Georg Winkler, BSc

Interaction effects on quantum wires with strong spin-orbit coupling

MASTER THESIS

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Graz, May 2014

Motivation

LETTERS
PUBLISHED ONLINE: 21 MARCH 2010 | DOI: 10.1038/NPHYS1626
nature physics

Observation of a one-dimensional spin-orbit gap in a quantum wire

C. H. L. Quay[1,2*†], T. L. Hughes[1†], J. A. Sulpizio[1], L. N. Pfeiffer[2†], K. W. Baldwin[2†], K. W. West[2†], D. Goldhaber-Gordon[1] and R. de Picciotto[2†]

Current interests:

- 1D systems
- Spintronics (spin filter) [P. Středa et al. 2003]
- Observation in GaAs/AlGaAs quantum wires
- Majorana Fermions [J. Alicea 2013]
- Luttinger liquid physics
- Phase diagrams

Hamiltonian and Dispersion relations

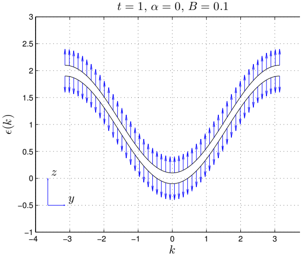
Extended Hubbard model with spin-orbit coupling and external B-field:

$$H_0 = \sum_j [-\frac{t}{2}(c_j^\dagger c_{j+1} + h.c.) - (\mu - t)c_j^\dagger c_j - \underbrace{\frac{\alpha}{2}(ic_j^\dagger \sigma^y c_{j+1} + h.c.)}_{\text{Rashba spin-orbit coupling}} + \underbrace{Bc_j^\dagger \sigma^z c_j}_{\text{ext. } B\text{-field}}]$$

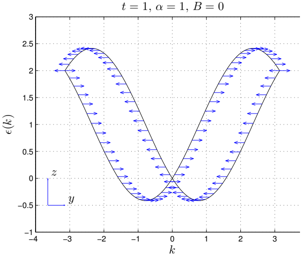
$$H_1 = \sum_j [Un_{j\uparrow}n_{j\downarrow} + U' n_j n_{j+1}]$$

(implizit summation over spin)

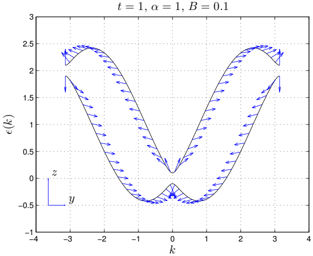
Non-interacting Hamiltonian H_0 can be straight-forward diagonalized:



$B \neq 0$, $\alpha = 0$: two branches for spin-$\uparrow$ and spin-$\downarrow$

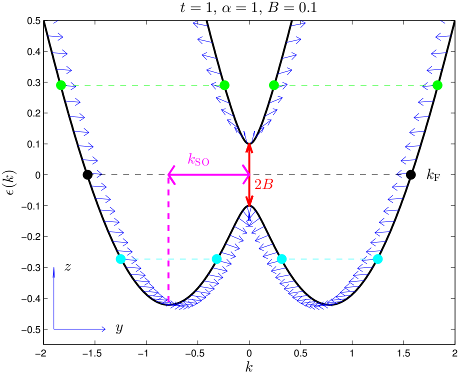


$B = 0$, $\alpha \neq 0$: two branches for spin-$\rightarrow$ and spin-$\leftarrow$



$B \neq 0$, $\alpha \neq 0$: B-field opens a spin-orbit gap

Dispersion of the noninteracting Hamiltonian H_0



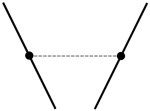
- The branches with spin parallel and antiparallel to the spin orbit coupling are shifted by $\pm k_{SO}$ with

$$k_{SO} = \arctan\left(\frac{\alpha}{t}\right).$$

- A spin-orbit gap of size $2B$ opens up.
- It is only a partial gap (pseudogap) in one half of the conduction modes.
- The gap appears in a superposition of spin and charge degrees of freedom.
- The system has 2- and 4-Fermi point phases, depending on the Fermi level.
- Reduced conductance in the 2-Fermi point phase [C. H. L. Quay et al. 2010]
- Spin dependent transport properties in the 2-Fermi point phase (Left movers and right movers with orthogonal spin)
  $\rightarrow$ 2-Fermi point phase is a spin filter

Luttinger liquid description

The interacting Hamiltonian can be described by the Tomonaga-Luttinger model, which is an effective low energy model.



