SCIENCE
PASSION
TECHNOLOGY

Bell‘s Theorem

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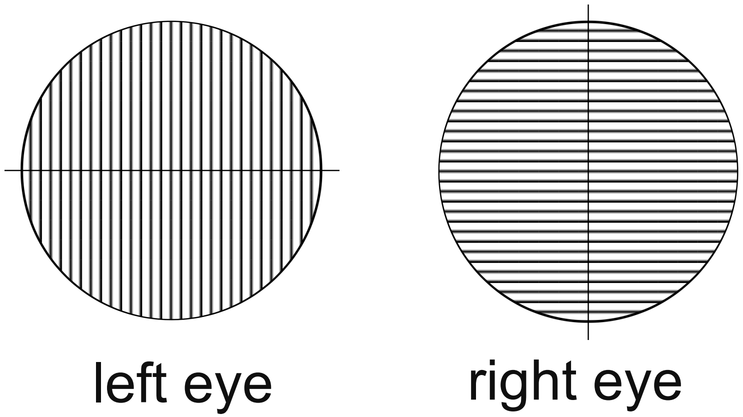
20.07.20

Contents

Entangled States

Product state $|\psi\rangle = |polarization\rangle \otimes |location\rangle$

Entangled state $|\psi\rangle = \frac{1}{\sqrt{2}}(|left, vertical\rangle + |right, horizontal\rangle$



Spin entanglement for two particles *A* and *B*

$$|\psi^-\rangle = \frac{1}{\sqrt{2}}(|\downarrow_A\uparrow_B\rangle_z - |\uparrow_A\downarrow_B\rangle_z$$

Entangled States

Entanglement in n degrees of freedom:

Definition 1.1 (PURE ENTANGLED STATE). *A pure state in two degrees of freedom (A and B) is entangled in a basis* $|\alpha_n\rangle_A$, $|\beta_n\rangle_B$, *if its basis representation is given by*

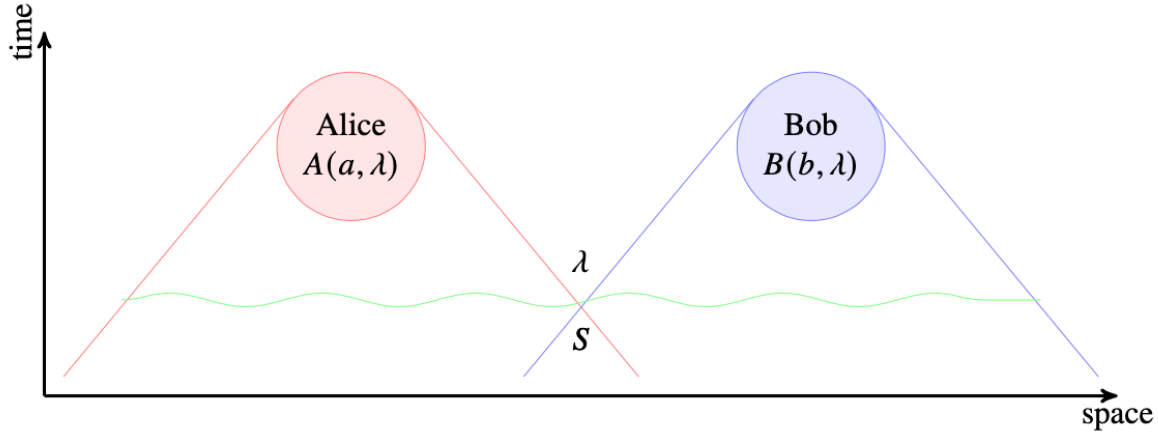
$$|\psi\rangle = \sum_n^N c_n\, |\alpha_n\rangle_A \otimes |\beta_n\rangle_B\,,\ N > 1$$

where $|\psi\rangle$ *must not reduce to* $N = 1$ *with any separate basis transformation in A and B.*

Evertz, https://itp.tugraz.at/~evertz/QM/qm_2017.pdf (20.07.2020)

EPR Paradox

Spacetime configuration considered by EPR



Assumptions

- Locality

  The result of a measurement in one system is unaffected by operations in a distant (spatially separated) system.

- Reality

  A physical quantity is real if it can be predicted without disturbing the system.

Einstein, Rosen, Podolsky https://doi.org/10.1103/PhysRev.47.777

EPR Paradox
Bohm's example of spin ½ particles

$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_A\downarrow_B\rangle_z - |\downarrow_A\uparrow_B\rangle_z\right)$$

If Alice measures $|\downarrow_A\rangle_z$, Bob's result $|\uparrow_B\rangle_z$ can be predicted.
This is also true if Alice and Bob are spatially separated.

Since no information could be exchanged, Bob's result must have been predetermined, but it can not be predicted by QM in Bob's system

$\Rightarrow$ *QM is incomplete*

A possible completion is offered by Hidden Variable (HV) theories.
Local HV theories are a subclass where the hidden variable is localized
→ EPR locality assumption

Wolf, Hornberger (p 138) https://www.yumpu.com/de/document/view/21137009/quantum-information-theory-coqus

EPR Paradox
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$$|\psi\rangle = \frac{1}{\sqrt{2}}\left(|\uparrow_A\downarrow_B\rangle_z - |\downarrow_A\uparrow_B\rangle_z\right)$$

If Alice measures $|\downarrow_A\rangle_z$, Bob's result $|\uparrow_B\rangle_z$ can be predicted.
This is also true if Alice and Bob are spatially separated.

How does this differ from probability theory, e.g. a coin flip?
→ Spin states allow measurements in arbitrary directions.

