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# CONFINED AND FREE CHARGES IN COMPACT SCALAR QED*

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Using new order parameters – one proposed by Fredenhagen and Marcu, and another one suggested by ourselves – we have investigated by Monte Carlo method several properties of the transition between the confinement-Higgs phase and the free charge phase of the U(1) lattice Higgs model. These different parameters are constructed by means of gauge invariant 2-point functions and of Wilson loops. The latter show perimeter decay in all phases in the presence of matter fields with unit charge. Nevertheless, appropriate ratios of such 2-point functions and Wilson loops provide sensitive order parameters which are unequal zero in the confinement-Higgs phase but vanish in the free charge phase. We give a heuristic interpretation of this behaviour in terms of dynamical and external charges.

## 1. Introduction

Compact scalar QED on the lattice, the so called U(1) lattice Higgs model [1–7], is a prototype of a gauge theory with matter fields which exhibits several characteristic properties of gauge theories within the same model. In various regions of the phase diagram in the space of the coupling constants one can find

– confinement in the sense that charged states do not occur in the physical spectrum,

– condensation of scalar matter fields, conventionally associated with the "Higgs phenomenon",

– free charges.

The U(1) lattice Higgs model is thus suitable for an investigation of these different features of gauge theories, in particular if one searches for observables and criteria allowing to identify and to distinguish between them. The lattice formulation permits a gauge invariant characterization of these various physical situations.

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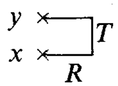
The number of practically calculable gauge invariant observables one can construct in gauge theories with matter is quite large. Correlation functions between various gauge invariant combinations of gauge and matter fields provide plenty of information about the spectrum of the theory, like the masses of the scalar Higgs boson, the vector boson, U(1) analogies of glueballs, and excited states. It turns out, however, that an excellent characterization of various features of the U(1) Higgs model, and presumably also of other gauge theories with matter fields, is achieved by studying those gauge invariant products of dynamical fields whose physical interpretation uses point-like external sources. These sources can also be looked at as additional, heavy charged fields, which makes their physical interpretation particularly transparent. For example, the Wilson loop criterion for confinement in pure gauge theories is formulated in terms of gauge fields alone, but interpreted physically in terms of external sources or heavy quarks.

The Wilson loop criterion [8] is known to fail in gauge theories with dynamical matter fields screening the confining linear potential [1,2,9]. However, in this case it is possible to find criteria for confinement by using a gauge invariant 2-point function of the matter fields. Again, a suitable physical interpretation can be found by means of external sources. The 2-point function of the matter field $\Phi$ [1,10,11] which we shall mainly consider in this paper is defined as follows:

$$G(T,R)=\left\langle \Phi_x^* \prod_{\ell\in\Gamma} U_\ell \Phi_y \right\rangle, \qquad |x-y|=T, \tag{1.1}$$

$\Gamma =$ 


where $U_\ell$ are link variables on an oriented path $\Gamma$ connecting points $x$ and $y$. In analogy to the Wilson criterion, the parallel transporter in the $T$ direction of the path $\Gamma$, when the $T$-direction is interpreted as (euclidean) time, is associated with the presence of an external charge q at the spatial point $\boldsymbol{x}+\boldsymbol{R}=\boldsymbol{y}+\boldsymbol{R}$ [2,12]. In the 2-point function the dynamical matter field is thus correlated with this charge at the spatial distance $R$ (a detailed explanation of this interpretation will be given in sec. 5).

The study of $G(T,R)$ provides information about the screening of an external source q by matter fields. This screening can either be due to the binding of one charged particle c associated with the $\Phi$-field to the external charge q, or it can be analogous to the Debye screening in a plasma of such particles. The presence of a screened external charge is associated with an energy increase $\mu \geqslant 0$ of the ground state of the system of the dynamical fields. This "*screening energy*" $\mu$ is obtained from the exponential decay of $G(T,R)$ at large distances $T=|x-y|$:

$$G(T,R) \xrightarrow[\substack{T\ \mathrm{large} \\ R\ \mathrm{fixed}}]{} f_G(R)\,\mathrm{e}^{-\mu T}. \tag{1.2}$$

It has been found by Fredenhagen and Marcu that one can use $G(T, R)$ in order to construct a criterion for the existence of free charged states with finite energy in gauge theories with matter fields [11]. This criterion has been investigated in several lattice Higgs models by numerical [13–15] and various analytic methods [16,17]. The function $G(T, R)$ has to be studied for both $T$ and $R$ large in this context. Motivated by our considerations of the screening energy $\mu$ we propose another criterion for confinement, for which $R$ in $G(T, R)$ can be zero.

We have performed a Monte Carlo investigation of the function $G(T, R)$ in lattice scalar QED with a scalar field of charge one. In our calculations we have concentrated on the vicinity of the Higgs phase transition (PT) which separates the free charge (Coulomb) phase from the Higgs region of the confinement-Higgs phase. The values of the parameter $\mu$ on both sides of the Higgs PT are determined and physically interpreted. We also point out that $\mu$ is a very suitable parameter to study the order of the Higgs PT. Although this PT seems to be continuous for our choice of coupling parameters if only local observables like $\langle \Phi^* \Phi \rangle$ are considered, $\mu$ exhibits a discontinuity, characteristic for a PT of first order. Our main result is the numerical support to the validity of the criterion proposed by Fredenhagen and Marcu [11] and of the one we propose in this paper. Both test for confinement or free charges.

The outline of this paper is as follows. In the next section we define lattice scalar QED and describe its phase structure in the space of three coupling parameters. In sect. 3 we then briefly summarize the main results of our investigation, whereas more detailed descriptions and interpretations are contained in the following sections: After discussing the static potential (sect. 4), we explain in detail the physical meaning of the function $G(T, R)$ in sects. 5 and 6. We describe its usefulness for studying the screening of the confining potential in the Higgs region (sect. 7), properties of charged particles (sect. 8) and the order of the Higgs PT (sect. 9). Finally, we discuss the confinement criteria (sects. 10 and 11) and the criterion for the existence of bound states of a charged particle and an external source due to Bricmont and Fröhlich (sect. 12). Sect. 13 contains conclusions.

## 2. Phase structure of the compact scalar QED

The compact lattice U(1) Higgs model with a scalar field $\Phi$ of charge one is defined by the action [18,6,7]

$$S = -\tfrac{1}{2}\beta \sum_{p \in \Lambda} \left(U_P + U_P^*\right)$$
$$-\kappa \sum_{x \in \Lambda} \sum_{\mu=1}^{4} \left(\Phi_x^* U_{x,\mu} \Phi_{x+\mu} + \text{c.c.}\right) + \lambda \sum_{x \in \Lambda} \left(\Phi_x^* \Phi_x - 1\right)^2 + \sum_{x \in \Lambda} \Phi_x^* \Phi_x . \quad (2.1)$$

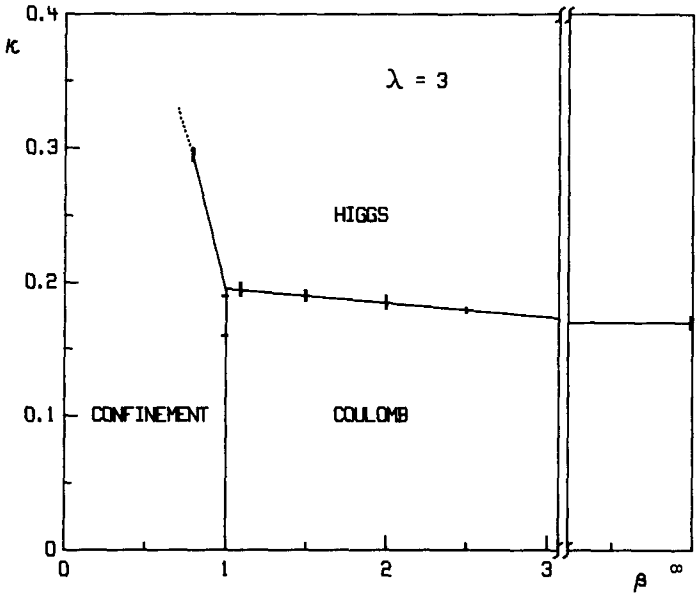



Fig. 1. Phase diagram of compact scalar QED as determined in our earlier [6] and present Monte Carlo calculations, at fixed quartic coupling $\lambda = 3$.

The relation to the bare parameters used in the continuum field theory,

$$S_{\mathrm{cont}} = \int \mathrm{d}^4x \left[ (D_\mu \varphi_x)^* (D_\mu \varphi_x) + \tilde{m}^2 \varphi_x^* \varphi_x + \tilde{\lambda} (\varphi_x^* \varphi_x)^2 \right], \tag{2.2}$$

is given by [18]

$$a\varphi_x = \Phi_x \sqrt{\kappa}\,, \qquad \tilde{\lambda} = \lambda/\kappa^2, \qquad (a\tilde{m})^2 = (1 - 2\lambda - 8\kappa)/\kappa. \tag{2.3}$$

We note that $\kappa = 0$ corresponds to $\tilde{m}^2 = +\infty$ and $\kappa = \infty$ to $\tilde{m}^2 = -\infty$ at a fixed $\tilde{\lambda}$.

For $\kappa = 0$ the model reduces to the pure U(1) lattice gauge theory with Wilson action. This theory has a deconfinement phase transition for $\beta \simeq 1$ [8,19,2,20]. In the limit $\beta \to \infty$ the pure lattice $\Phi^4$ theory with global U(1) symmetry is obtained, with spontaneous symmetry breakdown for $\kappa$ above the PT at a certain $\kappa_{\mathrm{PT}}(\lambda)$ [1,6]. For $\lambda = \infty$ the radial mode is frozen, $|\Phi| = 1$ [1,4]. In this case the model is trivial both for $\kappa = \infty$ and for $\beta = 0$, respectively. For $\beta \geqslant 0$ the model has two phases, depicted in fig. 1 for $\lambda = 3$. A similar phase structure is obtained for any $\lambda > 0.1$ [5–7].

The Coulomb phase ($\beta > \beta_{\mathrm{PT}} \simeq 1$, $\kappa < \kappa_{\mathrm{PT}}$) is expected to contain a massless photon [1]. Whereas the existence of the Coulomb phase with massless photon has been proven rigorously for the noncompact U(1) Higgs model [21], the corresponding proof does not exist, to the best of our knowledge, for the compact one. However, the MC results presented in this paper and in ref. [22] demonstrate the existence of such a phase, which is also expected to contain charged states of finite energy. This confirms, at least numerically, that for scalar QED Swieca's theorem [23, 24] holds on the euclidean lattice, too: free abelian gauge charges are only compatible with a vanishing mass of the vector gauge field coupled to the charges!