Spinless Fermions:

$$H_{\mathrm{LL}} = \frac{v}{2\pi} \int \mathrm{d}x [\frac{1}{K}(\nabla\phi)^2 + K(\nabla\theta)^2]$$

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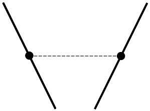

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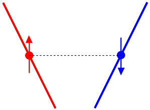

Helical Luttinger liquid:

$$H_{\text{HLL}} = \frac{v}{2\pi}\int dx[\frac{1}{K_{\text{HLL}}}(\nabla\Phi)^2 + K_{\text{HLL}}(\nabla\Theta)^2]$$

$$\Phi = \frac{\phi_{R\uparrow} + \phi_{L\downarrow}}{\sqrt{2}}, \quad \Theta = \frac{\theta_{R\uparrow} - \theta_{L\downarrow}}{\sqrt{2}}$$

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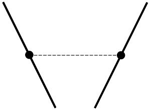

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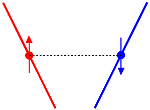

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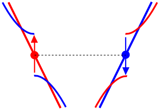


Spiral Luttinger liquid:

$$\begin{aligned} H_{\text{SLL}} &= H_+ + H_- \\ H_+ &= \frac{v_+}{2\pi}\int \mathrm{d}x[(\nabla\phi_+)^2 + (\nabla\theta_+)^2] + \frac{B_{\text{eff}}}{a}\int \mathrm{d}x\cos(\sqrt{2K}\phi_+) \\ H_- &= \frac{v_-}{2\pi}\int \mathrm{d}x[(\nabla\phi_-)^2 + (\nabla\theta_-)^2] \end{aligned}$$

Calculation of the interacting spectral densities

Calculated by the Fourier transformation of the real-time evolution:

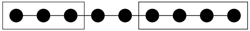
$$S_{aa^\dagger}(k,\omega) = \frac{1}{2\pi}\sum_{j=-\infty}^{\infty} \exp(-ikj)\int_{-\infty}^{\infty} \mathrm{d}t\, \exp(i\omega t)\; S_{cc^\dagger}(j,t)$$

$$S_{cc^\dagger}(j,t) = \langle 0|\, c_j(t) c_0^\dagger(0)\, |0\rangle = \langle 0|\, \exp(iHt) c_j \exp(-iHt) c_0^\dagger\, |0\rangle$$

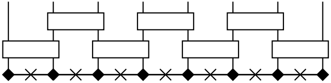
MPS formalism: $|\psi\rangle = \sum_{\sigma_1\ldots\sigma_L} A^{\sigma_1}A^{\sigma_2}\cdots A^{\sigma_{L-1}}A^{\sigma_L}\, |\sigma_1,\ldots,\sigma_L\rangle$

Procedure:

1. Obtain ground state $|0\rangle$ by DMRG



2. Apply operator $c_0^\dagger$ on ground state: $|\phi\rangle = c_0^\dagger\, |0\rangle$
3. Real-time evolution by TEBD:
   $|\phi(t)\rangle = \exp(-iHt)\, |\phi\rangle$

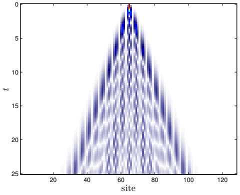


4. Calculate overlap with ground state $S_{cc^\dagger}(j,t) = \exp(iE_0 t)\, \langle 0|\, c_j\, |\phi(t)\rangle$

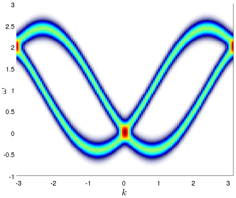
Linear prediction

Problem: finite time range, window function throws away data

Time evolution: (U=0)



FT + window:



Linear prediction

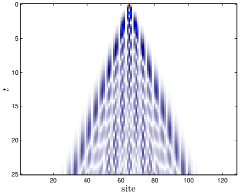
Problem: finite time range, window function throws away data

