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FINITE TEMPERATURE SU(2) HIGGS MODEL ON A LATTICE *

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By means of Monte Carlo simulations we investigate the finite temperature SU(2) lattice Higgs model with a doublet scalar field at large but finite quartic self-coupling. The lattices are asymmetric in space and time extensions and their spatial size is varied in order to study finite size effects. The second order deconfinement transition at high temperature of the pure SU(2) gauge theory changes into a crossover when the scalar field is coupled to the gauge field. The Higgs phase transition at zero temperature also changes into a crossover when the temperature gets high enough. Its position shifts slightly to larger values of the hopping parameter. This means that in the Higgs region of the phase diagram the system passes through this crossover when the temperature is raised at fixed values of the coupling parameters, in analogy to the symmetry restoring transition of Kirzhnits, Linde and Weinberg in the standard model.

1. Introduction

The properties of gauge field theories at high temperatures are of crucial importance for theories of the early universe. According to the present picture, the hadronic matter emerged from the hot deconfined plasma of gluons and quarks when its temperature decreased below the deconfining temperature [1]. Even earlier, and at a much higher temperature, the electroweak interactions underwent a phase transition (PT) with decreasing temperature after which the Higgs mechanism started to operate [2–4]. Correspondingly, finite temperature effects in gauge theories have received much attention in Monte Carlo (MC) calculations.

The deconfinement PT in the pure SU(2) and SU(3) gauge theories has been investigated quite intensively and it is by now rather well understood [1]. On the other hand the precise nature of the deconfinement PT in the presence of dynamical quarks has not yet been determined conclusively. In the case of the SU(2) gauge group, the question whether the second order PT vanishes when matter fields are introduced [5, 6] is still open.

In the electroweak theory the most important finite temperature effects are expected in the Higgs sector. Kirzhnits and Linde, Weinberg and other authors

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suggested more than ten years ago the occurrence of a "symmetry restoring" PT at high temperature by using perturbative methods [2–4]. The confirmation and further investigation of this PT by nonperturbative methods is desirable. This stimulated several analytic [7] and numerical [8–12] investigations of lattice Higgs models at finite temperature.

These models allow us to study simultaneously also the influence of scalar dynamical matter fields on the deconfinement PT in pure gauge theories. Since MC calculations with scalar matter fields are much easier than with fermionic ones, one can work on larger lattices and has a better control of finite size effects. The comparison of the results with those for fermionic matter contributes to the understanding of the deconfinement PT in QCD.

In this work we investigate the SU(2) lattice Higgs model with scalar fields in the fundamental representation. A finite temperature on isotropic lattices of size $N_s^3 \times N_t$ is achieved by choosing small values for N_t, the lattice size in the "temporal" direction. The main part of our data is for $N_t = 2$. The spatial lattice sizes we use are $N_s = 8$ and 16. This wide variation of N_s allows us to look for finite size effects. For a fixed finite quartic coupling λ we study the phase diagram and the nature of PT lines or crossovers found inside this diagram.

The value of λ has been chosen quite large, $\lambda = 0.5$. For this λ and moderate values of β the Higgs PT at zero temperature is either of a weak first order or of higher order [13, 14] so that some physical masses are small in lattice units in the vicinity of the Higgs PT. Thus the lattice constant a is small and the temperature $T = 1/(N_t a)$ is large in physical units. Only then we may expect observable finite temperature effects on the Higgs PT. On the other hand, for small λ the Higgs PT is of first order and without indications of a critical behavior [14]. The temperature would therefore be small in physical units even for the smallest N_t.

We find that the model has only one phase for $N_t = 2$ and for positive values of the gauge coupling β and of the hopping parameter κ. This phase is divided into three regions separated only partly by lines of *crossovers*. We define a crossover as a sudden, but smooth change of observables which shows no dependence on the lattice size for large lattices. The typical magnitude of finite size effects at PT's in the same model is used for comparison. However, since we cannot really distinguish between crossovers and very weak higher order PT's, which might not show finite size effects in variables we are calculating, the term crossover will be understood in this paper to include also the latter possibility. The phase diagram we found for $N_t = 2$ is shown in fig. 1.

One of the dividing lines is the extension of the deconfinement PT of the pure SU(2) theory into the κ direction. With growing κ the transition on this line rapidly weakens. We found no finite size effects at least for $\kappa \geqslant 0.15$ and thus conclude that the deconfinement PT of the pure SU(2) gauge theory changes into a crossover when scalar matter is present. Even this crossover becomes practically undetectable when κ approaches from below that value at which the Higgs PT occurs for $T = 0$.

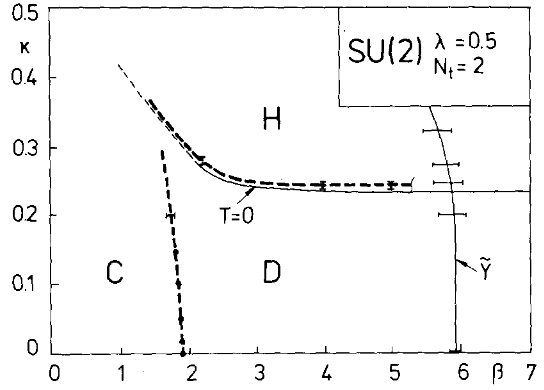


Fig. 1. Finite temperature phase diagram of the SU(2) Higgs system for $\lambda = 0.5$ and $N_t = 2$. Symbols H, C and D stand for the Higgs, confinement and deconfinement regions, respectively. The vertical broken line is the deconfinement crossover for $N_t = 2$. The vertical solid line at $\beta \simeq 6$ is the freezing transition of the icosahedron subgroup $\tilde{Y}$ observed on an $8^3 \times 2$ lattice. The partly horizontal solid line is the Higgs PT at $T = 0$. The data points and the broken line above this solid line indicate the approximate location of the Higgs crossover for $N_t = 2$.

The other dividing line is a finite temperature remnant of the Higgs PT line at $T = 0$. Again, the transition gets weaker on an $8^3 \times 2$ lattice in comparison with an 8^4 lattice. For $\beta = 2.25$ and 4 we found no difference between the transitions on $8^3 \times 2$ and $16^3 \times 2$ lattices. This means again that the Higgs PT at $T = 0$ changes into a crossover at high temperature at least for these β's and $\lambda = 0.5$.

Very precise data are required for the observation that the position of this crossover is at slightly higher κ than the Higgs PT at $T = 0$; e.g. for $N_t = 2$ at $\lambda = 0.5$ and $\beta = 4$ the shift is about 0.01. This tiny shift actually might be of major physical importance. When we increase the temperature $T = 1/(N_t a)$ of *a given physical system*, for N_t finite and fixed, by decreasing the lattice spacing a, we move in the coupling constant space along a renormalization group (RG) line. Such a line is originally defined as a line of constant physics *at zero temperature* (see sect. 3 for details). Since such RG lines are expected to approach the $T = 0$ Higgs PT line when a approaches zero, they may cross the shifted Higgs PT remnant at some finite temperature. This would correspond to the Kirzhnits-Linde-Weinberg PT, which appears here as a crossover due to nonperturbative effects. The mathematical difference between a true PT and a crossover might be physically unimportant as long as the crossover is sharp enough.

From our data and on the basis of various theoretical expectations we have sketched a schematic phase diagram, shown in fig. 2, for the finite temperature SU(2) Higgs model at a large fixed λ. The temperature is plotted in lattice units. This figure illustrates how the three regions of the phase diagram (confinement, deconfinement and Higgs regions), which appear to be of great difference physically, are actually analytically connected.

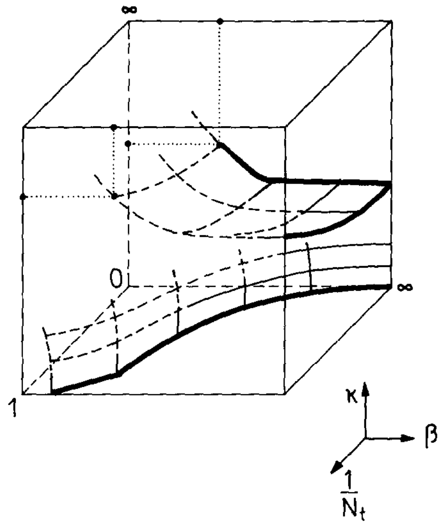


Fig. 2. Expected phase diagram of the SU(2) Higgs system for variable temperature in lattice units at a fixed large λ. The dashed lines indicate sheets of crossovers. The thin solid lines indicate either sharp crossovers or phase transitions. The thick solid lines are phase transition lines in the limiting cases of the model.

The outline of the paper is as follows: In the next section we introduce the SU(2) lattice Higgs model and discuss its most important limiting cases. In sect. 3 we describe the expected three-dimensional phase diagram shown in fig. 2 and discuss the physical meaning of temperature in lattice Higgs models. The definitions of various thermodynamic variables we are calculating, and some technical details of our MC calculations, are collected in sect. 4. In sect. 5 we investigate the effects of matter fields on the deconfinement PT. The influence of high temperatures on the Higgs PT is described in sect. 6. Sect. 7 is devoted to a short discussion on the large β behavior of the energy density. We list our conclusions and mention some open questions in sect. 8.