If Alice measures $|\downarrow_A\rangle_z$, Bob's result in *another direction* $\vec{a}$ can not be predicted!
When Bob measures in $\vec{a}$ (and there are no interactions), the spin would be defined in two directions which is again incompatible with QM as $[\hat{\vec{S}} \cdot \vec{a}, \hat{S}_z] \neq 0 \;\; \forall \, \vec{a} \neq \vec{e}_z$

Wolf, Hornberger (p 138) https://www.yumpu.com/de/document/view/21137009/quantum-information-theory-coqus

Contradiction between EPR assumptions and QM using GHZ States

Three particle spin state: $|\psi\rangle = \frac{1}{\sqrt{2}}(|\uparrow_A \uparrow_B \uparrow_C\rangle_z - |\downarrow_A \downarrow_B \downarrow_C\rangle_z)$

Measurement in *x*: $\{\sigma_x\}_A = \begin{cases} +1 & \text{for } |\uparrow_A\rangle \\ -1 & \text{for } |\downarrow_A\rangle \end{cases}$

$$|\psi\rangle = \frac{1}{2}(|\uparrow_A \uparrow_B \downarrow_C\rangle_x + |\uparrow_A \downarrow_B \uparrow_C\rangle_x + |\downarrow_A \uparrow_B \uparrow_C\rangle_x + |\downarrow_A \downarrow_B \downarrow_C\rangle_x)$$

Product of possible outcomes in *x*: $\{\sigma_x\}_A^I \{\sigma_x\}_B^I \{\sigma_x\}_C^I = -1$

In QM the outcome of $\{\sigma_x\}_A^I$ depends on $\{\sigma_x\}_B^I \{\sigma_x\}_C^I$.

Greenberger, Horne, Zeilinger https://doi.org/10.1119/1.16243
Bell, Gao (p195 ff) https://doi.org/10.1017/CBO9781316219393

Contradiction between EPR assumptions and QM using GHZ States

Measurement in *x* for *A* and *y* for *B*, *C*:

$$|\psi\rangle = \frac{1}{2}(|\uparrow_A\rangle_x \, |\uparrow_B\uparrow_C\rangle_y + |\uparrow_A\rangle_x \, |\downarrow_B\downarrow_C\rangle_y + |\downarrow_A\rangle_x \, |\downarrow_B\uparrow_C\rangle_y + |\downarrow_A\rangle_x \, |\uparrow_B\downarrow_C\rangle_y$$

Product of possible outcomes: $\{\sigma_x\}_A^{II}\{\sigma_y\}_B^{II}\{\sigma_y\}_C^{II} = 1$

Equivalent for the other permutations:

$$\{\sigma_y\}_A^{III}\{\sigma_x\}_B^{III}\{\sigma_y\}_C^{III} = 1$$
$$\{\sigma_y\}_A^{IV}\{\sigma_y\}_B^{IV}\{\sigma_x\}_C^{IV} = 1$$

EPR assumption: $\{\sigma_x\}_A^{I} = \{\sigma_x\}_A^{II}$, etc. (locality)
since results in *A*, may not depend on *B* and *C*

$$(\{\sigma_x\}_A\{\sigma_x\}_B\{\sigma_x\}_C\{\sigma_y\}_A\{\sigma_y\}_B\{\sigma_y\}_C)^2 = -1 \quad ϟ$$

Greenberger, Horne, Zeilinger https://doi.org/10.1119/1.16243
Bell, Gao (p195 ff) https://doi.org/10.1017/CBO9781316219393

Bell Inequality (1964)

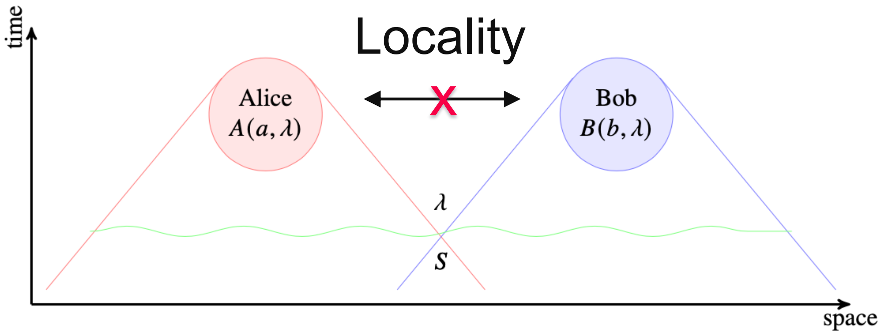
Definition 2.1 (Predetermination). *The result of an individual measurement is predetermined by the parameter λ.*

Definition 2.2 (Locality). *The result of a measurement in one system is unaffected by operations on a distant system.*

$$\forall \vec{b}, A, \vec{a}, \lambda \quad P(A \mid \vec{a}, \vec{b}, S, \lambda) = P(A \mid \vec{a}, S, \lambda)$$