The confinement-Higgs phase occurs for all $\kappa$ when $\beta < 1$ and for large $\kappa$ when $\beta > 1$. It consists of two regions, the confinement region ($\beta < 1$, small $\kappa$) and the Higgs region (large $\kappa$). Both are analytically connected [1–7]. Nevertheless, there are physical differences between both regions which require their separate consideration. In the confinement region, static charges are screened by a heavy (for small $\kappa$) but dynamical "constituent" field $\Phi$, in analogy with mesons consisting of different heavy quarks in QCD. In this region the expectation value of $\langle \Phi_x^* \Phi_x \rangle$ is small and only weakly $\kappa$-dependent. In the Higgs region the $\Phi$-field "condensates", which is indicated by a strong dependence of $\langle \Phi_x^* \Phi_x \rangle$ on $\kappa$ of the form

$$\langle \Phi_x^* \Phi_x \rangle \simeq \frac{4}{\lambda} \kappa + \text{const}. \tag{2.4}$$

Here charges are screened by a plasma-like behaviour of the $\Phi$-condensate. It is remarkable that for large $\lambda$ and sufficiently small $\beta$ the transition from the constituent to the condensate regimes is smooth. This is a consequence of the analytic connection mentioned above.

The positions of the PT's in the 3-dimensional space of coupling parameters have been investigated in detail in ref. [6]. Here we shall mainly be interested in the PT between the Coulomb phase and the Higgs region (Higgs PT). For $\lambda = 3$ and $\beta = 2.5$ the Higgs PT occurs on an $8^3 \times 16$ lattice at

$$\kappa = \kappa_{\text{PT}} = 0.179 \pm 0.001 . \tag{2.5}$$

Most of the results we shall present here have been obtained on both sides of the Higgs PT in the vicinity of this point. For comparison we also include some data in the Coulomb phase far below the Higgs PT.

## 3. Summary of our results

We have concentrated on the differences between the behaviour of physical observables in the Coulomb phase and the Higgs region of the confinement-Higgs phase. The choice of the values $\lambda = 3$, $\beta = 2.5$ for two of the coupling parameters was motivated by earlier results [6] indicating that the Higgs PT for these values of $\lambda$ and $\beta$ might be of higher order. As we shall explain later, our new results are inconclusive with respect to the order of this PT.

The phase transition between the confinement and Higgs regions for $\beta$ not far below $\beta = 1$ is also of considerable interest. However, here we expect results similar to those obtained recently for the SU(2) Higgs model with the scalar field in the fundamental representation [14]. There the Higgs PT separates the confinement and Higgs regions of the same phase, too. It turned out that it is very difficult to obtain asymptotic results for large distances in the confinement region with lattice sizes we can afford. So we decided not to attempt a similar investigation for the U(1) model.

Most of our data have been obtained on lattices of the size $8^3 \times 16$. But we also have some data samples on $12^4$ and $16^4$ lattices, which allow us to check that most of the results are not significantly dependent on the lattice size. Typically we have performed 150 000 iterations on the $8^3 \times 16$ lattice and 40 000 ones on the other lattices.

The Monte Carlo runs utilized in this paper have also been used for an analysis of the mass spectrum of the same model [22]. The total Cyber-205 CPU time spent for both purposes was about 350 hours. We used the Metropolis algorithm. For details of the program see ref. [6].

Let us summarize our results for $\lambda = 3$ and $\beta = 2.5$. They have been obtained for $\kappa$ in the vicinity of the Higgs PT, both in the Coulomb phase and in the Higgs region of the confinement-Higgs phase, $0.175 \leqslant \kappa \leqslant 0.64$. In addition we give results for small $\kappa$, i.e. deep in the Coulomb phase:

The static potential $V(R)$ is independent of $\kappa$ for all $\kappa < \kappa_{\mathrm{PT}}$. Its $R$-dependence is consistent with the Coulomb potential, and the value of the fine structure constant $\alpha$ agrees with that of the pure U(1) theory at the same $\beta$. For $\kappa > \kappa_{\mathrm{PT}}$ the potential slowly turns Yukawa-like. The Yukawa mass is consistent with the nonzero photon mass as calculated in ref. [22] by means of correlation functions. The lowest energy of one static source, $E_{\mathrm{q}} = \frac{1}{2}V(\infty)$, determined from the perimeter law of the Wilson loops, is constant for $\kappa < \kappa_{\mathrm{PT}}$ and shows no change at the Higgs PT.

The gauge invariant 2-point function $G(T, R)$, defined in eq. (1.1), decays for large $T$ and fixed $R$ as $\exp(-\mu T)$. The parameter $\mu$ is $R$-independent and has the physical meaning of the lowest energy contained in the dynamical matter and gauge fields screening a point-like external charge. We call $\mu$ the screening energy. In the Higgs region, where one has a Debye-Hückel-like screening, $\mu$ is equal to $E_{\mathrm{q}} = \frac{1}{2}V(\infty)$. In the Coulomb phase, however, we find $\mu$ to be substantially larger than $E_{\mathrm{q}}$. Their difference is interpreted as the mass $m_{\mathrm{c}}$ of the charged particle c (associated with the field $\Phi$), which can be present in the Coulomb phase, minus the binding energy $E_{\mathrm{b}}(\mathrm{cq})$ of the charged particle bound by the external source q. That is

$$\mu - E_{\mathrm{q}} = m_{\mathrm{c}} - E_{\mathrm{b}}(\mathrm{cq})\,. \tag{3.1}$$

We shall give an estimate for values of $m_{\mathrm{c}}$ at several $\kappa$ in the Coulomb phase.

The difference $\mu - E_{\mathrm{q}}$ vanishes in the Higgs region. On lattices of size up to $16^4$ it displays a remarkable discontinuity at the Higgs PT in spite of the fact that local observables like $\Phi^* U \Phi$ seem to behave smoothly. Therefore this quantity can be used as a sensitive parameter for the study of the Higgs PT, in particular of its order.

Furthermore, $\mu - E_q$ is expected to vanish not only in the Higgs region but also in the confinement region of the confinement-Higgs phase. Thus the difference $\mu - E_q$ can also be seen as an order parameter distinguishing between confinement (vanishing $\mu - E_q$) and free charge (positive $\mu - E_q$) phases of gauge theories with matter.

We have investigated in some detail this confinement criterion and also the one proposed by Fredenhagen and Marcu [11]. Both criteria can be formulated in terms of the asymptotic behaviour of ratios of the 2-point function $G(T, R)$ and of certain powers of expectation values of Wilson loops. In a self-explanatory pictorial notation we define

$$\rho_{AC}(T) = \frac{\langle \updownarrow T \rangle}{\langle \overset{T}{\Box}\, T \rangle^{1/4}}, \tag{3.2}$$

$$\rho_{FM}(R, T) = \frac{\langle \overset{R}{\sqcap}\, T \rangle}{\langle \overset{R}{\Box}\, 2T \rangle^{1/2}}. \tag{3.3}$$

Nonvanishing values

$$\rho_{AC}^{\infty} = \lim_{T \to \infty} \rho_{AC}(T), \qquad \rho_{FM}^{\infty} = \lim_{R \to \infty} \rho_{FM}(R, R/2) \tag{3.4}$$

signal confinement, whereas $\rho_{FM}^{\infty} = \rho_{AC}^{\infty} = 0$ signal a free charge phase. We have investigated both functions numerically and find full agreement of the data with these predictions in the Higgs region and in the Coulomb phase. We want to point out that our numerical results agree with the theoretical expectations even in the vicinity of the Higgs PT where no analytical proofs exist. In addition, it is remarkable that the approach of the functions $\rho_{FM}(R, T)$ and $\rho_{AC}(T)$ towards their asymptotic values $\rho_{FM}^{\infty}$ and $\rho_{AC}^{\infty}$, respectively, can already be seen clearly on a $16^4$ lattice. This is of considerable value for the practical use of those functions.

We have also investigated the ratio

$$\rho_{BF}(T) = \frac{\langle \updownarrow T \rangle^2}{\langle \updownarrow 2T \rangle} \tag{3.5}$$

suggested by Bricmont and Fröhlich, which signals absence of bound states of the charged particle with the external source, if $\rho_{BF}$ vanishes for large $T$ [10]. Our data indicate, unfortunately, rather slow convergence of $\rho_{\mathrm{BF}}(T)$ in the Coulomb phase. On the other hand, in the Higgs region $\rho_{\mathrm{BF}}(T)$ approaches a finite value $\rho_{\mathrm{BF}}^{\infty}$ equal to $\rho_{\mathrm{FM}}^{\infty}$.

Appendix B contains a derivation of the inequality

$$\langle \Box \rangle^{1/4} \geqslant c \langle \updownarrow \rangle, \qquad c > 0, \tag{3.6}$$

which is required for discussion of $\rho_{\mathrm{AC}}(T)$, by means of reflection positivity with respect to the planes $x^4 = \pm x^1$. The method employed might be useful for other purposes (see ref. [34]).

## 4. Static potential

The static potential $V(R)$ is one of the most fundamental observables in lattice gauge theories and it is important to understand the influence of matter fields on its shape. In the Higgs region we expect $V(R)$ to be Yukawa-like due to the Debye screening of charges by the matter fields,

$$V(R) = -\frac{\alpha}{R}\,\mathrm{e}^{-m_\gamma R} + V(\infty). \tag{4.1}$$

The Yukawa mass, $m_\gamma$, is expected to coincide with the nonvanishing photon mass. In the Coulomb phase, however, the photon is presumably massless and we expect a Coulomb potential.