2. The model and its limiting cases

The action of the SU(2) Higgs model we consider in this paper is parametrized following refs. [13, 14]:

$$S = -\tfrac{1}{2}\beta \sum_{\mathrm{p}} \mathrm{Re\,Tr}\, U_{\mathrm{p}} - \kappa \sum_{x,\mu} \rho_x \rho_{x+\mu}\, \mathrm{Re\,Tr}\left(\sigma_x^{\dagger} U_{x\mu} \sigma_{x+\mu}\right)$$

$$+\lambda \sum_{x} \left(\rho_x^2 - 1\right)^2 + \sum_{x} \rho_x^2, \qquad (2.1)$$

where σ_x and $U_{x\mu}$ are 2×2 SU(2) matrices and $\rho_x \in (0, \infty)$. The variables ρ_x and σ_x are the radial and the angular modes of the scalar field, respectively, whose conventional complex doublet form Φ_x is

$$\Phi_x = \rho_x \sigma_x \begin{pmatrix} 1 \\ 0 \end{pmatrix}. \tag{2.2}$$

This model has three coupling parameters: the gauge coupling $\beta \equiv 4/g^2$, the quartic self-coupling λ and the hopping parameter κ. The values $\kappa \approx 0$ and ∞ correspond to large positive and negative square bare masses of the scalar field, respectively. Connection with the standard continuum parametrization is discussed in ref. [13].

On a periodic $N_s^3 \times N_t$ lattice with a lattice spacing a, the temperature T is given by $T = 1/(N_t a)$. The action S is a function of T. The partition function is then defined as

$$Z(T) = N(T) \int [\mathrm{d}U \mathrm{d}\Phi] \mathrm{e}^{-S(T)}, \tag{2.3}$$

where $N(T)$ is a normalization factor which appears when the finite temperature Hamilton formalism is rewritten as a Lagrange formalism and corrects the ground state energy [15, 16].

There are three important limiting cases of the model, namely $T = 0$, $\kappa = 0$ and $\beta = \infty$, which have PT lines. We briefly summarize those known facts about these transitions which will help us to envisage the phase diagram of the whole model.

At $T = 0$, the SU(2) Higgs model with the scalar field in the fundamental representation has only one phase, which is divided into two physically different regions: the Higgs region and the confinement region. These two regions are analytically connected to each other at small β and large λ, and are partly separated from each other by a phase transition called the Higgs PT. The nature of this PT has been studied for finite λ in detail in refs. [9, 13, 14, 17, 18].

In the pure gauge limit ($\kappa = 0$), the system has a deconfinement PT between the low temperature confinement phase and the high temperature deconfinement phase. In the latter phase gauge bosons are liberated and in a plasma state. The deconfinement PT is known to be of second order for the SU(2) gauge group [16, 19, 20] and is expected to be in the same universality class as the Ising spin system in the same spatial number of dimensions [21]. This universality hypothesis is supported on both sides of this PT by comparisons of the numerical values for critical exponents associated with the corresponding order parameter, the susceptibility and the correlation length in both models [19, 20].

The limiting case $\beta = \infty$, the pure SO(4) Φ^4 theory at finite temperature, has not yet been investigated in MC simulations. Older perturbative calculations [2] suggest that the model has two phases, one with the global symmetry spontaneously broken, the other symmetric. A line of symmetry restoring PT's extends from the κ-point where SSB takes place for $T=0$ into the temperature direction and shifts to larger κ as T increases*.

3. The expected phase diagram

The effect of heavy matter fields on the pure gauge deconfinement PT can be studied by the hopping parameter expansion around $\kappa=0$. As will be demonstrated in sect. 5, the incorporation of a heavy matter field in the fundamental representation of the gauge group corresponds to the introduction of a weak external field in the spin system to which the gauge theory is related by the universality. In the Z(2) spin model, an infinitesimally small external field is enough to destroy the order-disorder PT which is of second order [22]. Thus the apparent common universality class of the pure SU(2) gauge theory and the Ising model [19, 20] suggests that the second order deconfinement PT disappears at $\kappa \neq 0$ [7, 23, 24], although a sharp crossover will remain at least for small κ.

In this paper we therefore call the extension of the deconfinement phase of the pure SU(2) model to finite κ below the Higgs PT the *deconfinement region*, which is separated by a crossover from the *confinement region*. The change of the deconfinement PT into a crossover is caused physically by the pair creation of matter. Since the matter is very heavy for small κ, the crossover will be indistinguishable on a finite lattice from a second order PT for small $\kappa>0$. An interesting question is whether this crossover can be distinguished from a PT for large κ values which are still below the Higgs PT.

What happens if we heat up the *Higgs region*? According to the standard perturbation theory in the continuum, the pure scalar Φ^4 theory ($\beta=\infty$) possesses a symmetry restoring PT at some finite temperature [2]. The interaction with gauge fields introduces only small modifications at large β [3, 4]. The typical leading finite temperature contribution to the effective potential for Φ is of the form $(c\lambda_{\mathrm{cont}}+c'g^2)T^2\Phi^2$, where c and c' are positive constants and $\lambda_{\mathrm{cont}} \equiv \lambda/\kappa^2$ is the self-coupling in the continuum parametrization. This contribution leads to a symmetry restoring PT (Kirzhnits-Linde-Weinberg PT) of second or first order at finite temperatures [2–4]. The possibility of such a PT motivated a series of cosmological scenarios for the evolution of the early universe [25]. There is no physical reason to distinguish the high temperature symmetric phase found in the perturbative approach from the deconfinement region. The heating can move the system from the Higgs region to the deconfinement region. We therefore expect the KLW transition at finite β to be an extension of the Higgs PT at $T=0$ into the T direction, the transition shifting to larger κ at $T>0$.

* We do not go into the problem of the triviality of the Φ^4 theory in the exact continuum limit. For definiteness, we assume a finite cut-off in our description of the Higgs system.

However, since only composite operators are gauge invariant and since it is difficult to control higher order effects in these perturbative calculations in a gauge theory at finite T [26], the detailed nature of the KLW transition, including the possibility that it becomes a crossover, may depend on nonperturbative properties of the model. There is indeed some reason to expect that at finite T the Higgs PT may partly be a crossover. A study of the gauge invariant radially frozen U(1) Higgs model with Villain action on the lattice using the Hamilton formalism indicates that this model with matter fields in the fundamental representation does not possess a PT for $T > 0$ at several limiting values of coupling parameters [7]*. Furthermore, MC investigations of some other lattice Higgs models at finite T suggest that the Higgs PT weakens with increasing T [11, 12].

We summarize our expectations for finite temperatures and for large fixed λ in the schematic phase diagram shown in fig. 2. For MC calculations it is more convenient to use $1/N_t$ instead of the physical temperature $T = 1/(N_t a)$. The particular shape of both PT/crossover sheets is inspired by the phase diagram shown in fig. 1. The $1/N_t = 0$ plane represents the zero temperature phase diagram, where at large λ the Higgs PT extends from $\beta = \infty$ to the end point at some finite β [13, 14, 18]. The shape of the deconfinement PT in the $\kappa = 0$ plane is based on the scaling of the pure gauge theory for $\beta > 2$. The interval of κ in which the deconfinement crossover is still sharp increases with growing β in the κ direction because the RG flow approaches the $T = 0$ Higgs PT. According to our discussion above, we expect that at $T > 0$ parts of the Higgs PT surface in fig. 2 actually represent crossovers. The dashed lines in fig. 2 indicate the regions where on finite lattices it may be possible to distinguish the crossovers from genuine phase transitions.

Before finishing this section, we shall discuss the physical meaning of the temperature T on a lattice and of RG lines at finite temperature. In a scaling region we have RG lines in coupling space, which are originally defined for $T = 0$ as lines of constant physics (lines of constant mass ratios etc.). At $T = 0$, points on the same RG line describe the same physical system on lattices with different lattice spacing a. *At each such point* we can heat the system by reducing N_t without changing the coupling parameters and the lattice spacing a. Thus each RG line at $T = 0$ is a base of a two-dimensional sheet in the space of coupling parameters and temperature T. Points on the same sheet describe the same system at various T and for various a. Lines of constant T in this sheet are again RG lines, i.e. lines of constant physics. The coordinates of these finite temperature RG lines in the subspace of couplings are independent of temperature and coincide with the position of the $T = 0$ RG line.

* This property is essentially due to the lack of an order parameter distinguishing the Higgs region and the high temperature deconfinement region. The 't Hooft loops can be screened by monopoles, whose existence is predicted also in the continuum limit in the form of monopole-antimonopole pairs [27]. However, since such monopoles are very heavy and couple only weakly, a sharp crossover is expected [7].

The position of constant $T = 1/(N_t a)$ lines in the sheet is determined by the magnitude of a and N_t, while the dependence of a on the coupling parameters can be determined by the magnitude of a physical mass scale, for example, by the string tension ($\kappa = 0$) or by the Higgs mass (for large κ) in physical units at $T = 0$*.

Knowing the RG lines defined at $T = 0$ in the coupling space, one could vary continuously the value of the physical temperature by moving along these lines *keeping* N_t *finite and fixed*. For finite fixed N_t these lines are no more lines of constant physics. They can cross PT's and the values of other physical quantities such as mass ratios at finite T can also vary along them. Unfortunately, at present very little is known about the precise position of RG lines in Higgs models.