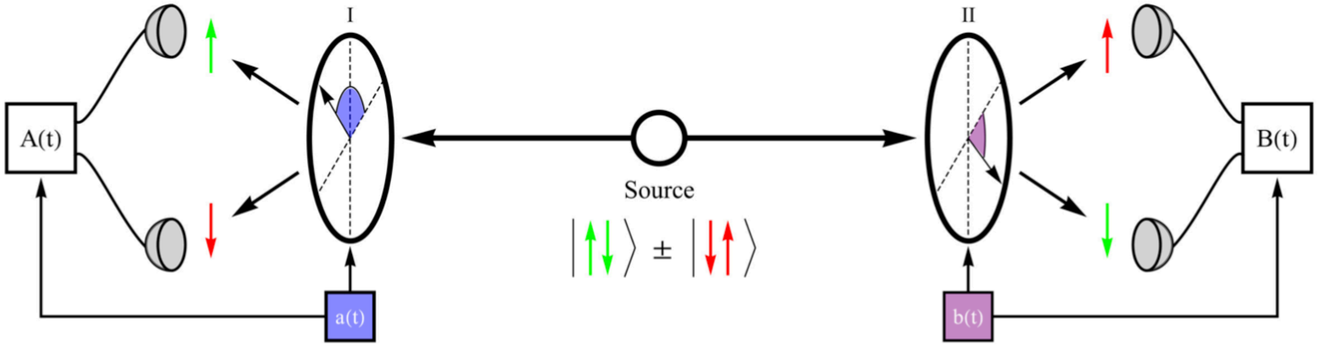
Expectation value for Alice’ result:

$$P(A \mid \vec{a}, \vec{b}, S, \lambda) = \int d\lambda \rho(\lambda) A(\vec{a}, \vec{b}, S, \lambda)$$



Bell https://doi.org/10.1103/PhysicsPhysiqueFizika.1.195,
Wiseman, Cavalcanti (p 122 ff) https://doi.org/10.1007/978-3-319-38987-5

Bell Inequality (1964)



Setup considered by Bell for spin ½ particles

Theorem 2.3 (Bell-1964). *A model satisfying* PREDETERMINATION *and* LOCALITY *must comply with the inequality*

$$|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})| \leq 1 + P(\vec{b},\vec{c}).$$

$$P(\vec{a},\vec{b}) \equiv P(A,B \mid \vec{a},\vec{b},S,\lambda) = \int d\lambda \rho(\lambda) A(\vec{a},\lambda) B(\vec{b},\lambda)$$

Bell https://doi.org/10.1103/PhysicsPhysiqueFizika.1.195,
Wiseman, Cavalcanti (p 122 ff) https://doi.org/10.1007/978-3-319-38987-5
Figure from Bertlmann https://arxiv.org/abs/1605.08081

Violation of the Inequality in Quantum Mechanics

Entangled state to maximize the violation: $|\psi^-\rangle = \frac{1}{\sqrt{2}}(|\uparrow_A\downarrow_B\rangle - |\downarrow_A\uparrow_B\rangle)$

QM expectation value $P(\vec{a},\vec{b}) = \langle\psi^-|\hat{\vec{S}}_A\cdot\vec{a}\otimes\hat{\vec{S}}_B\cdot\vec{b}|\psi^-\rangle = -\vec{a}\cdot\vec{b}$

Chosen settings $\vec{a}\cdot\vec{c}=0 \qquad \vec{a}\cdot\vec{b}=\vec{b}\cdot\vec{c}=\frac{1}{\sqrt{2}}$

→ Violation of the inequality

$$|P(\vec{a},\vec{b}) - P(\vec{a},\vec{c})| - P(\vec{b},\vec{c}) = \sqrt{2} \leq 1 \;\lightning$$

There exist QM phenomena, for which there is no model satisfying these assumptions.

Bell https://doi.org/10.1103/PhysicsPhysiqueFizika.1.195,
Wiseman, Cavalcanti (p 122 ff) https://doi.org/10.1007/978-3-319-38987-5
Bertlmann https://arxiv.org/abs/1605.08081

Loopholes - Overview

- Detection / Efficiency
  - inefficiencies in the experimental setup
- Locality
  - spacelike separation
- Freedom of Choice
  - choice of measurement settings
- Memory
  - outcome of a measurement depends on the previous measurement(s)

A theory satisfying the EPR assumptions can exploit a loophole to produce outcomes which violate the inequality.

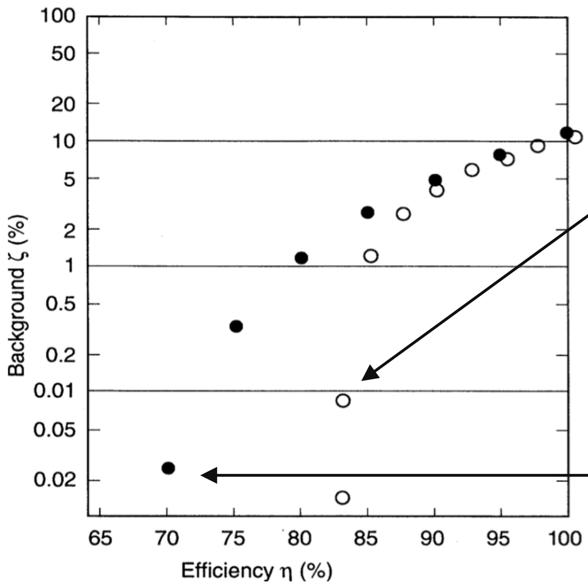
Myrvold et.al. https://plato.stanford.edu/archives/spr2019/entries/bell-theorem/ (20.07.2020)
Giustina et.al. https://doi.org/10.1103/PhysRevLett.115.250401

Detection Loophole

- Count rate normalization → CH Inequality

$$N_{12}(a,b) - N_{12}(a,b') + N_{12}(a',b) + N_{12}(a',b') - N_1(a') - N_2(b) \leq 0$$

- Detector efficiency → CH-Eberhard Inequality



CHSH-Inequality: the detection efficiency must be > 82.8%

CH-E-Inequality: for low backgrounds the detection efficiency must be > 66.7%

Clauser, Horne https://doi.org/10.1103/PhysRevD.10.526
Figure from Eberhard https://doi.org/10.1103/PhysRevA.47.R747