We have determined $V(R)$ by means of Wilson loops $W(T, R)$ which have been calculated for $T = 1,\ldots,8$ ($1,\ldots,6$ for $12^4$ lattice) and $R$ up to one half of the spatial size of our lattices. Since there is some ambiguity in determining $V(R)$ from the exponential decay of $W(T, R)$ with $T$, we have analyzed the data for $T \geqslant 4$ and $T \geqslant 5$ independently. The fits have been made by means of the lattice version of the Yukawa potential

$$V_{\mathrm{L}}(R) = -\frac{4\pi\alpha}{N_{\mathrm{s}}^3} \sum_{\substack{l_1=1\\ l_2,l_3=0}}^{N_{\mathrm{s}}-1} \frac{\cos(2\pi l_1 R/N_{\mathrm{s}})}{2\Sigma_{i=1}^3\left(1-\cos(2\pi l_i/N_{\mathrm{s}})\right) + m_\gamma^2} + \text{const}. \tag{4.2}$$

(The point $l_i = (0,0,0)$ has been left out in order to avoid a divergence for $m_\gamma = 0$.) The values of $m_\gamma$ are consistent with 0 for $\kappa < \kappa_{\mathrm{PT}}$, i.e. in the Coulomb phase, but rise as $\kappa$ increases above $\kappa_{\mathrm{PT}}$. In the Coulomb phase we have therefore fitted $V(R)$ also by the lattice Coulomb potential ($m_\gamma \equiv 0$ in eq. (4.2)). The fits are very good. Thus the photon mass is consistent with zero in the Coulomb phase and rises in the Higgs region. This result is in agreement with the behaviour of the photon mass determined by means of plaquette correlation functions in the context of the same Monte Carlo calculations and published in ref. [22].

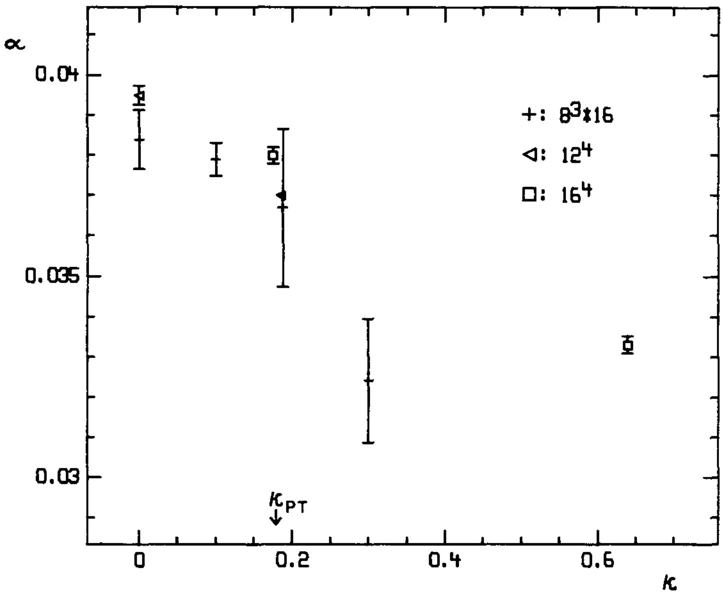



Fig. 2. Fine structure constant $\alpha$, determined from Wilson loops by means of lattice Yukawa and Coulomb potential fits, for various lattice sizes. In this and the following figures all error bars are displayed unless they are smaller than the size of the symbols.

The renormalized fine structure constant $\alpha$ has been determined in the Coulomb phase both by means of the lattice Yukawa and Coulomb potentials, both results being consistent. In the Higgs region we have only used the Yukawa potential. The values of $\alpha$ are shown in fig. 2. In the Coulomb phase $\alpha$ is independent of $\kappa$ and its value

$$\alpha = 0.0385(15) \tag{4.3}$$

is close to the value $\alpha = 0.036$ obtained in a weak coupling expansion [25] for $\beta = 2.5$. This expansion is known to agree very well with Monte Carlo data for the pure U(1) theory in the whole Coulomb phase [26]. Thus in the Coulomb phase of scalar QED the matter fields do not influence the renormalized charge in a noticeable way.

In the Higgs region, at $\kappa = 0.3$ and 0.64, our data indicate values of $\alpha$ which are slightly smaller than those for $\kappa \leqslant \kappa_{\mathrm{PT}}$. Thus the condensed $\Phi$-field may influence the static potential also at small distances.

Of particular interest for the later discussion of the 2-point function $G(T, R)$ is the value of the potential $V(R)$ at large distances $R$, when $V(R)$ gets constant. We introduce the quantity

$$E_{\mathrm{q}} = \tfrac{1}{2} V(\infty), \tag{4.4}$$

the physical meaning of which is the *lowest energy of fields around one external charge* q. (The energy is normalized to be zero in the ground state without external charges.) It is determined by the perimeter law of $I \times J$ Wilson loops,

$$W(I, J) \sim e^{-E_q 2(I+J)}, \qquad I, J \to \infty. \tag{4.5}$$

In the Coulomb phase $E_q$ is the energy in the Coulomb field produced by the external charge. Of course, on the lattice this self-energy is finite. The value of $E_q$ is $\kappa$-independent in the Coulomb phase as long as $\alpha$ is $\kappa$-independent.

In the confinement-Higgs phase, $E_q$ is the sum of the contributions from the interacting gauge fields and charged matter field screening the external source. We expect that in the confinement region, for small $\kappa$, this screening is realized by one "constituent" particle associated with the $\Phi$-field, whereas in the Higgs region the screening is due to the $\Phi$-condensate in analogy to the Debye-Hückel screening in a plasma [27].

The effect of the Higgs PT on the value of $E_q$ is physically similar to bringing a charge from the vacuum into a dilute plasma. The energy contained in a Coulomb field or in a Yukawa field of an external charge are nearly equal if the Yukawa mass is small. Since we know already that the photon mass in our data remains consistent with 0 even slightly above the Higgs PT [22], we expect $E_q$ not to change noticeably at this transition.

Our results for $E_q$ are indicated in fig. 3 by circles. It is apparent that $E_q$ is indeed insensitive to the Higgs PT.

## 5. Exponential decay of the gauge invariant 2-point function

The gauge invariant 2-point function $G(T, R)$ defined in eq. (1.1) can be seen as a gauge invariant generalization of spin-spin correlation functions which are used for characterizing long range order in statistical mechanical systems with global symmetries. However, it has been realized already a number of years ago that $G(T, R)$, as a function of the distance $T = |x - y|$, is bounded from above by functions decreasing exponentially with $T$ for large $T$ [28]. Thus, $G(T, R)$ cannot show any long range order, in agreement with the expectation that the fluctuations of the gauge degrees of freedom destroy any long range correlations [28]. The fast decay of $G(T, R)$ therefore is a reflection of the gauge invariance of the model and is not a consequence of an unsuitable choice of the correlation function.

Thus an investigation of $G(T, R)$ should provide us with genuine insight into the dynamics of gauge fields interacting with matter. Of particular interest is the precise form of the functional dependence of $G(T, R)$ on $T$ and $R$, and the physical interpretation of the energy parameter $\mu$ which determines the large distance behaviour of $G(T, R)$. The rest of the paper is devoted to various aspects of this general question.

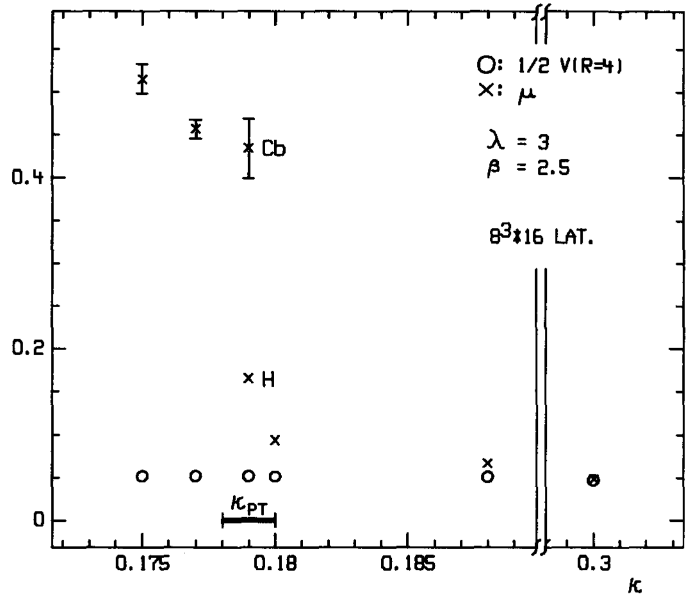



Fig. 3. Comparison of the potential $E_q = \frac{1}{2}V(\infty)$ at large distance, approximated by $\frac{1}{2}V(4)$ (circles), with the screening energy $\mu$ (crosses). $\mu$ has been determined from the exponential decay of the 2-point function $G(T, R=1)$. Data were obtained on an $8^3 \times 16$ lattice. At $\kappa = 0.179$ the system showed two long-living metastable states, one in the Coulomb phase (Cb), and one in the Higgs phase (H).

First we want to describe some of our Monte Carlo results, which will be interpreted physically in later sections. As the exponential decay of a correlation function in QFT is determined by the energy spectrum, we identify the distance $T$ between the points $x$, $y$ with the euclidean time. Accordingly we consider the decay of $G(T, R)$ as a function of "time" $T$ for fixed $R$. The distance $R$ of the "detour" in space direction characterizes the dependence of $G$ on different choices of the path $\Gamma$. For all $\kappa$ in both the Coulomb phase and the Higgs region for which we made calculations, $G(T, R)$ decreases exponentially as a function of $T$ for $T$ large,

$$G(T, R) \sim f_G(R)\,e^{-\mu T}; \qquad 4 \leqslant T \leqslant 8, \qquad R \text{ fixed}, \tag{5.1}$$

with $\mu$ being independent of $R$ to a high degree of accuracy. Analogous results have been obtained previously for the SU(2) Higgs model [14]. If both $T$ and $R$ become large, the 2-point function $G(T, R)$ behaves differently in the free charge phase and in the confinement-Higgs phase, respectively. According to the results by Fredenhagen and Marcu [11] one expects

$$G(T, R) \sim e^{-m_c T - E_q(T+2R)} \qquad \text{(Coulomb phase)}, \tag{5.2}$$

$$G(T, R) \sim e^{-\mu(T+2R)} \qquad \text{(confinement-Higgs phase)}. \tag{5.3}$$