4. Choice of λ and observables

There exist already several MC studies of the finite temperature SU(2) Higgs model with doublet scalar fields, especially for small λ [8–10]. Neither the change into a crossover nor a shift in κ has been reported for the Higgs PT at finite T. The weakening of the deconfinement PT/crossover at higher κ has been observed in ref. [9].

However, the dependence on the spatial lattice size has not been studied and the resources have not been large enough to detect a possibly small shift with κ of the Higgs PT. Actually it is quite obvious that such a shift must be very small even for $N_t = 2$. The reason is that the relevant measure of the temperature is the ratio $T/M_{phys} = 1/(N_t M_{phys} a)$, where M_{phys} is the smallest physical mass at $T = 0$. But for finite β, $M_{phys} a$ is small in the Higgs region only very close to the Higgs PT. If $M_{phys} a$ is not small enough, then even with the smallest N_t the value of T/M_{phys} cannot become sufficiently large to cause observable finite temperature effects. A large correlation length (small $M_{phys} a$) is therefore important for a study of finite T effects.

It is known [17,28] that at least for $\beta \leqslant 8$ the Higgs and scalar boson masses in the Higgs region are quite large except very close to the Higgs PT at large λ (i.e. $\lambda \gtrsim O(1)$). For small λ the Higgs PT is strongly of first order [14] and the masses are presumably large even at the Higgs PT. In the present paper we have therefore fixed λ at $\lambda = 0.5$, which corresponds to a high value of λ_{cont} in the continuum parametrization. At this λ, the Higgs PT is already associated with a large (though possibly still not critical [17,18]) correlation length at moderate β [17,28,29]. Furthermore, a strong first order PT cannot easily change into a crossover. Therefore at large λ we may have a better chance to observe such a change for the Higgs PT. For small λ we expect that the first order Higgs PT persists at finite tempera-

* Since physical masses may have different T dependence, it would be another test of scaling to check if their ratios remain the same on RG lines at the same finite T.

tures up to temperatures which cannot be achieved by decreasing N_t on isotropic lattices.

The thermodynamic quantity we study in this paper is the energy density.

$$\varepsilon = -\frac{1}{a^3 N_s^3}\frac{\partial}{\partial(1/T)}\ln Z(T)$$

$$\equiv \varepsilon_G + \varepsilon_H , \tag{4.1}$$

where ε_G is the energy density for the pure gauge system and ε_H is the additional contribution due to the scalar field:

$$a^4\varepsilon_G = 3\beta(\langle P_s\rangle - \langle P_t\rangle),$$

$$a^4\varepsilon_H = 2\kappa\big(\langle \Phi^\dagger U_t \Phi\rangle - 3\langle \Phi^\dagger U_s \Phi\rangle + 2\langle \Phi^\dagger U \Phi\rangle_0\big)$$

$$+\lambda\big(\langle \rho^4\rangle - \langle \rho^4\rangle_0\big) + (1-2\lambda-4\kappa)\big(\langle \rho^2\rangle - \langle \rho^2\rangle_0\big). \tag{4.2}$$

The subscript s (t) stands for the spatial (temporal) directions and

$$P_{s/t} \equiv \frac{1}{3N_t N_s^3}\sum_{p=s/t}\big(1-\tfrac{1}{2}\mathrm{Re\,Tr}\,U_p\big),$$

$$\Phi^\dagger U_\mu \Phi \equiv \frac{1}{N_t N_s^3}\sum_x \rho_x \rho_{x+\mu}\tfrac{1}{2}\mathrm{Re\,Tr}\big(\sigma_x^\dagger U_{x\mu}\sigma_{x+\mu}\big),$$

$$\Phi^\dagger U \Phi \equiv \tfrac{1}{4}\sum_\mu \Phi^\dagger U_\mu \Phi,$$

$$\rho^n \equiv \frac{1}{N_t N_s^3}\sum_x \rho_x^n . \tag{4.3}$$

Here $\langle \ldots \rangle_0$ denotes the expectation value at $T=0$ to correct the ground state energy and it may be approximated by the expectation value on a symmetric N_s^4 lattice. We refer to this approximation in the following as *symmetric subtraction*.

Since calculations on the symmetric lattice require a very large amount of computer time, $\langle \ldots \rangle_0$'s are sometimes approximated further by the expectation values of spatial operators on the same asymmetric lattice (*spatial subtraction*). In the latter approximation we have

$$a^4\varepsilon_H \simeq a^4\varepsilon_H^{(sp)} \equiv 2\kappa\big(\langle \Phi^\dagger U_t \Phi\rangle - \langle \Phi^\dagger U_s \Phi\rangle\big). \tag{4.4}$$

At the deconfinement PT of the pure gauge theory ($\kappa=0$) $\langle P_s \rangle$ is expected to be analytic [21,30], and we have found it and $\langle \Phi^\dagger U_s \Phi \rangle$ to be smooth also for $\kappa>0$ below the Higgs PT/crossover. We may thus use the spatial subtraction for the study of the deconfinement PT/crossover. On the other hand, at the Higgs PT/crossover both spatial and symmetric expectation values vary rapidly and in a substantially different way. In sect. 6 we shall discuss the quality and physical meaning of both the above approximations.

We have performed Monte Carlo calculations of the quantities (4.2) and (4.3) as well as of the Polyakov loop $\langle L \rangle$ and of its square $\langle L^2 \rangle$,

$$L=\left|\frac{1}{N_s^3}\sum_x \mathrm{Re\,Tr}\Big(\prod_{x_0} U_{x,0}\Big)\right| = \left|\frac{1}{N_s^3}\sum_x L_x\right|, \tag{4.5}$$

applying the standard Metropolis method. The SU(2) gauge group is approximated by its 120 element icosahedron subgroup $\tilde{\mathrm{Y}}$. Each of our main data points results from 4800–9600 measurements on an $8^3\times 2$ lattice, 6000 measurements on a $16^3\times 2$ lattice or 10000–12000 measurements on an $8^3\times 4$ lattice. For the subtraction in ϵ_{H}, calculations have been performed on an 8^4 lattice (28000–68000 measurements), too. Starting from an end configuration of a nearby point, we first performed an appropriate number of thermalization sweeps depending on the lattice size and the distance from the starting point in the β-κ plane. At large β more sweeps were used for thermalization, since the acceptance rate of the gauge configuration in the MC procedure is smaller there (at $\beta=5$ about 17%). At several points the calculations have been repeated with different start configurations in order to test the stability of expectation values and to look for hystereses. We found no hysteresis neither on the Higgs nor on the deconfinement PT/crossover. The freezing transition of the $\tilde{Y}$ subgroup has been localized by means of thermal cycles (with smaller number of measurements). The total amount of computer time spent on Cyber 205 was about 300 hours.

5. Deconfinement phase transition

Let us first discuss the deconfinement PT in the Higgs model. In the pure gauge limit ($\kappa=0$), the most convenient quantity to study is the Polyakov loop L. The deconfinement PT is identified here with a spontaneous breakdown of the global center symmetry Z(2) of SU(2), which transforms L_x into $-L_x$. Matter fields in the fundamental representation break this symmetry explicitly, so that $\langle L \rangle$ is no longer an order parameter at $\kappa>0$. $\langle L \rangle$ is not identically zero in the confinement region, but decreases as κ^{N_t} for $\kappa\to 0$.

The effect of a heavy scalar matter field can be estimated by the hopping parameter expansion. The effective action for the gauge sector to lowest order in κ

is

$$S_{\mathrm{eff}} = S_{\mathrm{G}} - 2\left(\tfrac{1}{2}\kappa f(\lambda)\right)^4 \sum_{\mathrm{p}} \mathrm{Re\,Tr}\, U_{\mathrm{p}} + \left[\mathrm{O}(\kappa^6) \text{ closed-loop corrections}\right]$$

$$- 2\left(\tfrac{1}{2}\kappa f(\lambda)\right)^{N_t} \sum_{x} L_x + \left[\mathrm{O}(\kappa^{N_t+2}) \text{ thermal-loop corrections}\right], \qquad (5.1)$$

where S_{G} is the pure gauge part of (2.1) and

$$f(\lambda) \equiv \frac{\int_0^\infty \mathrm{d}x\, x^2 \exp(-\lambda x^2 - (1-2\lambda)x)}{\int_0^\infty \mathrm{d}x\, x \exp(-\lambda x^2 - (1-2\lambda)x)}. \qquad (5.2)$$

Typical values of $f(\lambda)$ are $f(\infty) = 1$, $f(0.5) = \sqrt{\tfrac{1}{2}\pi}$ and $f(0) = 2$ ($f(0)$ counts the number of complex scalar fields). The κ^{N_t} term in (5.1) acts as an external field in the Ising system of "spins" $\{\mathrm{sgn}(L_x)\}$, to which S_{G} is related by universality [21].