Solution: extrapolate time series using linear prediction:

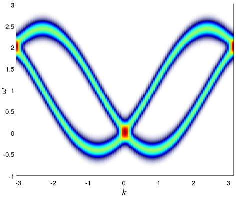
$y_i = \sum_{j=1}^{n} d_j y_{i-j}$ [White et. al 2008]

("linear" is a misnomer, it is a fit more like machine learning)

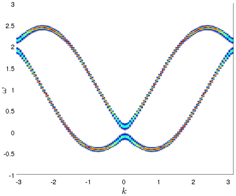
Time evolution:



FT + window:



FT + linear prediction:



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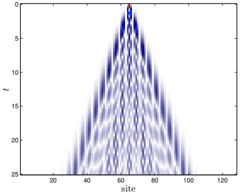

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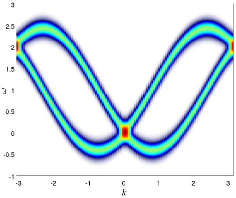
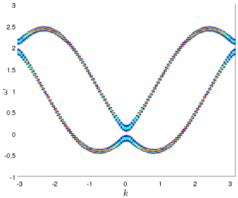
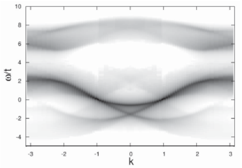


FT + linear prediction:

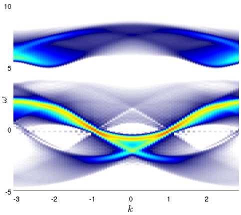


Test on a simple Hubbard model with $U = 4.9t$ and $n = 0.6$:

DDMRG [Jeckelmann 2008]:



Our method:



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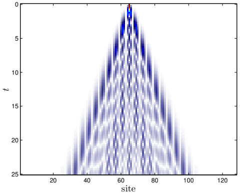
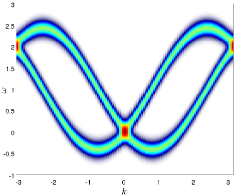


FT + window:



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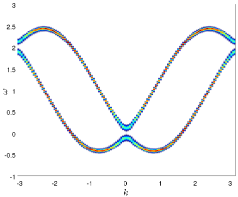

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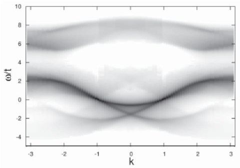

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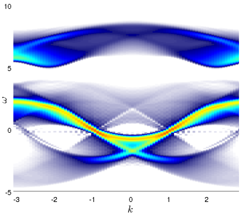
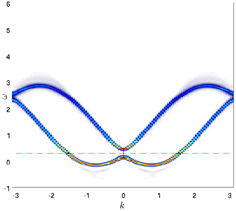


Advantages:

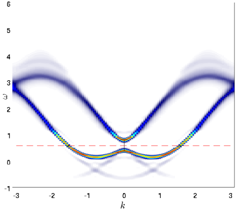
- Much less computational cost than DDMRG at comparable quality of the result.
- Simple implementation using the well known TEBD algorithm.