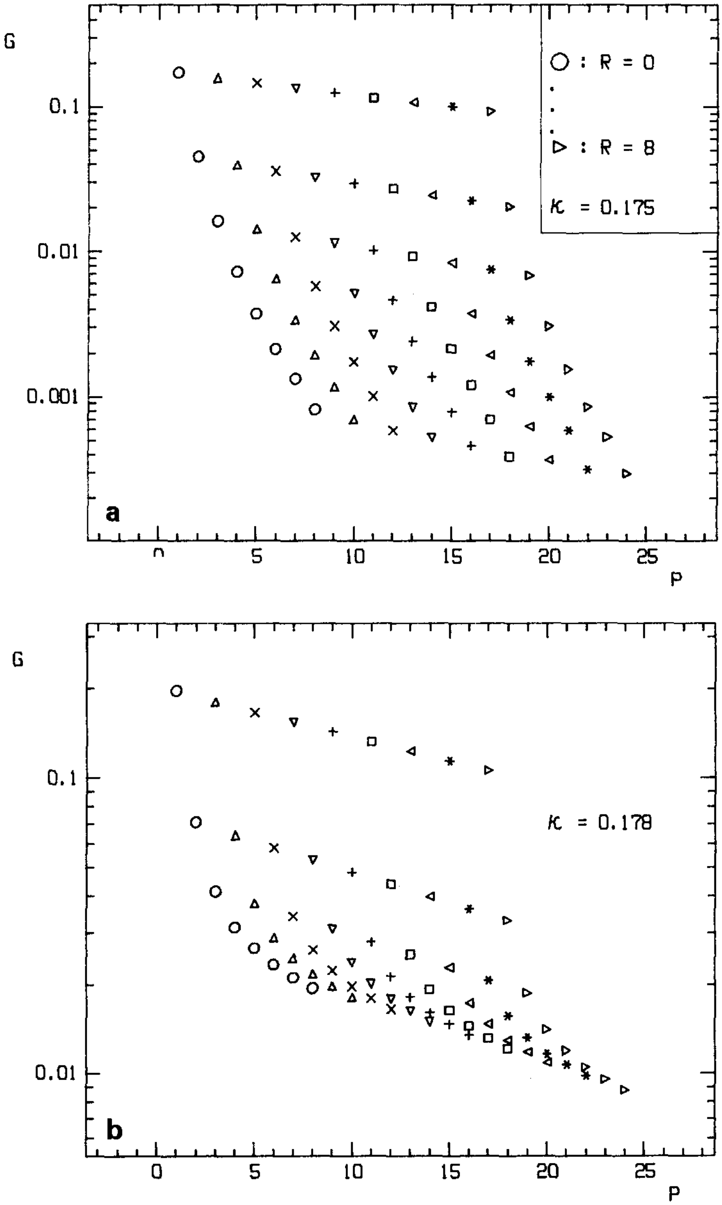



Fig. 4. The 2-point function $G(T, R)$ on a $16^4$ lattice as a function of the "perimeter" $P = T + 2R$ for fixed $R$ between 0 and 8. Different symbols represent different $R$. For each $R$, the distance $T$ ranges between 1 (top of figures) and 8 (bottom). Data are at (a) $\kappa = 0.175$ inside the Coulomb phase, (b) $\kappa = 0.178$ just above the PT inside the Higgs region, and (c) $\kappa = 0.64$ deep inside the Higgs region.

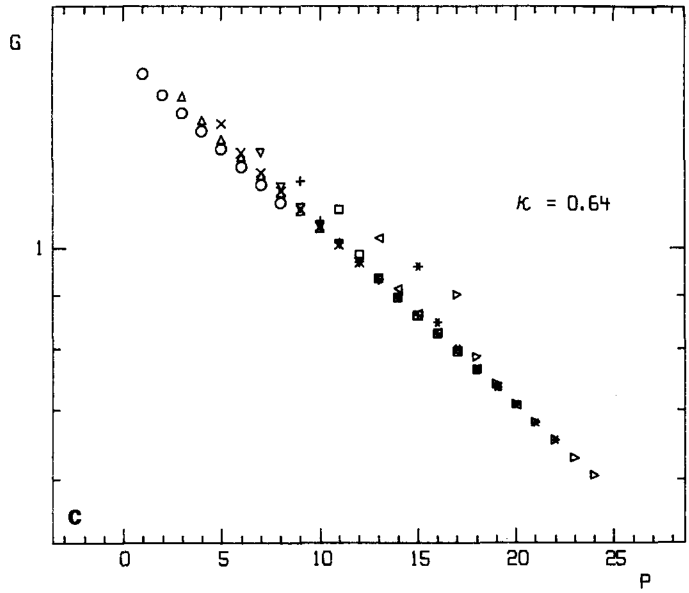



Fig. 4 (continued).

In appendix A we calculate $G(T, R)$ in the hopping parameter expansion. It provides heuristic support to the above dependence of $G(T, R)$ on $T$ and $R$ in both phases.

It is advantageous to plot $\log G(T, R)$ as a function of the "perimeter" $P = T + 2R$, the length of the path $\Gamma$ in eq. (1.1), for various fixed $R$. Figs. 4a–c show such plots of our data obtained on a $16^4$ lattice in the range $T = 1, \dots, 8$ and $R = 0, \dots, 8$ for $\kappa = 0.175$ in the Coulomb phase (fig. 4a), for $\kappa = 0.178$ just inside the Higgs region (fig. 4b) and for $\kappa = 0.64$ deep inside the Higgs region (fig. 4c). In all figures different symbols represent different values of $R$. At $\kappa = 0.64$ in the Higgs region one can observe the perimeter behaviour (5.3), which means that all points fall onto a single line. At $\kappa = 0.178$, near the PT, but still in the Higgs phase, the perimeter behaviour is found only for large $T$ and $R$. In the Coulomb phase at $\kappa = 0.175$ one can see that $G(T, R)$ does not depend on $P$ alone, in accordance with the relation (5.2).

The slope of $\log G(T, R)$ for fixed $R$ and large $T$ ($T \geqslant 4$) gives $\mu$. The equality of the slopes for different $R$ reflects the $R$-independence of $\mu$. This independence holds for all $\kappa$-points we have investigated. Whereas all our data show the asymptotic exponential behaviour (5.1) for $T \geqslant 4$ independent of $R$, there are significant deviations from this asymptotic form for $T < 4$. The $R$-dependence of $G(T, R)$ at large fixed $T$ gives the function $f_G(R)$ in the relation (5.1). The slope $C$ of $\log f_G(R)$ at fixed $T \geqslant 4$ has an approximate value of $C \simeq 0.1$, which is nearly the same for all

$\kappa$. This fact will be discussed further in sect. 8. We would like to point out that the calculation of $G(T, R)$ with small statistical errors requires only moderate statistics. It is about 10 times easier to calculate $\mu$ than to determine the Higgs boson mass with comparable statistical errors.

In sect. 11 we shall continue our discussion of $G(T, R)$ as a function of $T$ and $R$ when both variables grow simultaneously.

## 6. Screening energy μ

The $R$-independence of $\mu$ suggests that $\mu$ is a useful observable with the dimension of energy. Its physical meaning can be determined by using the temporal gauge [2, 12]. Then the parallel transporter in the time direction on the path $\Gamma$ reduces to 1 and $G(T, R)$ reduces to a usual correlation function in time direction between two field products of the form

$$D(\boldsymbol{x}, \boldsymbol{x}+\boldsymbol{R}) = \prod_{\ell} U_{\ell} \Phi_{\boldsymbol{x}, t}, \qquad \ell \in \text{line}\,((t, \boldsymbol{x}) \to (t, \boldsymbol{x}+\boldsymbol{R})) \tag{6.1}$$

for $t=0$ and $t=T$. These products are not gauge invariant and the standard interpretation of the states contributing to the correlation function is that an external source q is present at the space point $\boldsymbol{x}+\boldsymbol{R}$ forming a charge neutral system together with states created by $D$ [2, 12]. The behaviour of $G(T, R)$ for large $T$ projects out the lowest energy of this system. Therefore we conclude that $\mu$ *is the lowest energy in the fields screening a point-like external charge* q [11]. We call $\mu$ the screening energy.

In order to persuade the reader not familiar with the interpretation of gauge noninvariant states in the temporal gauge in terms of external sources, we give an alternative, fully gauge invariant argument, which leads to an identical physical interpretation of $\mu$. Suppose that we introduce into the theory an additional, heavy field $\Psi_x$ carrying the same charge as $\Phi_x$, which is therefore coupled minimally to the gauge field, too. We can form the gauge invariant correlation function

$$G_{\Psi}(T, R) = \left\langle \Psi^{\dagger}_{\boldsymbol{x}+\boldsymbol{R},0} D(\boldsymbol{x}, \boldsymbol{x}+\boldsymbol{R}, 0) \cdot \Psi_{\boldsymbol{x}+\boldsymbol{R},T} D^{\dagger}(\boldsymbol{x}, \boldsymbol{x}+\boldsymbol{R}, T) \right\rangle . \tag{6.2}$$

This is a correlation function between two gauge invariant products of fields. Its exponential decay for large $T$ is determined by the lowest total energy of the heavy particle $\Psi$ screened by the field $\Phi$. In the static limit for the field $\Psi$ we have approximately

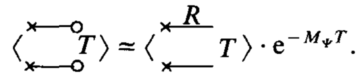
$$\Psi_{z,T} \simeq \mathrm{e}^{-M_{\Psi} T} \prod_{\ell} U_{\ell} \Psi_{z,0}, \qquad \ell \in \text{line}\,((z, 0) \to (z, T)), \tag{6.3}$$

where $M_{\Psi}$ is the bare mass of the particle $\Psi$.

Substituting this expression into (6.2) we obtain

$$G_\Psi(T, R) \simeq \mathrm{e}^{-M_\Psi T} \cdot G(T, R), \tag{6.4a}$$

or, pictorially, (circles represent the heavy particle $\Psi$)



(6.4b)

Combining this with eq. (1.2) we get

$$G_\Psi(T, R) \simeq f_G(R)\, \mathrm{e}^{-(\mu + M_\Psi)T}. \tag{6.5}$$

Thus $\mu$ is the lowest energy in the fields screening the heavy charged particle $\Psi$, in accordance with the above interpretation using the temporal gauge. Notice that $M_\Psi$ does not include the self-energy of the particle $\Psi$ caused by its interaction with the gauge field. This self-energy is contained in $\mu$.

The $R$-independence of $\mu$ now becomes quite obvious: One and the same lowest energy state contributes in (6.2) for all $R$. Its contribution is only weighted by the $R$-dependent function $f_G(R)$. If the parallel transporter along the line in (6.1) would not be present, at least in the Coulomb phase this function would measure the density of the dynamical charged particles screening the external source. However, the presence of the parallel transporter obscures such an interpretation, which is even more difficult in the confinement-Higgs phase.

