 + \sum_{x} \Phi_x^+ \Phi_x , \tag{13}$$

where $U_{x\nu}$, $U_p \in SU(2)$ are the usual link and plaquette variables. The two-component Higgs field Φ_x is conveniently parametrized by a pair (ρ, σ), where ρ is the length of the field Φ and $\sigma \in SU(2)$.

The values of the gauge coupling $\beta = 4/g^2$, the hopping parameter κ and the coupling λ of the quartic interaction of the Higgs field determine the state of the system. The phase structure is qualitatively as follows [13,10,3,15]:

The three-dimensional space of parameters ($\beta \geqslant 0$, $\kappa \geqslant 0$, $\lambda > 0$) contains a two-dimensional surface where phase transitions between the confinement and screening regions take place. One boundary of this surface is a critical line in the plane $\beta = \infty$, where, if on keeps ρ fixed ($\lambda = \infty$), the model reduces to a $SU(2) \times SU(2) \approx O(4)$ Heisenberg spin system, for which the existence of phase transitions has been proved rigorously [16]. It is essential that the surface of phase transitions in (β, κ, λ)-space does not divide this space into two separate parts but has a boundary inside, leaving a "hole" which allows for analytical continuations of physical quantities from below the surface to above. However, the behaviour of these quantities might be quite different on different sides of the surface. As the system reduces to a pure gauge theory for $\kappa = 0$, with a single confinement phase, its behaviour for small κ and finite β and λ is "confinement-like", whereas for large κ, beyond the phase transition surface, it is "screening-like".

Our Monte Carlo analysis has been done on a 16^4 lattice with periodic boundary conditions at fixed values of $\lambda = 0.5$ and $\beta = 2.4$, but for different κ. The confinement/screening transition occurs here at $\kappa_{PT} = 0.2590 \pm 0.0005$. At each κ value we performed between 15 000 and 28 000 standard Metropolis iterations using a vectorized checkerboard algorithm. The starting configurations were taken from runs at nearby values of κ and the first 2000 iterations were discarded for thermalisation. Measurements were taken after every 50 iterations.

The shape of the potential $V(R)$ was determined from the slope of $\ln W(T, R)$ as a function of T for $T \geqslant 4$, according to eq. (2). The string tension $\sigma(\kappa)$, determined from fits corre-

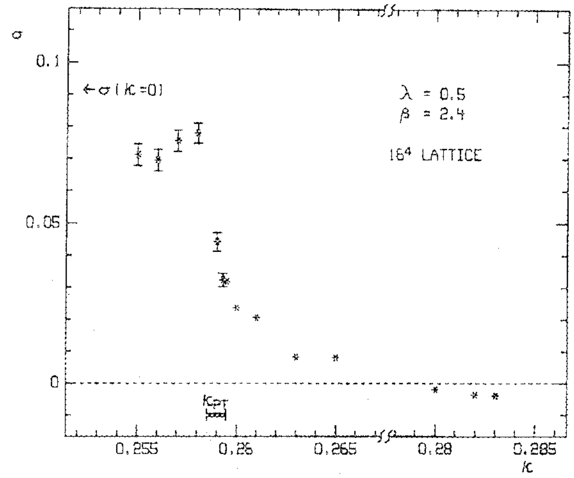

Fig. 1. String tension σ as determined from the fits to the potentials. The arrow denotes the value of σ for $\beta = 2.4$ and $\kappa = 0$, [17]. Slightly negative values of σ indicate that for larger κ the parametrization (14) becomes inappropriate.

sponding to

$$V(R) = C + \sigma R - \alpha/R, \tag{14}$$

is displayed in fig. 1. It clearly shows the "breakdown" of the linear term in the potential for $\kappa \geqslant \kappa_{PT}$.

The mass parameter μ can be determined, see eq. (10), from the asymptotic behaviour of $G_T(T, R)$ as a function of T with R fixed. We find that the resulting values of μ are not only

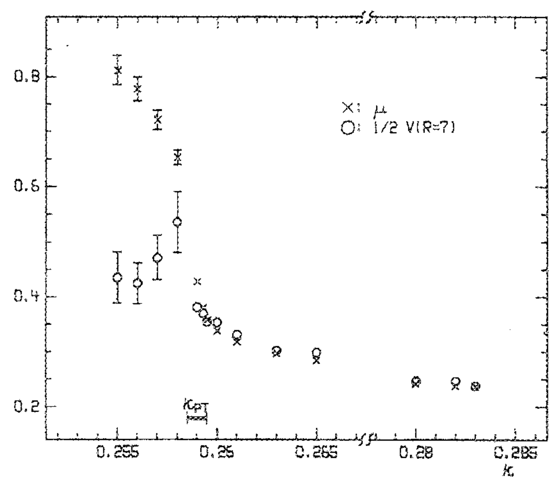

Fig. 2. The values of μ (crosses), defined by eq. (15). For comparison we include values of $V(R = 7)/2$ (circles).

insensitive to the choice of R but that in addition

$$G_T(T, R) \approx C_G(R)\exp(-\mu T) \approx c\exp[-\mu(2R+T)] \qquad (15)$$

depends only on the length of the path $T\sqcap R \sqcap$. Fig. 2 shows the value of $\mu(\kappa)$ obtained for different values of κ with $R = 1$.

We would like to stress that $\mu(\kappa)$ differs substantially from the Higgs boson and vector boson masses which have been calculated recently in the same model for the same values of λ and β [18]. Thus μ is a new mass parameter characterizing the spectrum of lattice Higgs models.