Locality Loophole

- Spacelike separation of the measurement interval

and

- Spacelike separation of setting choice generation

→ No information exchange at the speed of light can produce the measurement outcomes

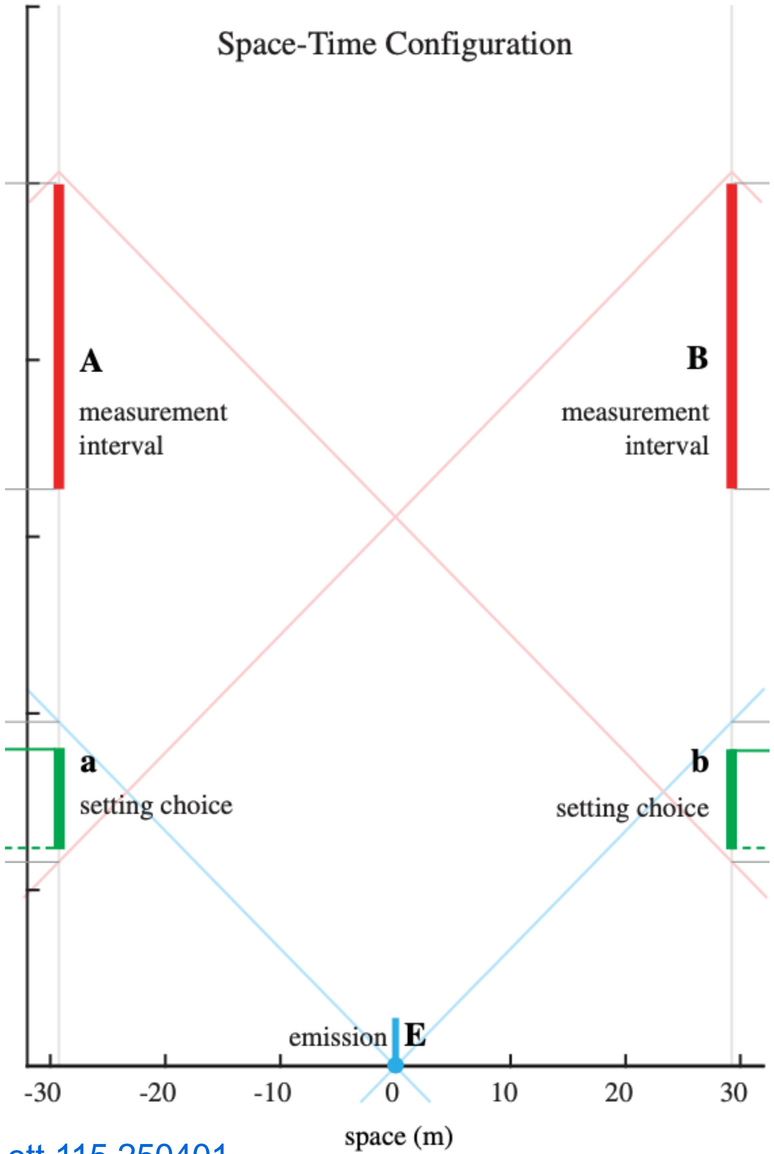
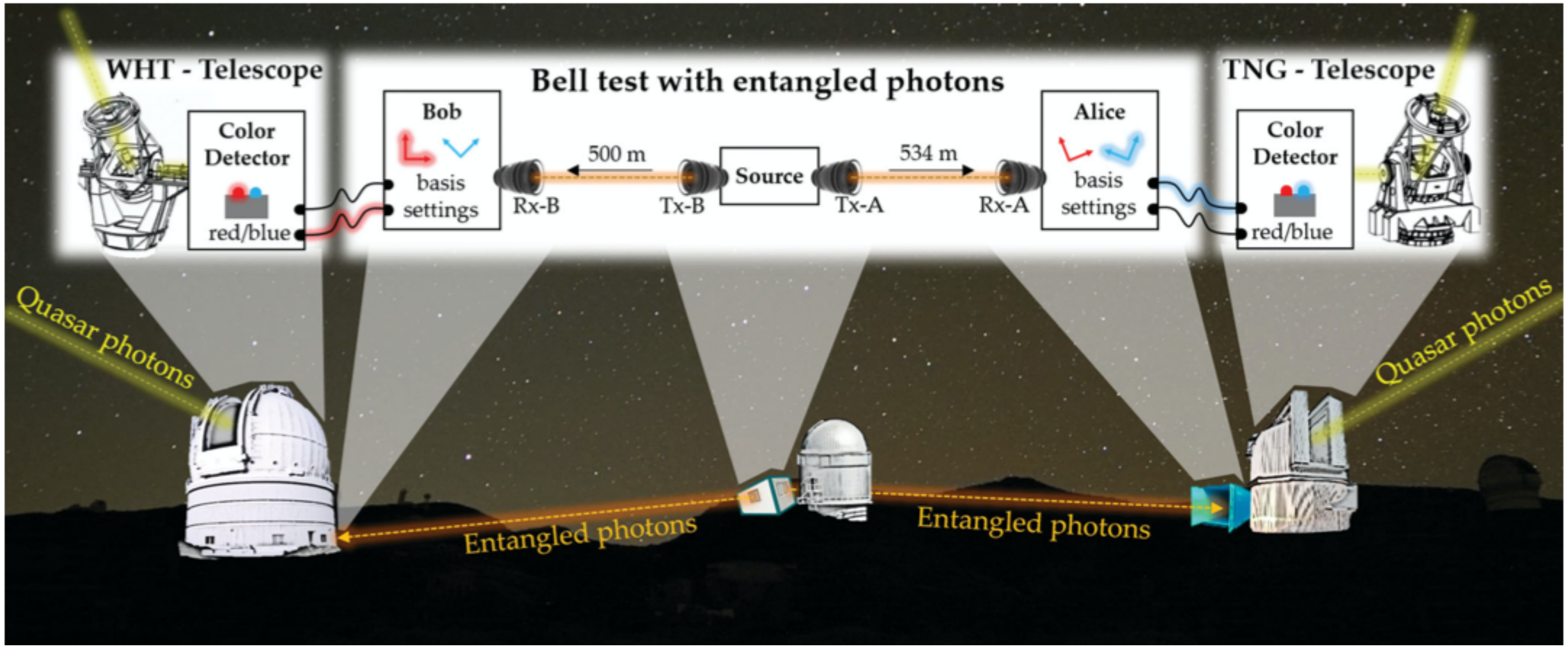