## 7. Screening in the Higgs region

In the confinement-Higgs phase, where the external charges are always screened, the physical meaning of the screening energy $\mu$ coincides with the lowest energy $E_\mathrm{q}$ of fields around external charges, eq. (4.4). Thus we expect

$$\mu = E_\mathrm{q} \qquad \text{in the confinement-Higgs phase}. \tag{7.1}$$

As $\mu$ and $E_\mathrm{q}$ are determined from conceptually different quantities, namely $G(T, R)$ and Wilson loops, respectively, the validity of the relation (7.1) provides a test of the physical interpretation of both quantities.

Unfortunately, it is very difficult to calculate $E_\mathrm{q} = \frac{1}{2}V(\infty)$ in the confinement region on lattices of the restricted size we can use. The static potential $V(R)$ is expected to rise linearly with the distance $R$ until complete screening through the quanta of the field $\Phi$ is possible. From our experience with the SU(2) Higgs model [14] we expect this screening distance to exceed the size of our lattice ($16^4$) for $\kappa < \kappa_{\mathrm{PT}}$ nearly everywhere in this region and thus only the relation $\frac{1}{2}V(R) \leqslant \mu$ might be tested.

In the Higgs region $\mu$ rapidly approaches $E_q$ with growing $\kappa$, as can be seen in fig. 3, and the relation (7.1) is satisfied quite well for $\kappa \geqslant 0.188$. As suggested by fig. 4b, the small difference between $\mu$ and $E_q$ for $\kappa_{PT} \leqslant \kappa \leqslant 0.188$ is probably caused by a slow approach of $G(T, R)$ to its asymptotic behaviour in $T$ in this $\kappa$-interval, and our values for $\mu$ obtained at $T = 4$–$8$ actually overestimate $\mu$.

It is interesting that the values of $\mu$ are very small in the Higgs region, e.g. $\mu \approx 0.05$ for $\kappa = 0.3$. Since this value is smaller than the inverse size of the lattice in the time direction ($16^{-1}$), one has to ask whether the finite size effects do not distort $\mu$. However, the inverse value of $\mu$ itself does not have the physical meaning of a correlation length. As seen from eq. (6.5), $\mu$ merely contributes to the correlation length $(\mu + M_{\Psi})^{-1}$, which for static charges actually vanishes. Thus, small $\mu$ does not mean long correlation. There is also no obvious relation between $G(T, R)$ and fluctuations of a quantity in the sense of a dissipation–fluctuation theorem.

## 8. Charged particles in the Coulomb phase

In the Coulomb phase we expect the existence of gauge invariant finite energy states associated with the complex field $\Phi$, which are charged in the sense of Gauss' law. That state with the lowest energy will be called the charged particle or briefly c-particle with dressed mass $m_c$. It is important to keep in mind that the c-particle states are not created by the local field $\Phi_x$ acting on the vacuum. Such a state would not be gauge invariant. Charged states cannot be created by local operators [24, 29, 30] and their construction represents a major challenge in QFT.

A convenient way to study the properties of a charged particle in the gauge invariant formalism is to consider it in the presence of an external source q. The whole system is charge neutral (like a hydrogen atom) and can thus be described by a localized state. The corresponding correlation function is just $G(T, R)$ (see, e.g. eqs. (6.2)–(6.5)). Thus the screening energy $\mu$ is the lowest energy of the system c-particle plus external source q [11], and one has

$$\mu = E_q + m_c - E_b(cq) \qquad \text{in the Coulomb phase}. \tag{8.1}$$

Here $E_b(cq) \geqslant 0$ is the largest binding energy of the c − q system and a function of $m_c$. It is the analogue of the binding energy of a hydrogen atom in the continuum, where we have

$$E_b(cq) = \tfrac{1}{2} m_c \alpha^2 . \tag{8.2}$$

On the lattice the energy $E_b(cq)$ can in principle also be determined as a function of $m_c$ for a given potential $V(R)$ by solving the corresponding Schrödinger equation. Thus, when $\mu$, $V(R)$ and $E_q$ are determined in Monte Carlo simulations, $m_c$ can be calculated from eq. (8.1).

Our data clearly show that $\mu - E_q > 0$ in the Coulomb phase, see figs. 3 and 7. Thus for $\lambda = 3$ and $\beta = 2.5$ the mass $m_c$ is nonzero immediately below the Higgs PT. A simple estimate using the continuum Bohr formula (8.2) and the value (4.3) for $\alpha$ shows that the binding energy is negligible in eq. (8.1) for $\beta = 2.5$. Thus the mass $m_c$ of the charged particle is practically equal to $\mu - E_q$. For $\kappa = 0.179$, 0.177 and 0.175 we find $m_c \simeq \mu - E_q = 0.35$, 0.40 and 0.46, respectively, with errors $\pm 0.05$, on an $8^3 \times 16$ lattice. On a $16^4$ lattice at $\kappa = 0.175$ we get $m_c \simeq 0.54$. As $\kappa \to 0$ the bare mass $\tilde{m}$ of the $\Phi$-field, eq. (2.3), will tend to infinity. According to eq. (8.1) and appendix A we expect the mass parameters $\mu$ and $m_c$ to become infinite, too.

If, as discussed at the end of sect. 6, $f_G(R)$ is at least a crude measure for the density of the charged particle, we can estimate the size of the "hydrogen atom". As seen in fig. 4a, $f_G(R)$ decreases very slowly with $R$, namely as $\exp(-CR)$, $C \simeq 0.1$. Such a low value of $C$ means a very broad cq state, as expected for bound states due to Coulomb forces. The Bohr radius is of the order of $20a$.

The mass $m_c$ of the charged particle can also be determined, as suggested by Fredenhagen and Marcu [11], from the decay of $G(T, R)$ divided by the square root of the Wilson loop. This ratio is taken as a function of $T$ with $R$ so large that the cq state becomes unbound. For this limit they predict for the Coulomb phase (up to a power law correction)

$$\rho_{\mathrm{FM}}(T, R) = \frac{G(T, R)}{\sqrt{W(T, 2R)}} \sim \frac{\mathrm{e}^{-m_c T - E_q(2R+T)}}{\mathrm{e}^{-E_q(2R+T)}} = \mathrm{e}^{-m_c T}. \tag{8.3}$$

Here $W(I, J)$ is the expectation value of the $I \times J$ Wilson loop with the perimeter behaviour (4.5).

In (8.3) no binding energy has to be considered, but the limit is more difficult, requiring large lattices in both space and time directions. We have attempted to determine $m_c$ from our data on a $16^4$ lattice according to eq. (8.3). The result for $\rho_{\mathrm{FM}}(T, R = 8)$ at $\kappa = 0.175$ is shown in fig. 5. The value for $m_c = 0.50(5)$ obtained in this way is consistent with the value $\mu - E_q = 0.54$ given above. Of course, due to the smallness of the binding energy an agreement was to be expected.

## 9. Discontinuity of $\mu$ at the Higgs phase transition

As seen in fig. 3, the screening energy $\mu$ has a distinct discontinuity at the Higgs PT for $\lambda = 3$ and $\beta = 2.5$. We have originally chosen this $(\lambda, \beta)$ point for a detailed investigation because the observable $\Phi^* U \Phi$ did not show any discontinuity here [6], contrary to its behaviour for lower $\lambda$, and we therefore expected that the Higgs PT might already be of higher order at this value of $\lambda$. In long MC runs at $\kappa \approx \kappa_{\mathrm{PT}}$ we have noticed two patterns of fluctuations of $\Phi^* U \Phi$, differing only in the range of

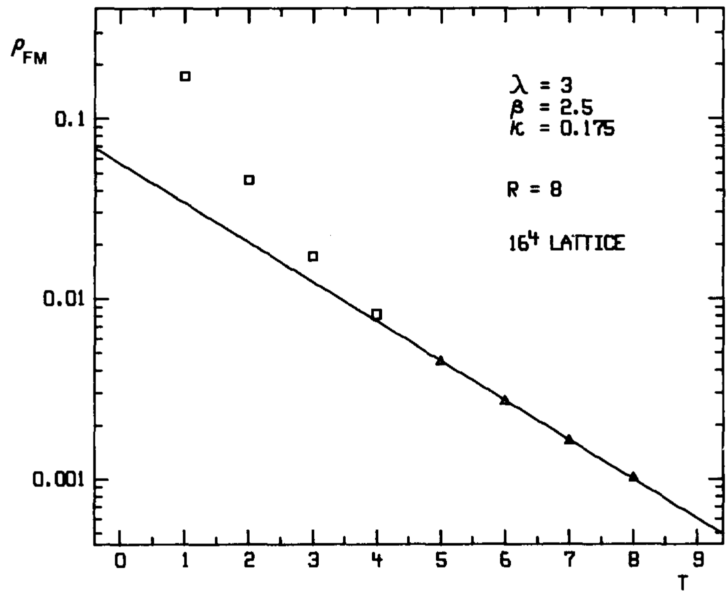



Fig. 5. Fredenhagen-Marcu parameter $\rho_{FM}(T, R=8)$, eq. (8.3), on a $16^4$ lattice at $\kappa=0.175$. The solid line represents a fit to the data points denoted by triangles using eq. (8.3). The result is $m_c = 0.50(5)$, the error representing an estimate of systematic effects.

fluctuations, the mean values being almost the same. Each pattern could persist for a very long time, sometimes for more than 10000 sweeps on an $8^3 \times 16$ lattice. Fig. 6 shows a typical example of this behaviour.

The astonishingly long lifetimes of these fluctuation patterns suggested that they are due to the presence of two long living metastable states. Since these states practically do not differ in the mean values of $\Phi^* U \Phi$, or of any other local observable we have analyzed, we have looked for a nonlocal observable which would distinguish between both states and explain their very low flip rate. The calculation of $\mu$ separately in each of the states revealed the two values of $\mu$ at $\kappa = 0.179$ shown in fig. 3. The state with larger fluctuations in fig. 6 has a lower value of $\mu$ and belongs to the Higgs region, whereas the state with smaller fluctuations has large $\mu$ and is in the Coulomb phase.