$U' = U/2$: $\alpha = 1$, $B = 0.1$, $n = 0.5$

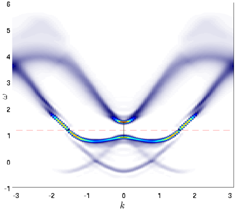
$U = 0.5$, $U' = 0.25$



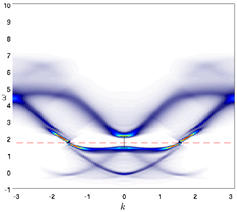
$U = 1$, $U' = 0.5$



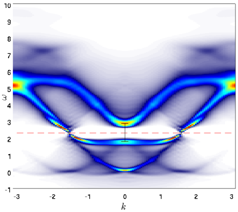
$U = 2$, $U' = 1$



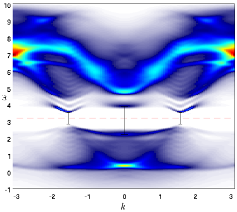
$U = 3$, $U' = 1.5$



$U = 4$, $U' = 2$



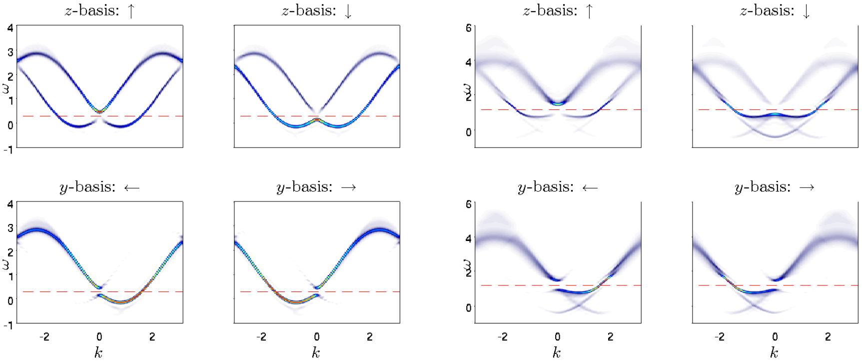
$U = 6$, $U' = 3$



Interaction effects:
spin-orbit gap enhanced, spinon branch, charge gap

Spin dependent spectral densities: $\alpha = 1, \;\; B = 0.2, \;\; n = 0.5$

$U = 0.5, \;\; U' = 0.25$: $\qquad\qquad$ $U = 2, \;\; U' = 1$:



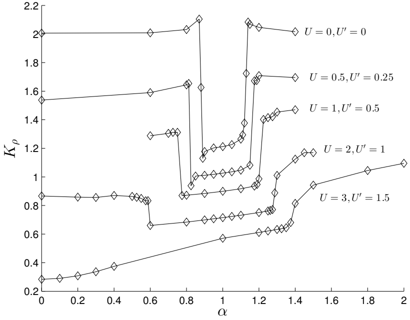
Helical spin order at Fermi points!

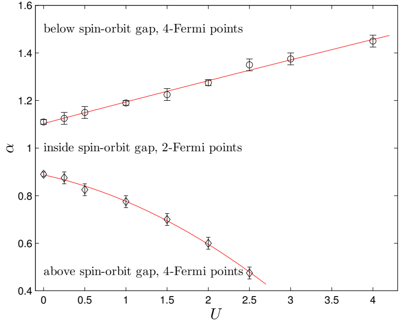
Phase boundaries between the 2- and 4-Fermi point phases

Luttinger liquid coefficient extracted from the density correlations of spinless fermions:

$$C^{\mathrm{NN}}(r) \sim -\frac{K_\rho}{2(\pi r)^2} + A\frac{\cos(2k_F r)}{r^{1+K_\rho}} \ln^{-\frac{3}{2}}(r) + \ldots$$

In the 4-Fermi point phase the system doesn't behave like spinless Fermions ⇒ discontinous jump

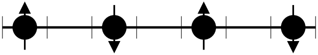




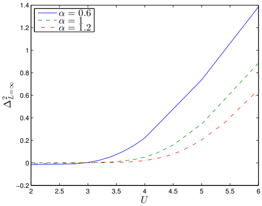
Spin-orbit gap gets greatly enhanced with interactions!

Mott phase: $U' = U/2, \; B = 0.1, \; n = 0.5$

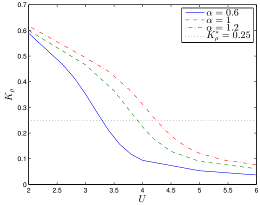
Mott insulating phase at quarter filling for finite-range interactions U, U'.



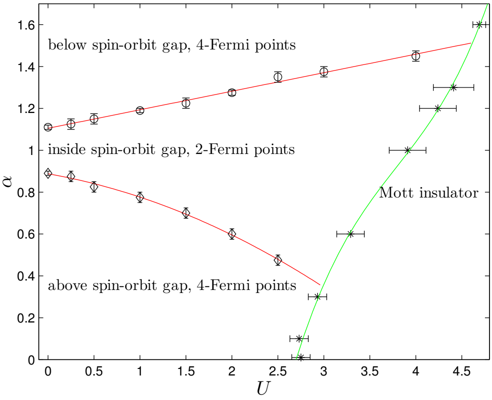
Identified by a charge gap,



and the critical value $K_\rho^* = 0.25$.