Figure from Giustina et.al. https://doi.org/10.1103/PhysRevLett.115.250401

Freedom of Choice Loophole

Measurement settings are assumed to be free parameters.

„… what really matters is that the hidden variable and the local parameters do not share a common cause." (Larsson)



Larsson https://doi.org/10.1088/1751-8113/47/42/424003
Figure from Rauch et.al. https://doi.org/10.1103/PhysRevLett.121.080403

Overview

Year	Author(s) [Ref.]	Bell Ineq.	System	closed loopholes[a]	violation
1972	Freedman and Clauser [24]	CHSH-F[b]	Ca cascade photons, UV excitation		6.3σ
1973	Holt [34]	CHSH-F[b]	Hg cascade photons, electron-beam excitation		None
1976	Clauser [17]	CHSH-F[b]	Hg cascade photons, electron-beam excitation		4.1σ
1982	Aspect, Dalibard, and Roger [4]	CHSH	Ca cascade photons, laser excitation	locality	5.1σ
1988	Shih and Alley [52]	CHSH-F[b]	parametric down conversion (PDC) photons		3σ
1998	Weihs et al. [55]	CHSH	Type II PDC photons	locality (> 400 m), memory, coincidence	37σ
1998	Tittel et al. [53]	CHSH	PDC photons, Franson Energy-Time entanglement	locality (> 10 km)	$> 16\sigma$
2001	Rowe et al. [45]	CHSH	Induced entanglement of 2 Be ions[c]	detection	8.3σ
2004	Go [27]	CHSH	KB meson decay (flavour entanglement)	detection	$> 3\sigma$
2007	Ursin et al. [54]	CHSH	Type II PDC photons	locality (144 km), coincidence	14σ
2008	Matsukevich et al. [40]	CHSH	Induced entanglement of trapped Yb ion	detection	$> 3.1\sigma$
2009	Ansmann et al. [2]	CHSH	Induced entanglement of Josephson phase qubits	detection	244σ
2012	Hofmann et al. [33]	CHSH	Induced entanglement of 2 trapped Rb atoms	detection	2.1σ
2013	Giustina et al. [25]	CH-E[d]	PDC photons (non maximally entangled)	detection, coincidence	69σ
2015	Hensen et al. [31]	CHSH	electron spin in diamond chip (event ready scheme)	'loophole free'[e]	$p = 0.039$[f]
2015	Giustina et al. [26]	CH-E[d]	PDC photons (non maximally entangled)	'loophole free'[e]	11.5σ
2015	Shalm et al. [51]	CH	PDC photons (non maximally entangled)	'loophole free'[e]	$p \leq 2.3 \cdot 10^{-7}$[f]
2017	Yin et al. [57]	CHSH	PDC photons	locality (1200 km)	4σ
2018	Abellán et al. [1]	multiple	multiple (ions, photons, atoms, solid state)	'loophole free'[eg]	9.3σ to 140σ
2018	Rauch et al. [44]	CHSH	Type 0 PDC photons (non maximally entangled)	'loophole free'[eh]	9.3σ

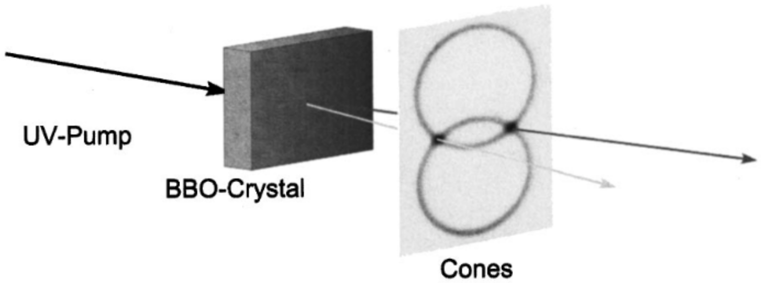
References as listed in the BSc-thesis.