Thus we find that the screening energy $\mu$ is a very sensitive parameter, suitable to distinguish between the Coulomb phase and the Higgs region of the confinement-Higgs phase. Its sizable discontinuity indicates that for $\lambda = 3$, $\beta = 2.5$ the Higgs PT might be of first order, in spite of the continuous behaviour of local observables. The discontinuity has been found on lattices $8^3 \times 16$, $12^4$ and $16^4$, but our data, shown in fig. 7, are not sufficient for determining the dependence of this discontinuity on the lattice size. We cannot exclude that the data obtained in one phase are contaminated by short flips into the other phase. As described in ref. [22] an

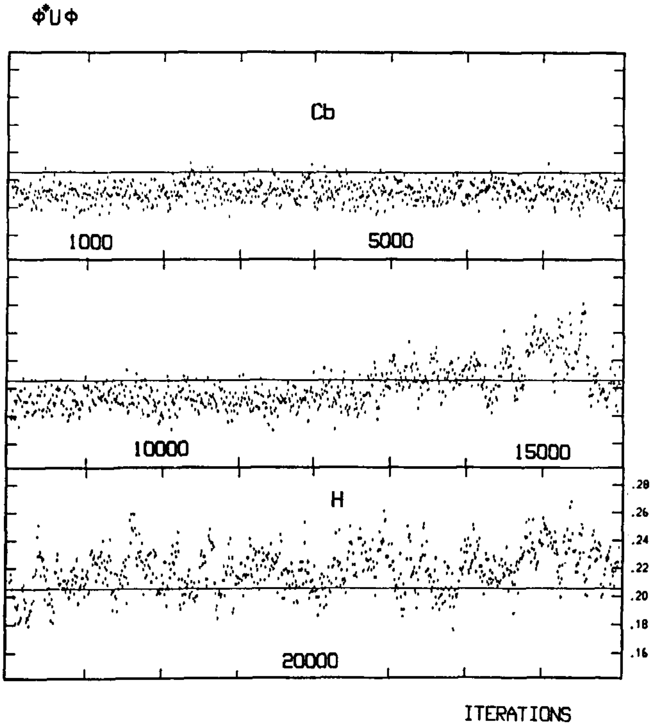



Fig. 6. Monte Carlo evolution of the link operator $\Phi^* U \Phi$ over 24000 iterations, on an $8^3 \times 16$ lattice at $\kappa = 0.179$, exhibiting two long-living metastable states. The state with smaller fluctuations has large $\mu$ (cf. fig. 3) and belongs to the Coulomb phase (Cb), whereas the state with larger fluctuations has lower $\mu$ and belongs to the Higgs region (H).

increase of the lattice size shifts the Higgs PT to lower $\kappa$ and narrows the peak of the specific heat. It might be that the discontinuity of $\mu$ decreases with growing lattice size. Thus a definite conclusion on the order of the PT would be premature.

Since, as discussed in sect. 4, the energy $E_q$ practically does not change at the Higgs PT, the discontinuity of $\mu$ at the PT is caused by a sudden decrease of the quantity $m_c - E_b(\mathrm{cq})$ in eq. (8.1). Neglecting $E_b$, as suggested by its small value in eq. (8.2), and taking into account that $V(R)$ does not change noticably at the PT, we conclude that it is actually the mass $m_c$ of the charged particle which vanishes discontinuously or at least drops substantially at the Higgs PT.

## 10. Screening energy criterion for confinement

It is well known that the Wilson loop criterion for confinement in pure gauge theories fails to distinguish between phases with confinement and with free charges when matter fields are introduced which can screen the confining potential. It is a

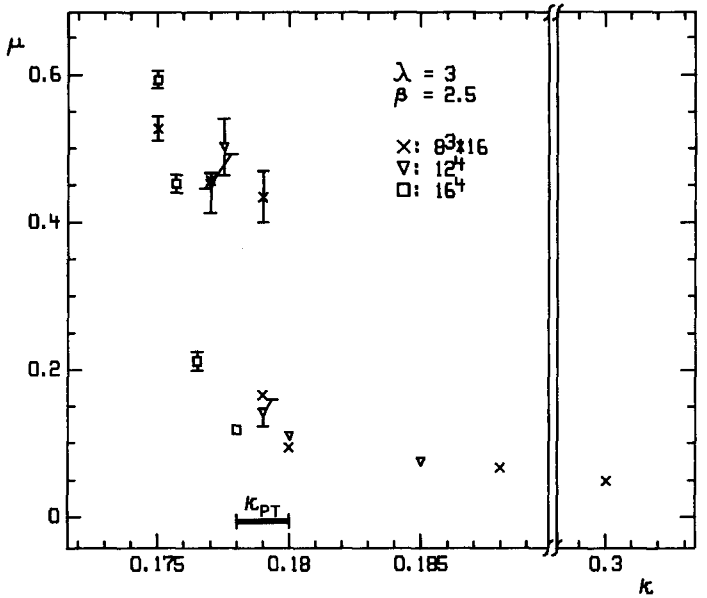



Fig. 7. Comparison of the values of $\mu$ on lattices of sizes $16^4$ (squares), $12^4$ (triangles) and $8^3 \times 16$ (crosses). At $\kappa = 0.179$ on the $8^3 \times 16$ lattice two values for $\mu$ are given, corresponding to two metastable states.

challenge [10, 11, 31] to construct a field theoretical observable which could be used as a criterion for the presence or absence of confinement in that case.

We want to propose that the energy difference $\mu - E_q$ is suitable for this purpose. The physical interpretations of the quantities $\mu$ and $E_q$ described in the preceeding sections makes it clear that $\mu - E_q$ vanishes in the confinement-Higgs phase. As any charge must be screened, the lowest energy state with an external charge is identical with the screened external charge. On the other hand, in a free charge phase an external charge can exist in both screened (i.e. bound) and unscreened states with, in general, different energies $\mu$ and $E_q$. Provided the difference $\mu - E_q$ is nonnegative, the criterion for confinement or free charges can be expressed as the question whether the lowest energy of a *screened* external charge is equal to or greater than the lowest energy of an external charge.

In the Coulomb phase of the U(1) Higgs model, the quantity $\mu - E_q = m_c - E_b(cq)$ is positive when the continuum formula (8.2) is used for an estimate of $E_b(cq)$,

$$\mu - E_q \approx m_c\left(1 - \tfrac{1}{2}\alpha^2\right). \tag{10.1}$$

Namely it has been shown in analytic [25, 32] and numerical [26] calculations that the renormalized charge in the pure U(1) lattice gauge theory with Wilson and Villain actions stays finite and is actually quite small ($\alpha < 0.2$ [26]) in the whole

Coulomb phase. As we have seen in sect. 4, $\alpha$ is numerically independent of $\kappa$ so that the expression on the r.h.s. of eq. (10.1) is expected to be positive and very close to $m_c$ in the whole Coulomb phase of the U(1) Higgs model.

For U(1) Higgs models with other actions or for other gauge theories with matter fields having a free charge phase, the positivity of $\mu - E_q$ is less obvious. One can image $E_b(cq)$ being larger than $m_c$. Thus it is necessary to formalize the criterion in order to make it accessible to rigorous treatment in QFT. For this purpose we introduce the ratio

$$\rho_{AC}(T) = \frac{G(T,0)}{W(T,T)^{1/4}}. \tag{10.2}$$

Using the perimeter behaviour (4.5) of $W(I, J)$ for large $I$, $J$, we obtain from (5.1)

$$\rho_{AC}(T) \sim e^{-(\mu - E_q)T}. \tag{10.3}$$

Thus we expect

$$\rho_{AC}^{\infty} = \lim_{T \to \infty} \rho_{AC}(T) \begin{cases} \neq 0 & \text{(confinement)} \\ = 0 & \text{(free charge)} \end{cases} \tag{10.4}$$

and $\rho_{AC}^{\infty}$ is an order parameter vanishing in the free charge phase. The ratio $\rho_{AC}(T)$ is different from – but inspired by – the order parameter introduced by Fredenhagen and Marcu [11], which provides another criterion for confinement, and which will be discussed in the next section.

It follows from reflection positivity (see appendix B) that

$$W(T,T)^{1/4} \geqslant cG(T,0), \tag{10.5}$$

where $c$ is positive, bounded and independent of $T$. (Pictorial form of (10.5) is given in (3.6).) Thus the parameter $\mu - E_q$ is indeed nonnegative.

In fig. 8 we show our data for $\rho_{AC}(T)$ in the U(1) Higgs model for $\kappa = 0.175$ and 0.1757 (Coulomb phase) and $\kappa = 0.178$ (Higgs region) on a $16^4$ lattice. The complete agreement with the prediction (10.4) is not surprising when we insert the values of $\mu$ and $E_q$ from fig. 3 into eq. (10.3). It is remarkable that $\rho_{AC}(T)$ approaches its asymptotic values already for small $T$, both in the Higgs region and in the Coulomb phase. In order to determine the asymptotic values $\rho_{AC}^{\infty}$, we have fitted the data by

$$\rho_{AC}(T) = A + B\,e^{-CT}. \tag{10.6}$$

The fits are shown in fig. 8, too. The resulting values $\rho_{AC}^{\infty} = A$, displayed in fig. 9, are in full agreement with eq. (10.4).

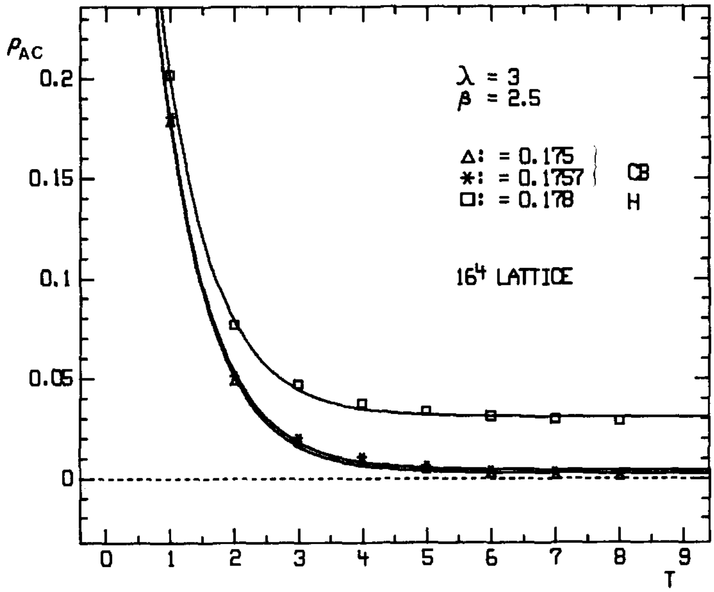



Fig. 8. The ratio $\rho_{AC}(T)$, eq. (10.2), on a $16^4$ lattice at three values of $\kappa$: $\kappa = 0.175$ and 0.1757 in the Coulomb phase, and $\kappa = 0.178$ in the Higgs region. The solid lines represent fits to the points at $T \geqslant 1$ using eq. (10.6). Note the rapid convergence of the data to the asymptotic values.