We may compare this effective action with that of gauge theory with fermionic matter [31]:

$$S_{\mathrm{eff}}^{\mathrm{Fermi}} = S_{\mathrm{G}} - 16\kappa^4 n_{\mathrm{f}} \sum_{\mathrm{p}} \mathrm{Re\,Tr}\, U_{\mathrm{p}} - 2^{N_t+2}\kappa^{N_t} n_{\mathrm{f}} \sum_{x} L_x$$

$$+ \left[\mathrm{O}(\kappa^6) \text{ and } \mathrm{O}(\kappa^{N_t+2}) \text{ corrections}\right], \qquad (5.3)$$

for n_{f} Wilson fermions. At present the properties of the deconfinement transition in the presence of fermions are not well known, but its location or that of its remnant crossover is observed to shift to smaller β with increasing κ [5,6]*. In the case of Kogut-Susskind fermions the deconfinement PT/crossover merges with the chiral transition at larger κ [6,33]. For Wilson fermions, a different behavior has been reported [34].

For scalars, we expect a similar but smaller shift of the PT/crossover to smaller β with increasing κ. Comparing (5.1) and (5.3) we see that the signs of the leading corrections to S_{G} are the same. Scalars act similar to fermions for small κ, but much more weakly due to the different numerical coefficients in front of the κ-dependent terms in (5.1) and (5.3). At small κ the system will behave more like the pure gauge theory than in the fermion case. Since we have no chiral symmetry in the Higgs model, the behavior near the Higgs PT will probably be different from that of the theory with fermions in the chiral symmetric limit.

In fig. 3 we collect our results for $\langle L \rangle$ at small κ on an $8^3 \times 2$ lattice. The pure gauge theory goes through the deconfinement PT at $\beta = 1.89 \pm 0.01$ for $N_t = 2$. We

* The shift of the effective β due to the κ^4 correction in (5.3), though in the same direction, is not sufficient to explain the shift of the deconfinement PT/crossover observed in MC simulations with unquenched fermions. A quenched MC study combined with the hopping parameter expansion according to (5.3) for $N_t = 3$ shows that the κ^{N_t} correction dominates the shift [32].

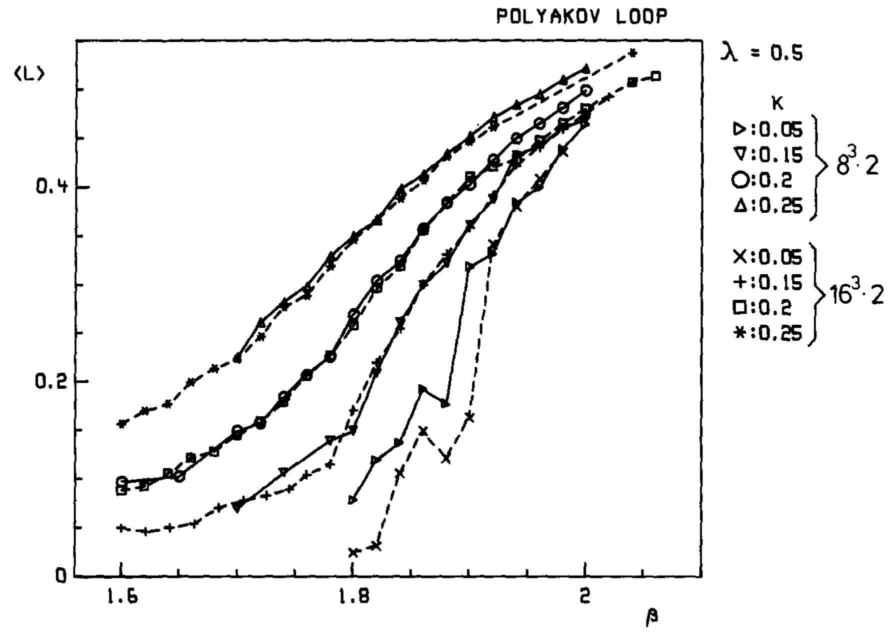


Fig. 3. Polyakov loop near the deconfinement PT/crossover for several values of κ on $8^3 \times 2$ and $16^3 \times 2$ lattices. The statistical errors determined by the blocking method are approximately of the same size as the symbols.

find that on this lattice the behavior of $\langle L \rangle$ is almost the same as in the pure gauge theory up to $\kappa \simeq 0.05$. With increasing κ, fig. 3 suggests that the deconfinement PT/crossover shifts to smaller β and weakens. The same tendency is also observed in the susceptibility (fig. 4),

$$\chi = N_s^3 (\langle L^2 \rangle - \langle L \rangle^2). \tag{5.4}$$

We have also studied the energy density. The deconfinement PT/crossover can be characterized by a sudden increase of $\langle P_t \rangle$ [21, 30] and, we found, also by a sudden decrease of $\langle \Phi^\dagger U_t \Phi \rangle$. We can thus trace it by both ε_G and $\varepsilon_H^{(sp)}$. Their behavior, shown in figs. 5 and 6, confirms the structure suggested by the properties of $\langle L \rangle$. The deconfinement PT/crossover shifts slightly to smaller β. As expected, this shift with κ is much smaller than in the fermion case [6]. The rapid change of observables becomes a milder one around $\kappa \approx 0.20$ far below the Higgs PT at $\kappa \simeq 0.29$ for $\beta = 1.9$. (The peak of $\varepsilon_H^{(sp)}$ at large κ's corresponds to the Higgs PT/crossover. See sect. 6.)

The question is whether this rapid variation of observables is due to a PT or is just a crossover. To answer it, we have studied the same quantities on a $16^3 \times 2$ lattice. Our guiding principle is as follows: If the rapid variation is due to a PT, the

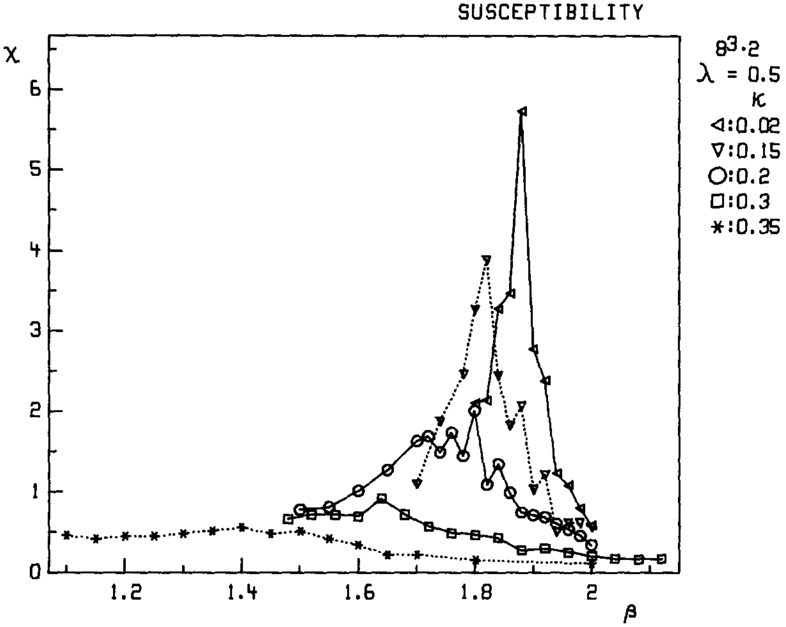


Fig. 4. Susceptibility near the deconfinement PT/crossover on an $8^3 \times 2$ lattice.

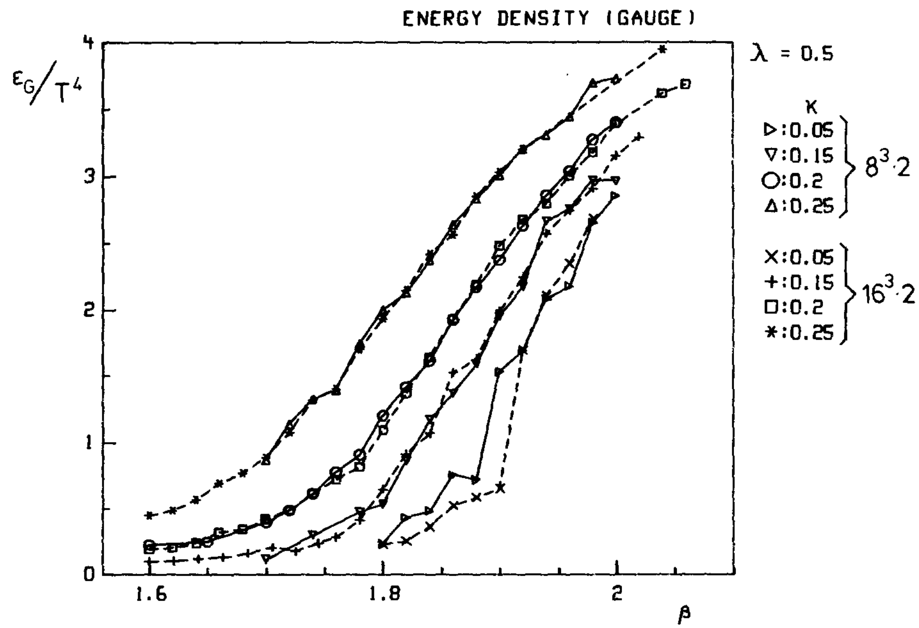


Fig. 5. The gauge part ε_G of the energy density in units of T^4. Errors are approximately of the same size as the symbols.