‚Loophole Free‘ Experiments

- Hensen et.al. (2015)
  - nitrogen vacancy in a diamond chip
  - ‚heralding‘ approach
  - 245 trials over 220 h
- Shalm et.al. (2015)
  - parametric down conversion photons
  - 12,127 trials over 30 min
- Giustina et.al. (2015)
  - paramtric down conversion photons
  - approx. 12,300,000 trials over 58.5 min

Hensen et.al. https://doi.org/10.1038/nature15759
Shalm et.al. https://doi.org/10.1103/PhysRevLett.115.250402
Giustina et.al. https://doi.org/10.1103/PhysRevLett.115.250401

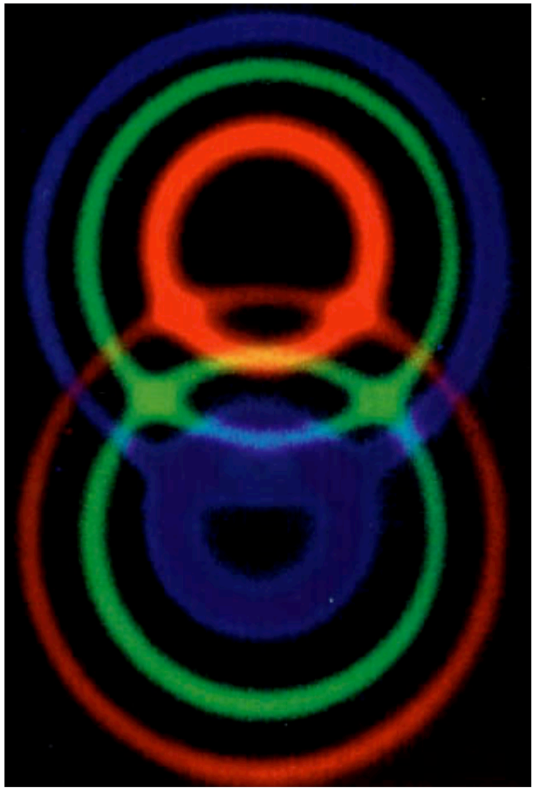
Parametric Down Conversion (PDC)



Decay of a single photon into two photons:

$$\vec{k}_p = \vec{k}_1 + \vec{k}_2,$$
$$\omega_p = \omega_1 + \omega_2.$$

Type I: photons have the same polarization
Type II: photons have different polarization

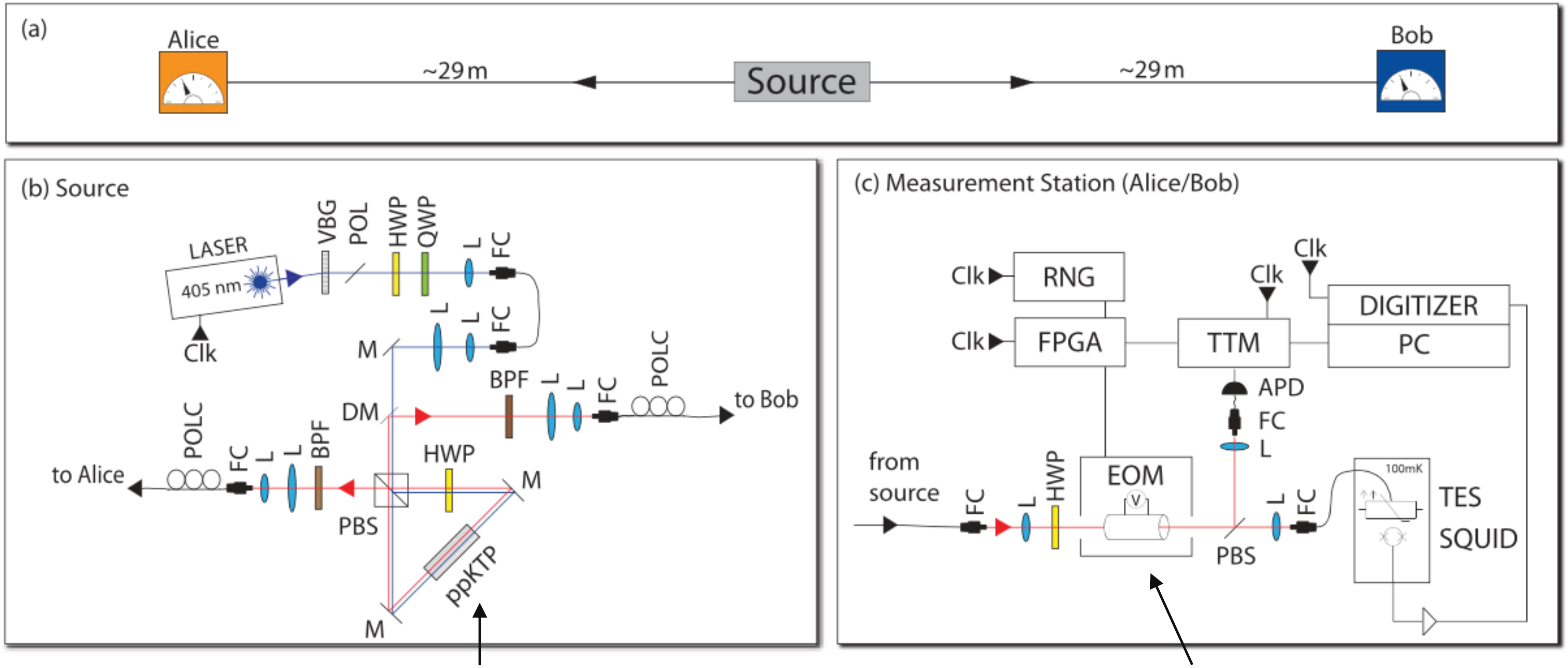


Type II PDC with artificial colour for different wavelengths

BBO … barium borate

Figures from Zeilinger https://doi.org/10.1103/RevModPhys.71.S288

Experiment performed by Giustina et.al. (2015)



ppKTP (periodically poled potassium titanyl phosphate) crystal excited by 405 nm laser produces type II PDC photons

electro optical modulator (Plockels cell) changes the polarization based on the settings generated via random number generator