## 11. Fredenhagen-Marcu criterion for confinement

Fredenhagen and Marcu proposed a different criterion distinguishing between confinement and free charge phases. Its validity has been shown rigorously by series expansions in various regions of the phase diagrams for Z(2) and U(1) lattice Higgs models [11, 16] and confirmed by other analytic methods [17]. Furthermore, Monte Carlo investigations of the Z(2) model [13], the U(1) model with charge two matter fields [15] and the SU(2) model [14] support the analytic predictions. Here we extend the numerical verification of the criterion to the Higgs region of the confinement-Higgs phase and to the Coulomb phase of the U(1) Higgs model with matter fields of charge one, especially in the neighbourhood of the Higgs PT.

The criterion is formulated by means of the ratio

$$\rho_{FM}(R,T) = \frac{G(R,T)}{W(R,2T)^{1/2}}. \tag{11.1}$$

Note that the distance between the endpoints of the path $\Gamma$ is now denoted by $R$, since the physical motivation for the criterion requires this distance to be space-like. In the limit in which both $R$ and $T$ grow proportionally to each other (e.g. $T = \frac{1}{2}R$)

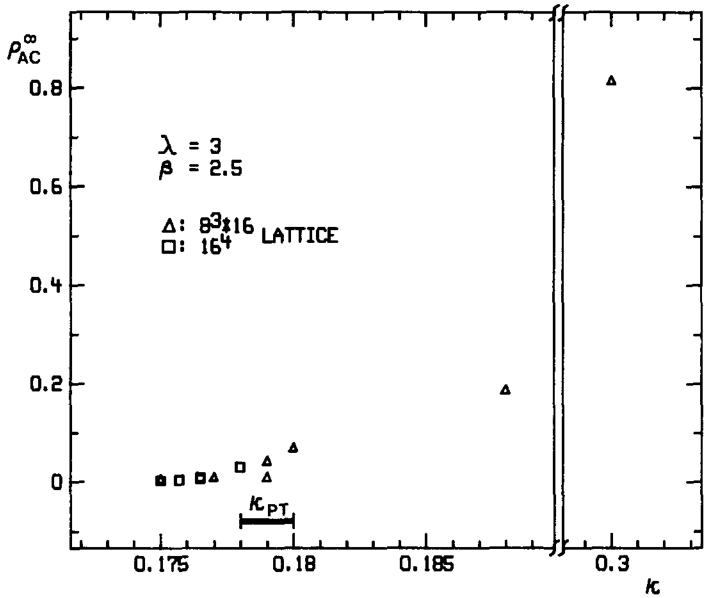



Fig. 9. Asymptotic values $\rho_{AC}^{\infty}$ of $\rho_{AC}(T)$, determined from fits according to eq. (10.6). These values coincide with those of $\rho_{FM}^{\infty}$, described in sec. 11. The distance of the two triangles at $\kappa = 0.179$ gives an indication of the size of systematical errors at the PT.

it is expected that

$$\rho_{FM}(R,T) \xrightarrow[T=\frac{1}{2}R]{R\to\infty} \begin{cases} \rho_{FM}^{\infty} \neq 0 & \text{(confinement)} \\ 0 & \text{(free charge)}. \end{cases} \tag{11.2}$$

Thus $\rho_{FM}^{\infty}$ is an order parameter (called "vacuum overlap order parameter" by Fredenhagen and Marcu) distinguishing between confinement and free charge phases.

The motivation for this criterion [11] is based on the physical picture of a pair of opposite charges separated by a spatial distance $R$. The state is made gauge invariant by means of a parallel transporter connecting both charges. As one of the charges is removed to $R \to \infty$, the vacuum overlap of the remaining state is signaling whether this state is charged (no overlap) or not. The simultaneous shift of the parallel transporter in the time-like distance $T$ projects out the low energy states. Division by the square root of the expectation value of the Wilson loop removes the perimeter law decay factor $\exp(-E_q(R+2T))$ in $G(R,T)$ in the limit performed in eq. (11.2). In the free charge phase the numerator in (11.1) decays faster than the denominator by the factor $\exp(-m_c R)$, eq. (8.3) [11]. Thus $\rho_{FM}$ also vanishes in the free charge phase. The additional energy is analogous to the positive difference $\mu - E_q = m_c - E_b(\mathrm{cq})$ which we have used for the free charge criterion (10.4). Therefore the basic ingredients of both criteria, namely properties of the energy spectrum, are closely related. Also the equality $\rho_{AC}^{\infty} = \rho_{FM}^{\infty}$ is expected from results contained in ref. [11].

For all values of $\kappa$ at $\lambda = 3$, $\beta = 2.5$ our results for $\rho_{FM}(R, R/2)$ turn out to be very close to the values of $\rho_{AC}(T)$ when compared for $T = R$. Except for fig. 5 we do not plot these data independently, since they can be read off from fig. 8. For the above value of $\beta$ at $\kappa \simeq 0.3$ and for $\beta = 1.1$ we found larger, but not substantial, differences between $\rho_{AC}(T)$ and $\rho_{FM}(T, T/2)$ for small $T = 1,2$. In any case the equality $\rho_{AC}^{\infty} = \rho_{FM}^{\infty}$ is satisfied to a very good accuracy, so that also the values $\rho_{FM}^{\infty}$ can be read off from fig. 9.

## 12. Criterion for bound states of the charged particle

In general, the ground state of a charged particle in a free charge phase in presence of an external charge of opposite sign may be either a bound state or a state in which the particle of mass $m_c$ moves freely after sufficiently long time. The difference is reflected by the asymptotic behaviour of $G(T,0)$ which in the free case is not given by (5.1) but by

$$G(T,0) \sim e^{-\mu T} \frac{1}{T^{(d-1)/2}}, \qquad \mu = m_c + E_q. \tag{12.1}$$

This power law (Ornstein-Zernike) correction has been discussed by Bricmont and Fröhlich as a signal for the absence of bound states [10]. Its physical meaning has also been pointed out in ref. [11].

The purely exponential decay (5.1) is expected to hold in the presence of a bound state and in the whole confinement phase (rigorous proofs exist for the Z(2) model [11]). Unfortunately, it is very difficult to distinguish between the asymptotic behaviour of the type (12.1) or (5.1) in Monte Carlo calculations.

In a slight generalization of the proposal by Bricmont and Fröhlich [10] we define the ratio

$$\rho_{BF}(T_1 + T_2) = \frac{G(T_1,0)G(T_2,0)}{G(T_1 + T_2,0)}, \tag{12.2}$$

where $T_2 = T_1$ or $T_1 + 1$. For $T \to \infty$ this ratio should behave as

$$\rho_{BF}(T) \xrightarrow[T \to \infty]{} \begin{cases} \rho_{BF}^{\infty} \neq 0 & \text{(bound state, confinement)} \\ 0 & \text{(no bound state)}. \end{cases} \tag{12.3}$$

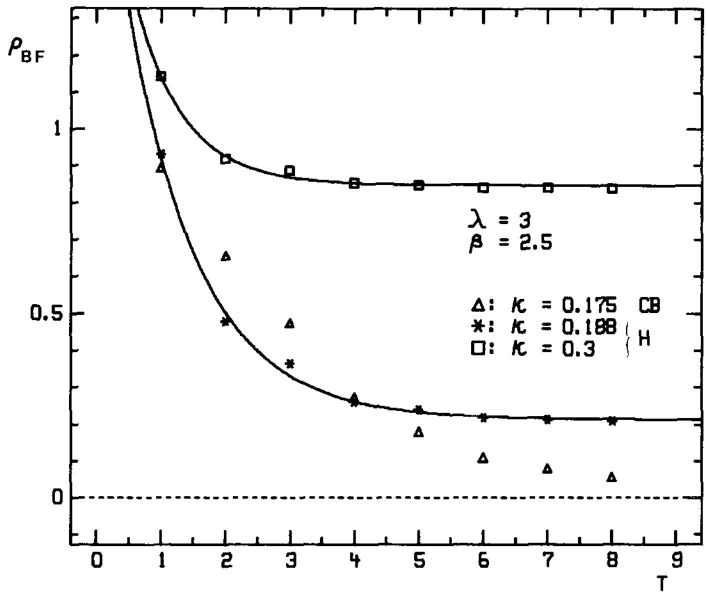



Fig. 10. The Bricmont-Fröhlich parameter $\rho_{BF}(T)$ at $\kappa = 0.175$ in the Coulomb phase and $\kappa = 0.188$ and 0.3 in the Higgs region. The solid lines represent fits to the points at $T \geqslant 1$ again using eq. (10.6). At $\kappa = 0.175$ no such fit is possible.

In addition, in the confinement-Higgs phase the relation

$$\rho_{BF}^{\infty} = \rho_{FM}^{\infty} \tag{12.4}$$

should hold [11].

In the U(1) Higgs model we expect the presence of cq bound states in the Coulomb phase, and thus $\rho_{BF}^{\infty} \neq 0$ everywhere. Our results confirm the expectation $\rho_{BF}^{\infty} \neq 0$ in the Higgs region. The equality (12.4) is satisfied to a high accuracy there. In the Coulomb phase, however, the ratio $\rho_{BF}(T)$ does not yet show its asymptotic behaviour for $\kappa = 0.175$ even at the distance $T = 8$ (fig. 10). But it gets very small at this distance. The cause is a small value of $f_G(R)$ for all $R$, due to the broad probability distribution of the c-particle bound to an external charge in the Coulomb phase. Large values of $\rho_{BF}(T)$ for small $T$ are due to very high values of $G(T, R)$ for small $T$ (fig. 4a) and are thus a nonasymptotic effect.

We conclude that the ratio $\rho_{BF}(T)$ is strongly influenced by nonasymptotic effects in the Coulomb phase at least up to the distance $T \approx 8$ and it is therefore of small practical value in the context of presently available lattice sizes.

## 13. Conclusions

Our results, summarized in sect. 3, show that the screening energy $\mu$ which determines the exponential decay of the gauge invariant 2-point function $G(T, R)$

for large $T$ and (or) $R$ is very sensitive to the phase structure of the U(1) Higgs model. It is therefore a very useful quantity for constructing order parameters which distinguish between the confinement-Higgs phase and the free charge (Coulomb) phase, respectively.

Motivated by heuristic physical arguments the construction of appropriate order parameters essentially compares the exponential decay of 2-point functions with that of Wilson loops. Thus, whereas Wilson loops alone are no longer useful as an order parameter in the presence of matter fields in the fundamental representation, because they have perimeter decay in all phases, their combination with gauge invariant 2-point functions provides order parameters with the desired properties.