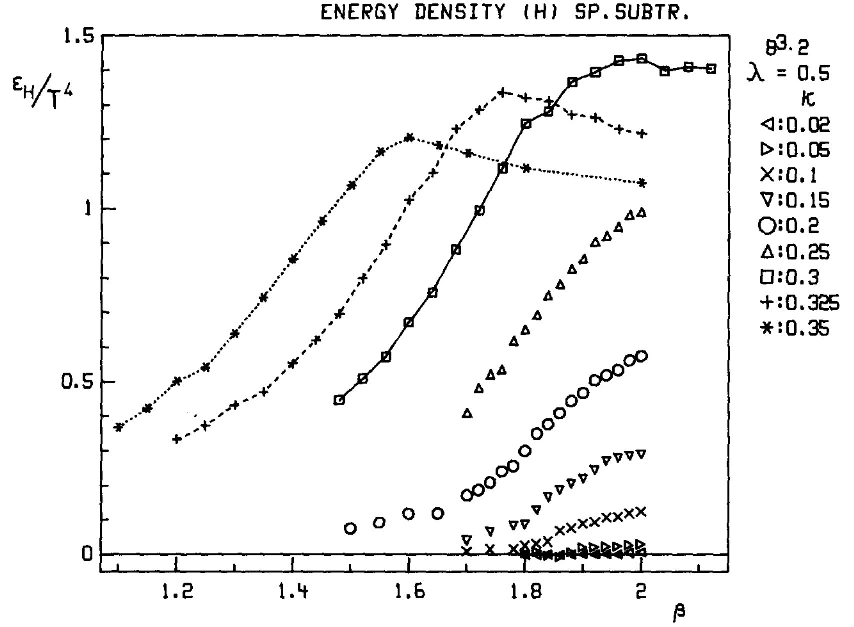


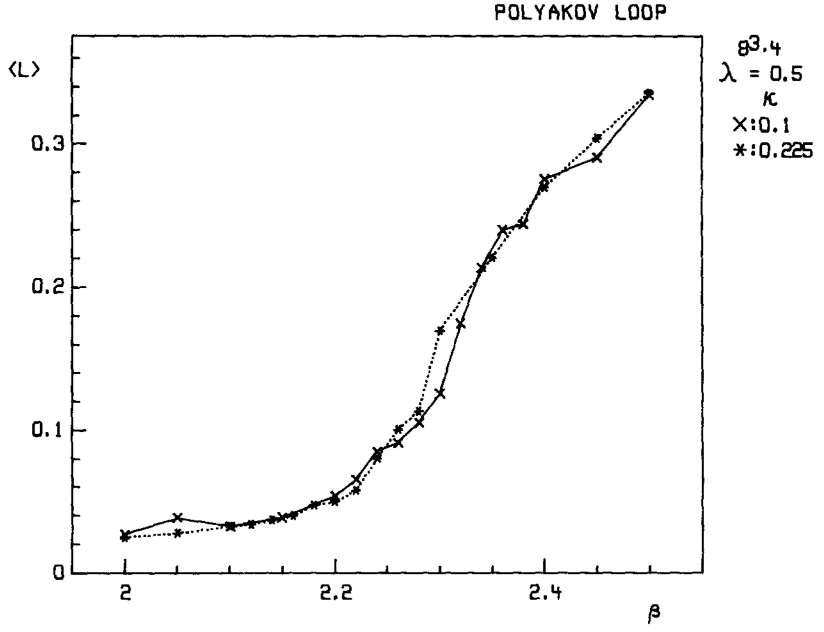
Fig. 6. The scalar field part $\varepsilon_{\mathrm{H}}^{(\mathrm{sp})}$ of the energy density in units of T^4 with the spatial subtraction.

signal will become clearer on a larger lattice. On the other hand, if the smooth behavior is independent of the lattice size, it originates from a crossover. The converse of these statements is not always true: For a crossover very near to a second order PT, the correlation length will not be sufficiently small in comparison with the lattice sizes under consideration, so that the crossover may appear like a PT.

Our results for $\langle L \rangle$ on the larger lattice are shown in fig. 3, too. A similar size dependence is also observed for the gauge energy density (fig. 5). For $\kappa = 0.05$, the data show a clear finite size effect. As discussed in the previous paragraph, we cannot deduce from it the nature of the deconfinement PT/crossover at this κ. The steeper behavior of $\langle L \rangle$ and ε_{G} on the $16^3 \times 2$ lattice may be explained by the nearby second order deconfinement PT for $\kappa = 0$. Nevertheless, we can learn the typical magnitude of the finite size effects we might expect for a PT when the lattice size changes from 8^3 to 16^3. For $\kappa \geqslant 0.15$ we find no finite size effects at all. We thus conclude that here the rapid variation of $\langle L \rangle$ and ε observed is a crossover*. The resulting deconfinement PT/crossover for $N_{\mathrm{t}} = 2$ is shown in fig. 1.

In order to study the N_{t}-dependence of the deconfinement PT/crossover, we have performed a rough survey on an $8^3 \times 4$ lattice. In the pure gauge limit, the PT shifts

* As mentioned in the introduction, it is outside of our computational accuracy to exclude the possibility of a higher order PT if it does not show finite size effects. Since this does not happen in the Ising model, the universality arguments suggest this alternative to be unlikely for the Higgs model, too.



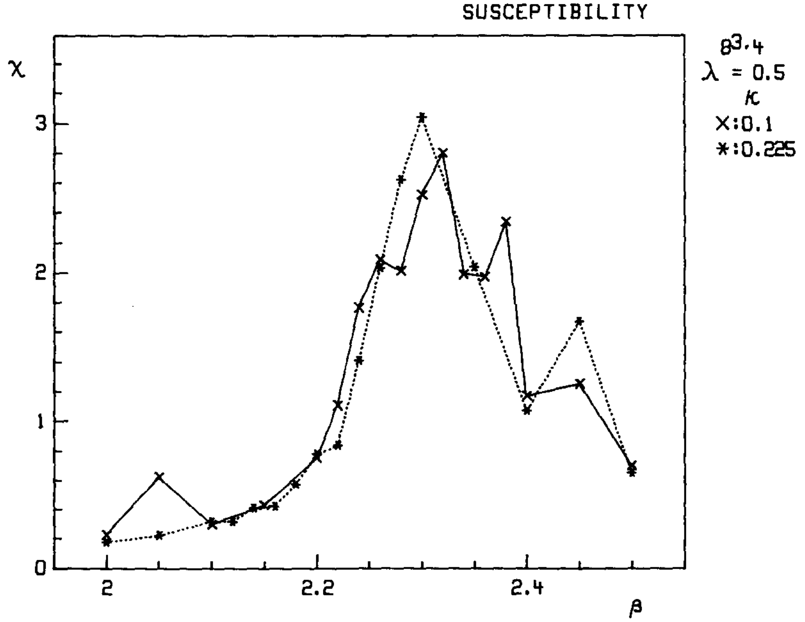


Fig. 7. Polyakov loop (a) and susceptibility (b) near the deconfinement PT/crossover below the Higgs PT on an $8^3 \times 4$ lattice.

to a larger β with N_t due to scaling. At $\kappa \neq 0$ we expect the κ-interval with sharp crossover ($0 \leqslant \kappa \leqslant 0.1$ for $N_t = 2$) to increase with N_t, because the RG lines presumably approach the $T = 0$ Higgs PT with growing β. The same effect is also suggested by the hopping parameter expansion (5.1), in which the power of κ in the Z(2) symmetry breaking term becomes higher at larger N_t. In fig. 7 we show our results for $\langle L \rangle$ and χ. As expected, the sharp variation persists until larger κ. The same effect has also been observed previously for the Higgs model [10] as well as in the fermion case [6]. The behavior of the energy density is consistent with the behavior of both observables shown in fig. 7.

6. Higgs phase transition

We now turn our attention to the Higgs PT at high temperature. At $T = 0$ and $\lambda = 0.5$, the Higgs PT line has an end point at $\beta = 1.5 \pm 0.5$ (fig. 1). In the interval $3 \leqslant \beta \leqslant 6$ its position decreases slightly with growing β. The strongest PT on this line is observed at $\beta \simeq 2.25$ [18], where it seems to be weakly of first order. At larger β the PT becomes weaker (second order or still weakly first order).

We apply the same method as in the previous section to determine the nature of the Higgs PT at finite T. The convenient local observables to study it are $\langle \Phi^\dagger U_s \Phi \rangle$ and $\langle \Phi^\dagger U_t \Phi \rangle$. We shall compare them with $\langle \Phi^\dagger U \Phi \rangle_0$ on the symmetric lattice. In fig. 8 we show these quantities at $\beta = 4$ on $8^3 \times 2$, $16^3 \times 2$ and 8^4 lattices as functions of κ. The $T = 0$, Higgs PT at $\kappa = 0.235 \pm 0.005$ causes a sudden bend in the κ-dependence of $\langle \Phi^\dagger U \Phi \rangle_0$. On the other hand, both $\langle \Phi^\dagger U_s \Phi \rangle$ and $\langle \Phi^\dagger U_t \Phi \rangle$ show a much milder change on asymmetric lattices. The data on $8^3 \times 2$ and $16^3 \times 2$ lattices are consistent with each other and are thus independent of the lattice size. Applying the same reasoning as in the previous section, we therefore conclude that the Higgs PT on the symmetric lattice has changed into a crossover at $N_t = 2$.