Figure from Giustina et.al. https://doi.org/10.1103/PhysRevLett.115.250401

Experiment performed by Giustina et.al. (2015)

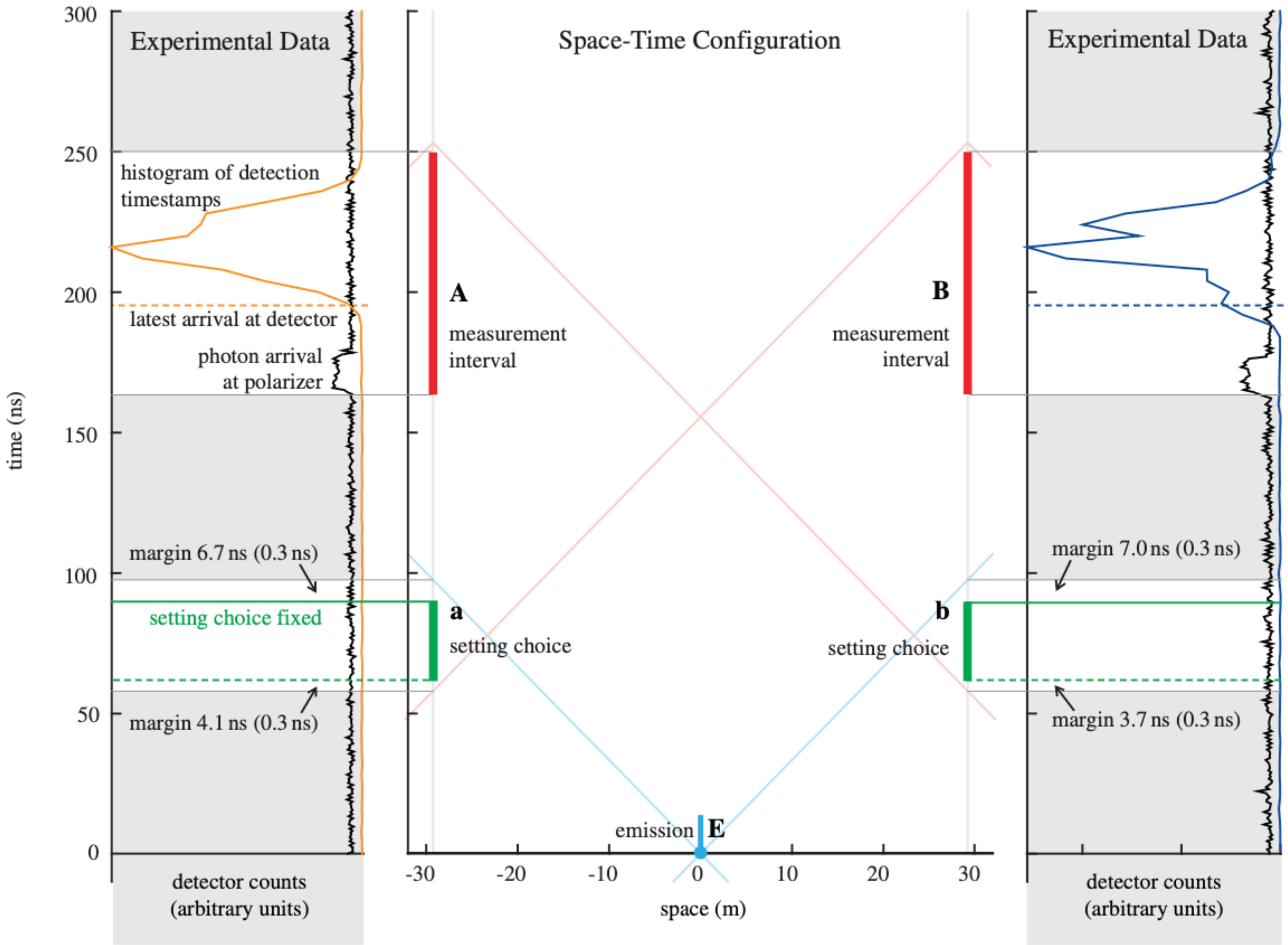


Figure from Giustina et.al. https://doi.org/10.1103/PhysRevLett.115.250401

Experiment performed by Giustina et.al. (2015)

CH-E type inequality $p_{++}(a_1, b_1) - p_{+0}(a_1, b_2) - p_{0+}(a_2, b_1) - p_{++}(a_2, b_2) \leq 0$