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## Appendix A

For the convenience of the reader we give in this appendix a simple argument, based on the hopping parameter expansion, which elucidates the different behaviour of $G(T, R)$ for both $T$ and $R$ large in the Coulomb and confinement-Higgs phases, eqs. (5.2)–(5.3). A more rigorous (and more involved) derivation can be found in ref. [11]. For simplicity we consider the case of the frozen radial mode $|\Phi_x| = 1$, i.e. $\lambda = \infty$. The expansion of $G(T, R)$, eq. (1.1), in powers of $\kappa$ consists of terms of two types. Either the hopping terms complete the path $\Gamma$ to a closed loop or all the fields in the expectation value (1.1) on the path are multiplied by their complex conjugates.

In the Coulomb phase of the pure U(1) gauge theory the Wilson loop decays according to the perimeter law (4.5). The leading term in the hopping parameter expansion for large $T$ and $R$ and $R \sim T$ will therefore be the term $\kappa^T$ which completes the path in eq. (1.1) to the closed Wilson loop $T \times R$. This term gives

$$G(T, R) = \mathrm{e}^{-\tilde{m}T} W(T, R), \qquad \tilde{m} = \ln\frac{1}{\kappa}, \tag{A.1}$$

where $W(T, R)$ is determined by the pure gauge theory alone. Using (4.5) we obtain for larger $T$ and $R$

$$G(T, R) \sim \exp\big(-(\tilde{m} + E_{\mathrm{q}})T - E_{\mathrm{q}}(T + 2R)\big). \tag{A.2}$$

The quantity $\tilde{m} + E_q$ is the charged particle mass $m_c$ split into the bare mass $\tilde{m}$ and the energy of the Coulomb field. Thus we have obtained the relation (5.2).

In the confinement phase of the pure U(1) lattice gauge theory the Wilson loops obey the area law and the leading term of the $\kappa$ expansion for larger $T$ and $R$ which turns the parallel transporter on the path $\Gamma$ into unity is of the order $\kappa^{T+2R}$. Thus we obtain the perimeter behaviour (5.3) of $G(T, R)$ with

$$\mu = \ln \frac{1}{\kappa}\,. \tag{A.3}$$

Of course, in this way we have not shown that the relation (5.3) also holds in the Higgs region where $\kappa$ is not small [11].

## Appendix B

In this appendix we want to provide the arguments for the inequality (10.5). Our main tool is reflection positivity with respect to the 3-dimensional hyperplanes $\Pi_y^{\pm}$: $x^4 - y^4 = \pm(x^1 - y^1)$ through the point $y$.

Usually reflection positivity – which ensures the existence of a Hilbert space – is proved for lattice gauge theories [2] with respect to the hyperplane $x^4 = \frac{1}{2}$ which cuts the links connecting the lattice sites in the planes $x^4 = 0$ and 1. The planes $\Pi_y^{\pm}$ do not cut any link but pass through lattice sites instead. Reflection positivity of the U(1) Higgs model with respect to these planes follows immediately as a generalization of the corresponding investigations of systems in statistical mechanics [33].

Each of the planes $\Pi_y^{\varepsilon}$, $\varepsilon = \pm 1$, divides the hypercubic lattice $\bar{\Lambda}$, which for the present purpose consists of sites *and* links, into three disjoint sets $\bar{\Lambda}_0$, $\bar{\Lambda}_+$ and $\bar{\Lambda}_-$, where $\bar{\Lambda}_0$ consists of the sites and links which lie in the plane, $\bar{\Lambda}_+$ contains the sites and links "above" and $\bar{\Lambda}_-$ those "below" the plane. The reflections $r_\varepsilon$ with respect to the plane $\Pi_y^{\varepsilon}$ map $\bar{\Lambda}_+$ onto $\bar{\Lambda}_-$ and vice versa and leave the elements of $\bar{\Lambda}_0$ invariant. The division $\bar{\Lambda} = \bar{\Lambda}_0 \cup \bar{\Lambda}_+ \cup \bar{\Lambda}_-$ of the lattice decomposes the algebra of fields A into 3 corresponding subalgebras $\mathrm{A}_0$, $\mathrm{A}_+$ and $\mathrm{A}_-$ with supports in $\bar{\Lambda}_0$, $\bar{\Lambda}_+$ and $\bar{\Lambda}_-$, respectively. The reflections $r_\varepsilon$ of the lattice are associated with the following mappings $\theta_\varepsilon$ of the algebras $\mathrm{A}_0, \mathrm{A}_+, \mathrm{A}_-$ generated by the field variables $\Phi_x$ and $U_{x,\mu}$:

$$\theta_\varepsilon \Phi_x = \Phi^*_{r_\varepsilon(x)}\,, \qquad \theta_\varepsilon U_{x,\mu} = U^*_{r_\varepsilon(x,\mu)}\,. \tag{B.1}$$

$\theta_\varepsilon$ maps $\mathrm{A}_+$ onto $\mathrm{A}_-$ and vice versa and $\mathrm{A}_0$ onto itself. The action $S$ in eq. (2.1) is invariant under the reflections $\theta_\varepsilon$. This action can be split into the following parts:

$S_0$: all terms containing variables with support in $\bar{\Lambda}_0$ only,

$S_+$: terms with support in $\bar{\Lambda}_+ \cup \bar{\Lambda}_0$, except for those contained in $S_0$,

$S_-$: terms with support in $\bar{\Lambda}_- \cup \bar{\Lambda}_0$, except for those contained in $S_0$,

$S_c$: sum over plaquettes that are cut by $\Pi_y^\varepsilon$,

$$S = S_0 + S_+ + S_- + S_c . \tag{B.2}$$

The part $S_c$ has the form $-\Sigma G_+ \theta_\varepsilon G_+$ with $G_+$ being link products on the $\bar{\Lambda}_+$ side of the cut plaquettes. Therefore the series expansion of $\exp(-S_c)$ is also of the form

$$e^{-S_c} = \sum_j H_+^j \theta_\varepsilon H_+^j . \tag{B.3}$$

Let us now show the positivity of the expectation value $\langle F\theta_\varepsilon F\rangle$, where $F \in A_0 \cup A_+$. The integration measure is

$$\prod_{\bar{\Lambda}} d\Phi\, dU = \prod_{x,(x,\nu)\in\bar{\Lambda}} d\rho_x\, \rho_x\, d\mu(\sigma_x)\, d\mu(U_{x,\nu}), \tag{B.4}$$

where $\Phi_x = \rho_x \cdot \sigma_x$, $\rho_x \in R^+$, $\sigma_x \in \mathrm{U}(1)$. The idea is to separate the integration over $\bar{\Lambda}_0$ and use the fact that the integrations over $\bar{\Lambda}_+$ and $\bar{\Lambda}_-$ factorize for each $j$ in (B.3):

$$\begin{aligned}\langle F\theta_\varepsilon F\rangle &= \int \prod_{\bar{\Lambda}} d\Phi\, dU e^{-S} F\theta_\varepsilon F \\ &= \int \prod_{\bar{\Lambda}_0} d\Phi\, dU e^{-S_0} \prod_{\bar{\Lambda}_+} d\Phi\, dU e^{-S_+} \prod_{\bar{\Lambda}_-} d\Phi\, dU e^{-S_-} \\ &\quad \times F\theta_\varepsilon F \cdot \sum_j H_+^j \theta_\varepsilon H_+^j \\ &= \int \prod_{\bar{\Lambda}_0} d\Phi\, dU e^{-S_0} \sum_j \left| \int \prod_{\bar{\Lambda}_+} d\Phi\, dU e^{-S_+} H_+^j F \right|^2 . \end{aligned} \tag{B.5}$$

Since the integrand in the integral over $\bar{\Lambda}_0$ is obviously positive, we get the reflection positivity

$$\langle F\theta_\varepsilon F\rangle \geqslant 0 . \tag{B.6}$$

For functions $F_1, F_2 \in \mathrm{A}_0 \cup \mathrm{A}_+$ this implies the Schwartz inequality

$$|\langle F_1\theta_\varepsilon F_2\rangle| \leqslant \langle F_1\theta_\varepsilon F_1\rangle^{1/2} \langle F_2\theta_\varepsilon F_2\rangle^{1/2} . \tag{B.7}$$

We now apply this last result to the 2-point function

$$G(T,0) = \left\langle \rho_z \sigma_z^* \prod_{\ell\in\Gamma} U_\ell \rho_y \sigma_y \right\rangle, \qquad T = z^4 - y^4, \qquad z = y .$$

The planes $\Pi^{\pm}$ here pass through the points $y$ and $z$, respectively. Applying the inequality (B.7) three times, we get

$$
\begin{aligned}
G(T,0) &\leqslant \langle \rho_z^2 \rangle^{1/2} \left\langle \rho_y \sigma_y \prod_\Gamma U \theta_- \left( \rho_y \sigma_y \prod_\Gamma U \right) \right\rangle^{1/2} \\
&= \langle \rho_z^2 \rangle^{1/2} \left\langle \rho_y \sigma_y \theta_- (\rho_y \sigma_y) \prod_\Gamma U \theta_- \left( \prod_\Gamma U \right) \right\rangle^{1/2} \\
&\leqslant \langle \rho_z^2 \rangle^{1/2} \left\langle \left( \rho_y \theta_- \rho_y \right)^2 \right\rangle^{1/4} \left\langle \prod_\Gamma U \prod_{r_-(\Gamma)} \theta_- U \theta_+ \left( \prod_\Gamma U \prod_{r_-(\Gamma)} \theta_- U \right) \right\rangle^{1/4} \\
&= \langle \rho_z^2 \rangle^{1/2} \langle \rho_y^2 \theta_- \rho_y^2 \rangle^{1/4} W(T,T)^{1/4} \\
&\leqslant \langle \rho_z^2 \rangle^{1/2} \langle \rho_y^4 \rangle^{1/4} W(T,T)^{1/4}. 
\end{aligned} \tag{B.8}
$$

Pictorially the first two reflections mean:


The last step in (B.8) again uses the reflection $r_+$. Since $\langle \rho_z^2 \rangle$ and $\langle \rho_y^4 \rangle$ are independent of $z$ and $y$, we finally obtain the required result

$$
G(T,0) \leqslant c W(T,T)^{1/4}, \qquad c = \langle \rho^2 \rangle^{1/2} \cdot \langle \rho^4 \rangle^{1/4} > 0. \tag{B.9}
$$

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