Another important feature of the data shown in fig. 8 is a small shift of the location of the crossover to a larger κ by about 0.01 with respect to the location of the Higgs PT at $T = 0$. The direction of this shift with T is consistent with the expectation for the KLW transition discussed in sect. 3. Its smallness explains why it has not been observed in previous studies of the model [8–10]. The physical importance of this shift was discussed in the introduction.

We have performed a similar analysis at $\beta = 2.25$ and some further calculations at $\beta = 2$ and 5 on $8^3 \times 2$ and 8^4 lattices. Qualitatively the same change of $\langle \Phi^\dagger U_\mu \Phi \rangle$ at finite temperature is always observed. For $N_t = 2$ and $\lambda = 0.5$, the whole Higgs PT line seems to change into a crossover. For these values of β, the position of the Higgs crossover relative to the $T = 0$ Higgs PT is shown in fig. 1.

We found no remarkable variation of $\langle L \rangle$ at the Higgs crossover. This feature is different from a previous observation [10], presumably due to the larger value of our λ. In plots for constant β, the gauge part ε_G of the energy density shows a peak at the Higgs crossover. This peak becomes softer at larger β.

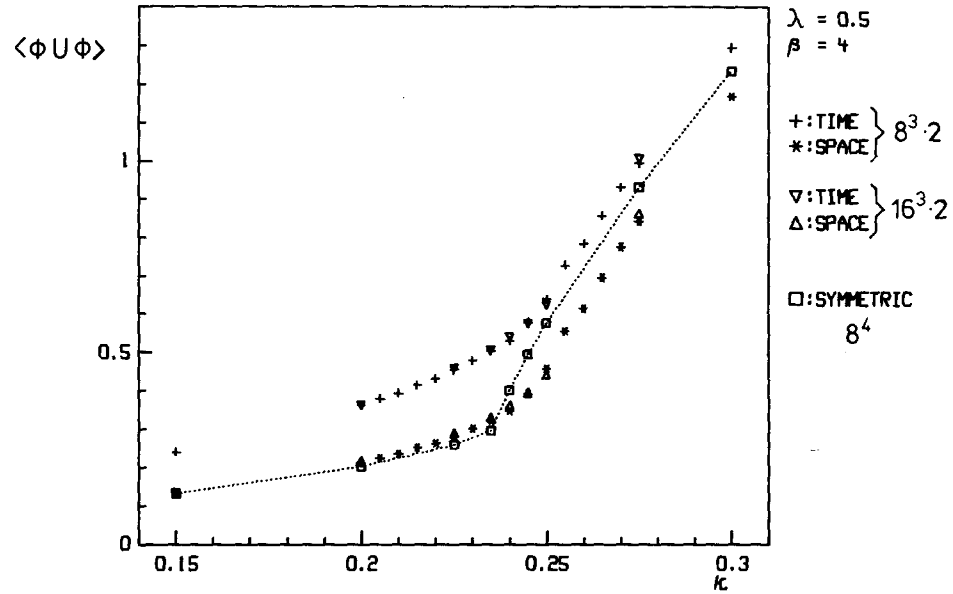
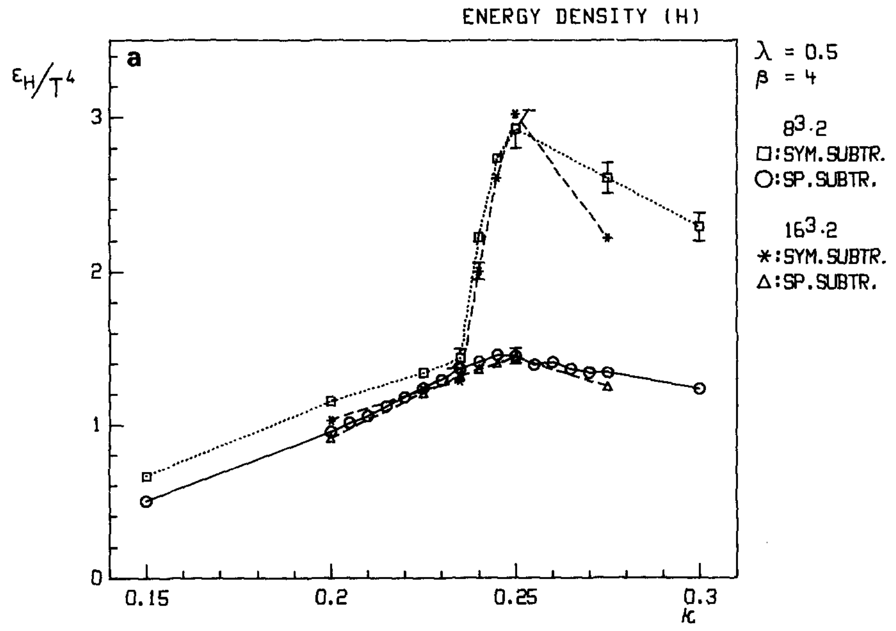


Fig. 8. Link variables $\langle \Phi^{\dagger} U_{\mu} \Phi \rangle$ in spatial and temporal directions on $8^3 \times 2$ and $16^3 \times 2$ lattices near the Higgs PT/crossover at $\beta = 4$. For comparison we also show the $T = 0$ symmetric link variable $\langle \Phi^{\dagger} U \Phi \rangle_0$ on an 8^4 lattice.

Let us now discuss the Higgs energy density ε_{H} at the Higgs crossover. As explained in sect. 4, ε_{H} requires a subtraction of the $T = 0$ expectation values (*symmetric subtraction*). The gauge part ε_{G} requires a subtraction, too, if we go beyond the tree approximation [35]. We have learned from the data shown in fig. 8 that for $T \neq 0$ near the Higgs crossover the expectation value $\langle \Phi^{\dagger} U_{\mathrm{s}} \Phi \rangle$ can be quite different from $\langle \Phi^{\dagger} U \Phi \rangle_0$ at $T = 0$. We have found similar differences between the $T = 0$ and $T \neq 0$ behavior of $\langle \Phi^{\dagger} \Phi \rangle$ and $\langle (\Phi^{\dagger} \Phi)^2 \rangle$. Therefore the *spatial subtraction*, in which the $T = 0$ expectation values are approximated by corresponding spatial expectation values on the asymmetric lattice, turns out to be a bad approximation to the symmetric subtraction at and above the Higgs crossover. In fig. 9 we show the difference between the two subtraction schemes. The symmetric subtraction leads to a much larger ε_{H} and total energy density ε in the Higgs region. This difference will be important for an analysis of the asymptotic Stefan-Boltzmann (SB) behavior.

Here we encounter an interesting question: The energy density with the symmetric subtraction at finite temperature shows a singular behavior at the κ-location of the zero temperature Higgs PT! This singularity appears only through the subtracted expectation values at $T = 0$. One could suspect that there is something wrong with the symmetric subtraction which seems to produce a fake singularity of the energy density at finite temperature. However, we think that the correct physical interpretation of the MC data avoids such a problem: The symmetric subtraction corresponds



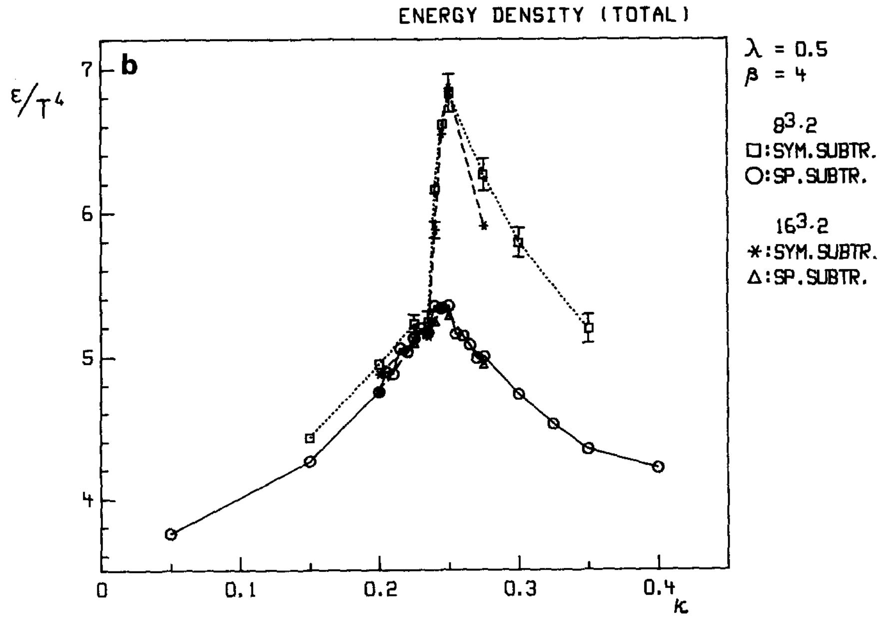


Fig. 9. The scalar field part ε_H of the energy density (a) and the total energy density ε (b) at $\beta = 4$ in units of T^4 with spatial and symmetric subtractions on $8^3 \times 2$ and $16^3 \times 2$ lattices. For the symmetric subtraction the data obtained on an 8^4 lattice have been used.

to the normalization condition for the energy density to be zero at $T=0$. With this condition it is physically sensible to compare the magnitude of the renormalized energy density *only for the same physical system*, i.e. on the same RG line. RG lines never cross the PT of the system at $T=0$, thus the singularity of ε at the $T=0$ Higgs PT is never encountered when a given physical system is heated.