Complete Phase diagram:



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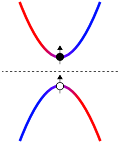
Breather bound states

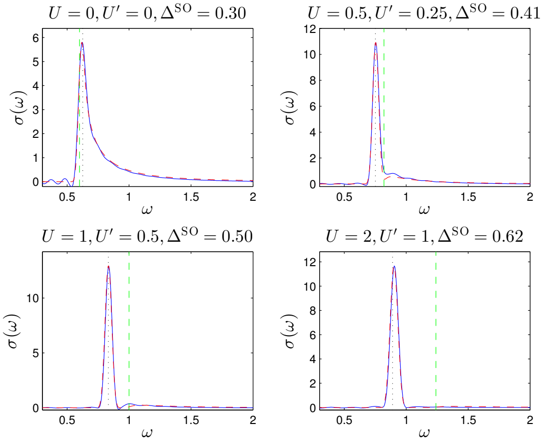
Bound states of an electron and a hole in the gapped modes. Similar to an Exciton.



Optical conductivity:

$$\sigma(\omega > 0) = \frac{\mathrm{Im}(\chi(\omega > 0))}{\omega},$$

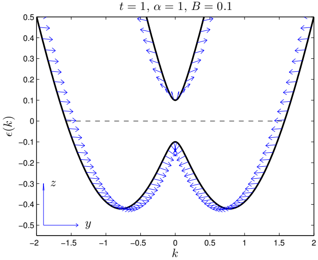
$$\chi(t) = \frac{1}{L}\sum_i \langle j_i(t) j_0 \rangle$$



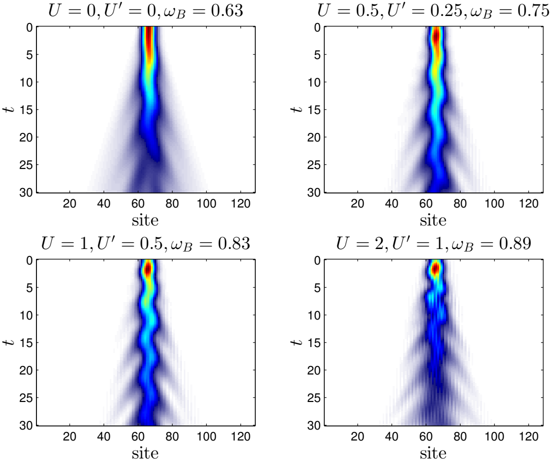
Direct excitation of Breathers

Suitable Excitation with $g^\dagger$ a gaussian particle creation operator acting on $k = 0$:

$$(g_\uparrow^\dagger + g_\downarrow^\dagger)(g_\uparrow + g_\downarrow)\,|0\rangle$$



Time evolution of the magnetization $\hat{S}^z$ after excitation:



Conclusions

Conclusions:

- Spin-orbit gap gets greatly enlarged by interactions.
  - Wider range of μ-values inside spin-orbit gap
- Helical spin order is preserved as long as the system is inside the Luttinger liquid phase.
- Strong interactions destroy the Luttinger liquid phase.
- Different phases identified with the use of the Luttinger liquid coefficients.
- Breather bound states, predicted by the spiral Luttinger liquid theory, were verified.

Thank you for your attention!

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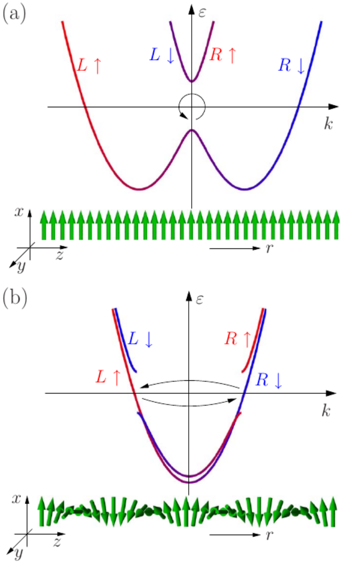

[Braunecker et. al 2010]