Result

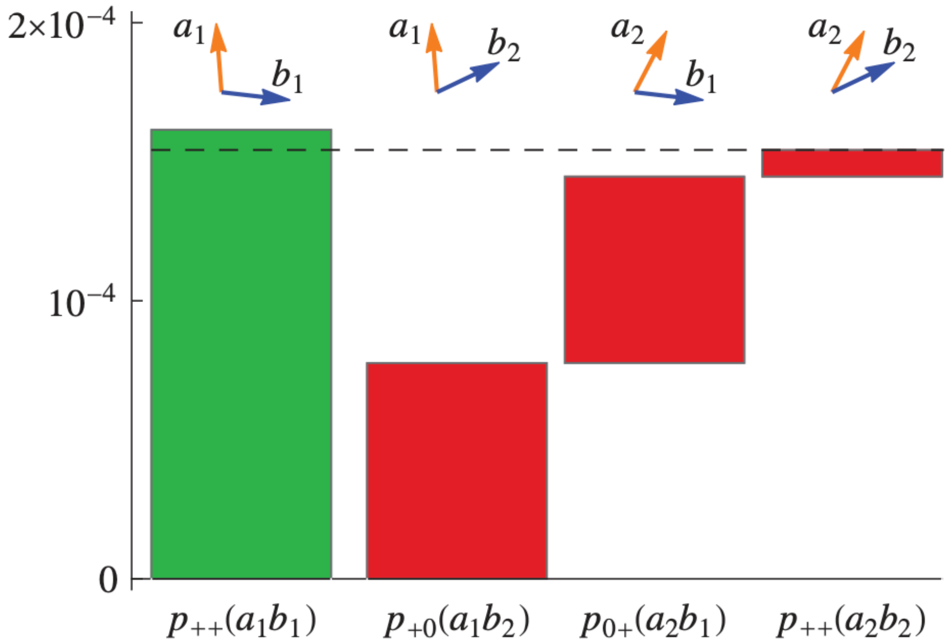


Figure from Giustina et.al. https://doi.org/10.1103/PhysRevLett.115.250401

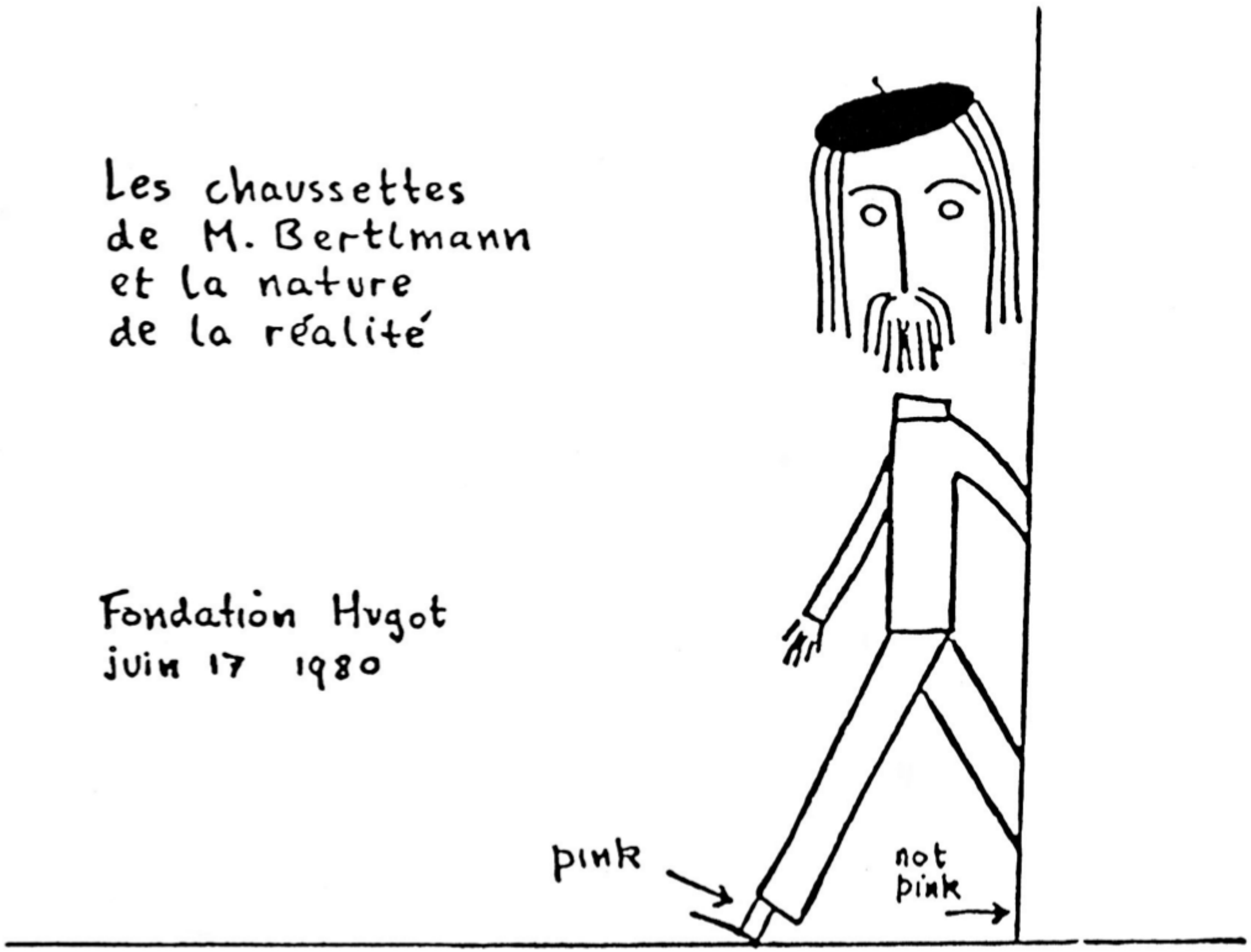
Summary

- Bell‘s Theorem shows a contradiction between the EPR assumptions and Quantum Mechanics.

- Experiments performed show a contradiction between the EPR assumptions and nature.
  - Further loopholes have been proposed where nature conspires against the experiment.
  - Some options (e.g. Superdeterminism) are untestable

- If one believes the experimental evidence, one of the EPR assumptions can be rejected. Different philosophies exist which to reject.

- Bell’s Theorem does not exclude all Hidden Variable Theories as a completion to QM!
  - Functioning HV Theories exist (Bohmian Mechanics).
  - Bell’s Theorem only excludes Local Hidden Variable Theories.


Les chaussettes
de M. Bertlmann
et la nature
de la réalité
Fondation Hugot
juin 17 1980
pink
not
pink