7. Large β behavior of the energy density

Finally, we discuss the large β behavior of the energy density. The spurious freezing PT due to the approximation of SU(2) by the icosahedron group Y is located at $\beta = 5.8 \pm 0.3$ for $\kappa \leqslant 0.325$ (fig. 1). By inspecting the hystereses of $\langle P_\mu \rangle$, $\langle \Phi^\dagger U_\mu \Phi \rangle$ etc. obtained in thermal cycles around the freezing transition, we have convinced ourselves that for our κ's the values $\beta \leqslant 5$ are sufficiently outside the freezing PT region. In fig. 10, we show the large β behavior of energy densities at several typical fixed κ-values compared with the ideal gas limit (SB limit)

$$\varepsilon_{SB}/T^4 = \tfrac{1}{30}\pi^2 N_f C(N_s, N_t) . \tag{7.1}$$

Here N_f is the number of fields (the total number in our case is $3 \times 2 + 4$) and $C(N_s, N_t)$ is the exactly calculable finite size correction [36]*. Because of limited computed time, we have results for the scalar field energy density ε_H with the spatial subtraction only. As has been discussed above, the ambiguity due to the subtraction schemes is small in the deconfinement region. In the Higgs region at $\beta = 4$, the values of ε_H with symmetric subtraction are larger than those with spatial subtraction by about one. We expect the difference at other values of β to be similar.

We have plotted the energy densities as functions of β at constant values of κ, since the positions of the RG lines are not yet known. Only along these lines we would be able to study the temperature behavior of a given physical system. When we increase β at constant κ, the physical system changes so that the physical meaning of the curves in fig. 10 is not a priori clear. However, we expect that, at least for a comparison with the universal SB behavior, the dependence on the variable $T/M_{phys} = 1/(N_t M_{phys} a)$ will be more relevant than that on the ratios of physical masses. Then, provided that $M_{phys} a$ becomes sufficiently small at large β for fixed κ, we may expect the universal SB behavior at such β regardless of the (small) difference between physical systems on a constant κ line.

In the confinement and deconfinement regions, the temperature measured in units of the string tension at $T=0$ is expected to rise when β increases for constant κ, as in the pure SU(2) case. But the scale M_{phys} relevant for the validity of the SB law in the total system of gauge and matter fields will be large for small κ.

* In fig. 10, the values of ε_{SB} for the $16^3 \times 2$ lattice are practically identical with those for $8^3 \times 2$. We found $C(16,2)$ to be 0.16% larger than $C(8,2)$.

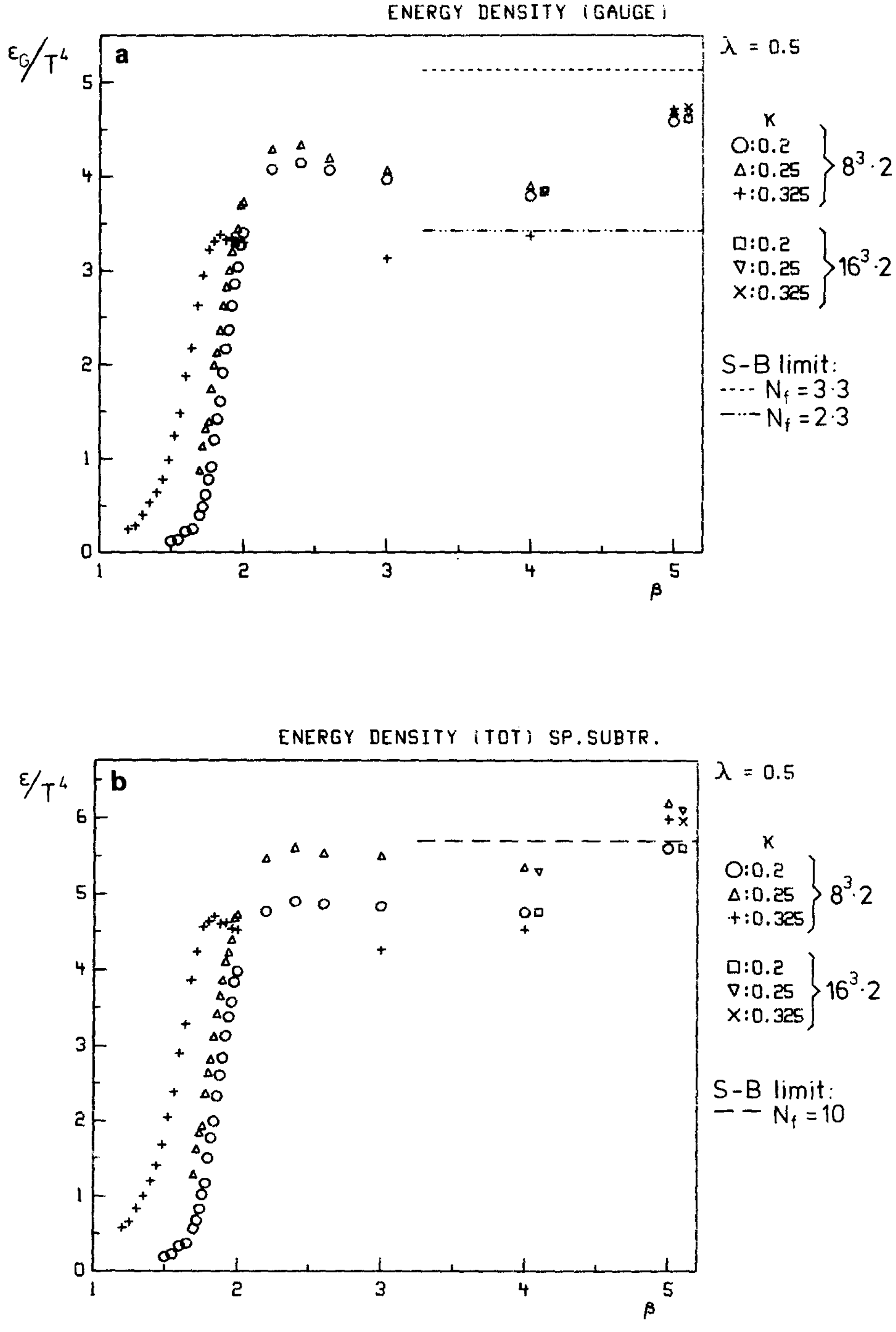


Fig. 10. Large β behavior of ε_G/T^4 (a) and $\varepsilon^{(sp)}/T^4$ (b) with spatial subtraction. Results from a $16^3 \times 2$ lattice at $\beta = 4$ and 5 are shifted slightly to the right for the sake of clarify of the figure.

Furthermore, along the lines of constant κ the physics changes and this M_{phys} may rise with increasing β. Therefore it might be difficult or even impossible to find the SB behavior of the Higgs system varying β with fixed κ. From fig. 10a we see that for $\kappa = 0.2$ the gauge energy density ε_G overshoots the free massless SU(2) gas limit ($N_f = 3 \times 2$) like in the fermion case [6, 32], but unlike in the latter case, ε_G does not converge to this limit at large β but increases again for $\beta \geqslant 4$. Results from the $16^3 \times 2$ lattice at $\beta = 4$ and 5 are consistent with those from $8^3 \times 2$, as shown in fig. 10. They confirm that this increase is not due to the smaller total physical lattice sizes for these β's. Also the total energy density $\varepsilon^{(\text{sp})}$ (fig. 10b) changes at least until $\beta = 5$. This indicates that for $\kappa = 0.2$ the SB limit has not been achieved even for this value of β.

Also in the Higgs region, the physical interpretation of our results for $\varepsilon^{(\text{sp})}$ is difficult. According to fig. 10, ε_G and $\varepsilon^{(\text{sp})}$ seem to approach the same large β limits as those in the deconfinement region. But since $\varepsilon^{(\text{sp})}$ is expected to be smaller than ε with the symmetric subtraction, the latter might actually overshoot the SB limit. Again, it might be that in the Higgs region T/M_{phys} does not increase sufficiently with β for fixed κ: If we can simply extend the result obtained at small β and at $T = 0$ [28], $M_{\text{phys}}a$ will not decrease much along the constant κ lines in the Higgs region. It may even increase there. In that case we would not have high T/M_{phys} at large β and fixed κ. One should increase the temperature by increasing β and simultaneously by approaching the $T = 0$ Higgs PT line. Further studies of RG lines without the icosahedron approximation will be necessary. An interesting question to be studied is whether we could see an SB behavior on these lines before we pass through the Higgs PT/crossover at high T.

8. Concluding remarks

From our MC investigation of the finite temperature SU(2) lattice Higgs model with a doublet scalar field we draw the following conclusions:

(i) *For positive β and λ the system has only one phase in the 4-dimensional space of the parameters β, λ, κ and T.* Though our data are restricted to the hyperplane $\lambda = 0.5$, an analytic continuation of the regions connected analytically on this hyperplane to other λ-values is expected to be possible in analogy to the $T = 0$ model [14].