In the same figure we have plotted the values $V(7)/2$ of the potential $V(R)$ for the largest R reasonably considered on a 16^4 lattice. It is obvious that $\mu = V(7)/2$ (eq. 11) to a remarkable degree of accuracy for $\kappa > \kappa_{PT}$, that is to say, in the screening region, in agreement with theoretical discussions above. In the confinement region the situation is quite different, which had to be expected, because there the linear part of the potential has not yet been "screened" and the potential still grows. The expected perimeter-law behaviour of $W(R, T)$ for large T is not yet realized on a 16^4 lattice.

The equality of μ and $V(7)/2$ for $\kappa > \kappa_{PT}$ and the perimeter behaviour of both $G_T(T, R)$, eq. (15), and of the Wilson loop are the reasons for the predicted behaviour $\rho^{\infty}_{FM} \neq 0$ in the screening region. More generally, fig. 3 shows the function

$$\rho_{FM}(R, T_1, T_2) = [G_R(R, T_1)G_R(R, T_2)/W(R, R)]^{1/2},$$
$$T_1 + T_2 = R, \quad |T_1 - T_2| \leqslant 1, \qquad (16)$$

plotted with respect to the perimeter $P = 4R$ of the Wilson loop for different values of κ. The slight generalization (16) of the ratio $\rho_{FM}(R, T)$ in eq. (5) has been introduced in order to be able to plot ρ_{FM} also for those cases, where R is an odd multiple of the lattice constant. The plot shows a remarkable difference in the behaviour of ρ_{FM} in the confinement and screening regions: Whereas ρ_{FM} decays to a constant rather fast in the screening region, it drops practically to 0 in the confinement region above $P \geqslant 20$ on our 16^4 lattice. This behaviour can be interpreted similarly as that of μ and $V(7)$ in fig. 2: For R not too large and for κ in the confinement region the potential still rises linearly because hadronization by means of pair creation has not yet set in and the charges appear unshielded at the distance we are able to consider on a 16^4 lattice. However, $\rho_{FM}(R, T)$ is expected to increase again to a constant $\neq 0$ for sufficiently large R.

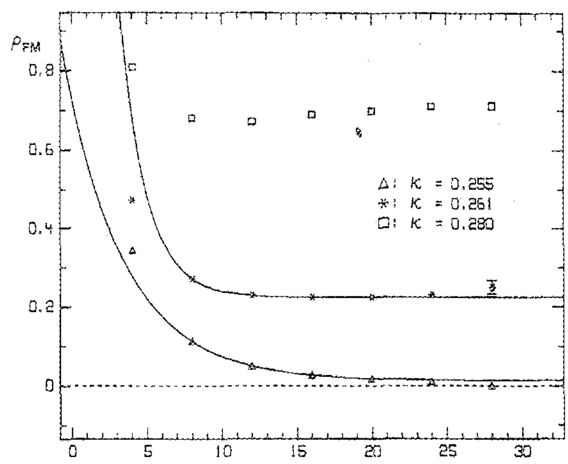

Fig. 3. The values of $\rho_{FM}(R, T_1, T_2)$, defined by eq. (16), for $T_1 + T_2 = R$, plotted against the perimeter of the Wilson loop ($P = 4R$). The curves represent fits according to eq. (17).

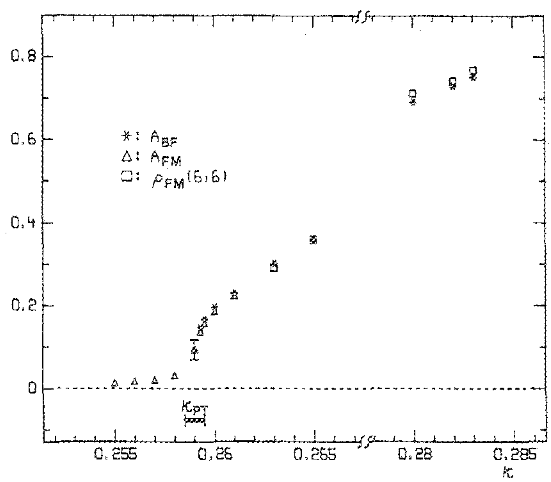

Fig. 4. The asymptotic values of ρ_{BF} and ρ_{FM}, namely A_{BF} (asterisks), A_{FM} (triangles), from fits (17), and for $\kappa \geqslant 0.263 \times \rho_{FM}(R = 6, T = 6)$ (squares).

The fits of $\rho_{FM}(P)$ in fig. 3 are made with the help of the ansatz

$$\rho_{FM} = A_{FM} + B_{FM} \exp(-C_{FM} P), \quad \text{for } P \geqslant 8. \tag{17}$$

For $\kappa \geqslant 0.263$ the second term in (17) is not measurable, since ρ_{FM} is practically constant. The Bricmont–Fröhlich parameter ρ_{BF}, eq. (7), was fitted correspondingly. Fig. 4 finally shows a comparison of the asymptotic values ρ_{BF}^{∞} and ρ_{FM}^{∞} as a function of κ. Clearly both parameters coincide in the screening region, as expected on theoretical grounds.

We conclude that the properties of the gauge invariant two-point functions in the SU(2) model with scalar fields in fundamental representation are in agreement with analytical predictions based on the screening picture of confinement in gauge theories with matter fields. A more detailed account of this work will be published elsewhere.

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