(ii) *The coupling to the scalar field changes the second order deconfinement PT of the pure $SU(2)$ gauge theory into a crossover.* The crossover, verified by the finite size analysis, shifts to lower β and weakens as κ increases. It gets very smooth long before κ reaches the values at which the Higgs PT takes place. We note that a similar property is reported for the deconfinement transition in the SU(3) gauge theory with Wilson fermions [34], which has no chiral symmetry for finite lattice constant a. For Kogut-Susskind fermions the deconfinement PT or crossover is well observable also for κ-values close to the chiral limit both in SU(2) and SU(3) gauge theories [6, 33]. Our results confirm the expectation based on the hopping parameter expansions that scalars and fermions influence the deconfinement transition very similarly except for chiral symmetry effects.

(iii) *At large λ and high temperatures the Higgs PT changes into a crossover*. We have observed a smoothening of the Higgs PT at $\lambda = 0.5$ for $N_t = 2$. No finite size effects have been seen at $\beta = 2.25$ and 4. For much smaller λ, we expect the following difference: Since at $T = 0$ the Higgs PT is of first order, it persists up to some finite temperature. But we cannot achieve a really high temperature on isotropic lattices even with the smallest N_t because of the smallness of the correlation length. Indeed, for small λ and for $\beta \simeq 2$–5, no change of the nature of the Higgs PT at small N_t has been observed so far [9, 10].

(iv) *The crossover remnant of the Higgs PT slightly shifts to larger κ relative to the Higgs PT at $T = 0$*. Thus, at large λ, RG lines in the Higgs region very close to the $T = 0$ Higgs PT can cross the Higgs crossover at finite T. This crossover therefore corresponds to the Kirzhnits-Linde-Weinberg phase transition [2, 3] smoothened by nonperturbative effects.

Our work also reveals some difficulties arising in studies of Higgs models at finite temperature: Since at moderate β-values the vector boson and Higgs boson masses in lattice units grow when κ increases above the Higgs PT [17, 28, 29], the temperature measured in units of physical masses soon gets very low even for the smallest N_t. Thus, in the Higgs region, the only way to achieve a high T in physical units on isotropic lattices is to approach the Higgs PT closely at a location where this transition shows signs of critical behavior at $T = 0$. This requires a large λ and a fine tuning of κ. Far from the Higgs PT line a higher physical temperature can only be achieved on nonisotropic lattices which have a smaller lattice constant in the temporal direction. Furthermore, a reliable knowledge of RG lines and of the scaling regions in the 3-dimensional coupling space is still lacking. The qualitative results obtained in our paper will not depend on the scaling properties of the model. However, in order to extract quantitative information relevant for the electroweak theory from the finite temperature simulations on the lattice, a better understanding of the zero temperature lattice Higgs model will be required.

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References

[1] T. Çelik, J. Engels and H. Satz, Nucl. Phys. B256 (1985) 670, and references therein
[2] D.A. Kirzhnits and A.D. Linde, Phys. Lett. 42B (1972) 471
[3] S. Weinberg, Phys. Rev. D9 (1974) 3357;
L. Dolan and R. Jackiw, Phys. Rev. D9 (1974) 3320
[4] D.A. Kirzhnits and A.D. Linde, Ann. of Phys. 101 (1976) 195;
J. Iliopoulos and N. Papanicolaou, Nucl. Phys. B111 (1976) 209
[5] A. Nakamura, Phys. Lett. 149B (1984) 391
[6] U. Heller and F. Karsch, Nucl. Phys. B258 (1985) 29;
J.B. Kogut, J. Polónyi, H.W. Wyld and D.K. Sinclair, Phys. Rev. D31 (1985) 3307; Phys. Rev. Lett. 54 (1985) 1475;
U. Heller, Phys. Lett. 163B (1985) 203
[7] T. Banks and E. Rabinovici, Nucl. Phys. B160 (1979) 349
[8] J. C. Sexton, Fermilab preprint-Pub-85/168-T
[9] T. Munehisa and Y. Munehisa, Z. Phys. C32 (1986) 531
[10] P.H. Damgaard and U.M. Heller, Phys. Lett. 171B (1986) 442; preprint CERN-TH 4481/86
[11] D. Espriu and J.F. Wheater, Nucl. Phys. B258 (1985) 101
[12] F. Karsch, E. Seiler and I.O. Stamatescu, Phys. Lett. 131B (1983) 138; Phys. Lett. 157B (1985) 60;
M. Kikugawa, T. Maehara, J. Saito, R. Sasaki, H. Tanaka and Y. Yamaoka, Progr. Theor. Phys. 74 (1985) 553
[13] H. Kühnelt, C.B. Lang and G. Vones, Nucl. Phys. B230 [FS10] (1984) 16
[14] J. Jersák, C.B. Lang, T. Neuhaus and G. Vones, Phys. Rev. D32 (1985) 2761
[15] C. Bernard, Phys. Rev. D9 (1974) 3312;
M. Suzuki, Progr. Theor. Phys. 56 (1976) 1454
[16] J. Engels, F. Karsch, H. Satz and I. Montvay, Nucl. Phys. B205 [FS5] (1982) 545
[17] W. Langguth and I. Montvay, Phys. Lett. 165B (1985) 135;
W. Langguth, I. Montvay and P. Weisz, Nucl. Phys. B277 (1986) 11
[18] H. G. Evertz, J. Jersák, D.P. Landau, T. Neuhaus and J.-L. Xu, in preparation
[19] L. McLerran and B. Svetitsky, Phys. Rev. D24 (1981) 450;
J.B. Kogut, M. Stone, H.W. Wyld, W.R. Gibbs, J. Shigemitsu, S.H. Shenker and D.K. Sinclair, Phys. Rev. Lett. 50 (1983) 393;
R.V. Gavai and H. Satz, Phys. Lett. 145B (1984) 248;
G. Curci and R. Tripiccione, Phys. Lett. 151B (1985) 143;
K. Kanaya, *in* Non-perturbative methods, ed. S. Narison (World Scientific, 1986);
J. Kiskis, Phys. Rev. D33 (1986) 2380; D35 (1987) 1456
[20] K. Kanaya and H. Satz, Phys. Rev. D34 (1986) 3193;
J. Engels, J. Jersák, K. Kanaya, E. Laermann, C.B. Lang, T. Neuhaus and H. Satz, Nucl. Phys. B280 [FS18] (1987) 577
[21] B. Svetitsky and L.G. Yaffe, Nucl. Phys. B210 [FS6] (1982) 423;
J. Polónyi and K. Szlachányi, Phys. Lett. 110B (1982) 395
[22] T.D. Lee and C.N. Yang, Phys. Rev. 87 (1952) 410;
R.B. Griffiths, *in* Phase transitions and critical phenomena, vol. 1, ed. C. Domb and M.S. Green (Academic Press, 1974)
[23] L. Susskind, Phys. Rev. D20 (1979) 2610
[24] T. Banks and A. Ukawa, Nucl. Phys. B225 [FS9] (1983) 145
[25] R.H. Brandenburger, Rev. Mod. Phys. 57 (1985) 1, and references therein
[26] A. D. Linde, Phys. Lett. 96B (1980) 289;
D.J. Gross, R.D. Pisarski and L.G. Yaffe, Rev. Mod. Phys. 53 (1981) 43
[27] Y. Nambu, Nucl. Phys. B130 (1977) 505
[28] H.G. Evertz, J. Jersák, C.B. Lang and T. Neuhaus, Phys. Lett. 171B (1986) 271
[29] I. Montvay, Phys. Lett. 150B (1985) 441
[30] H. Yoneyama, Z. Phys. C30 (1986) 267;
M. Imachi, Progr. Theor. Phys. 76 (1986) 192

[31] A. Hasenfratz and P. Hasenfratz, Phys. Lett. 104B (1981) 489;
I.O. Stamatescu, Phys. Rev. D25 (1982) 1130;
C.B.Lang and H. Nicolai, Nucl. Phys. B200 [FS4] (1982) 135
[32] T. Çelik, J. Engels and H. Satz, Phys. Lett. 133B (1983) 427
[33] R.V. Gavai, M. Lev and B. Petersson, Phys. Lett. 140B (1984) 397; 149B (1984) 492;
F. Fucito and S. Solomon, Phys. Lett. 140B (1984) 387; Phys. Rev. Lett. 55 (1985) 2641;
M. Fishler and R. Roskies, Phys. Lett. 145B (1984) 99;
J. Polónyi, H.W. Wyld, J.B. Kogut, J. Shigemitsu and D.K. Sinclair, Phys. Rev. Lett. 53 (1984) 644;
F. Fucito, C. Rebbi and S. Solomon, Nucl. Phys. B248 (1984) 615;
F. Fucito, S. Solomon and C. Rebbi, Phys. Rev. D31 (1985) 1460;
F. Fucito, R. Kinney and S. Solomon, Nucl. Phys. B225 (1985) 727;
R.V. Gavai and F. Karsch, Nucl. Phys. B261 (1985) 273;
S. Duane and J.B. Kogut, Phys. Rev. Lett. 55 (1985) 2774;
R.V. Gavai, Nucl. Phys. B269 (1986) 530;
M. Fukugita and A. Ukawa, Phys. Rev. Lett. 57 (1986) 503
[34] M. Fukugita, S. Ohta and A. Ukawa, Phys. Rev. Lett. 57 (1986) 1974
[35] F. Karsch, Nucl. Phys. B205 [FS5] (1982) 285
[36] J. Engels, F. Karsch and H. Satz, Nucl. Phys. B205 [FS5] (1982) 239