# §.17.1.2.4: Konvergenz der GF zwischen zwei parallelen leitenden Platten

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Here all coordinates are taken as relative values, so z is z/h, z' is z'/h,
r is P/h ; h is the width of the plane condensor, $0 \le z, z' \le 1$.

Rho = $P = \sqrt{(x - \mathrm{xp})^2 + (y - \mathrm{yp})^2}$ , eq.(17.10).

$$P = \sqrt{(x - \mathrm{xp})^2 + (y - \mathrm{yp})^2}$$

$$\sqrt{(\mathrm{x} - \mathrm{xp})^2 + (\mathrm{y} - \mathrm{yp})^2}$$

- **Fig2a: In the integral representation of the Green's function, Eqs.(17.12), (17.13), $\lambda = \tau$.**

```
g0 = Sinh[τ (1 - zg)] Sinh[τ zk] / Sinh[τ]
```

```
Csch[τ] Sinh[(1 - zg) τ] Sinh[zk τ]
```

```
g1 = g0 /. zk → .3
```

```
Csch[τ] Sinh[0.3 τ] Sinh[(1 - zg) τ]
```

```
p39 = Table[ Plot[Evaluate[g1 /. zg → 0.1 k], {τ, 0, 10}, PlotLabel →  .1 k], {k, 3, 9, 3}];
```

```
g2 = g0 /. zg → .3
```

```
Csch[τ] Sinh[0.7 τ] Sinh[zk τ]
```

```
p1 = Plot[Evaluate[g2 /. zk → 0.1`], {τ, 0, 10}, PlotStyle → Dashing[{0.02`, 0.03`}]]
```

```
fig2a = Show[p39, p1, PlotRange → All, PlotLabel → None,
  AxesLabel → {"κ", "g0(κ;z,z'= 0.3)"}, Epilog → {Text["z=0.3", {8, 0.46`}],
    Text["z=0.6", {2.2`, 0.2`}], Text["z=0.9", {4, 0.052`}], Text["z=0.1", {8, 0.13`}]}]
```

- **Modifikation des Integranden, Gl.(17.15)**

```
n1 =  Numerator[TrigToExp[g0 ]] Exp[- τ ] // ExpandAll
```

$$-e^{-zg\,\tau - zk\,\tau} + e^{-2\,\tau + zg\,\tau - zk\,\tau} + e^{-zg\,\tau + zk\,\tau} - e^{-2\,\tau + zg\,\tau + zk\,\tau}$$

```
d1 =  Denominator[TrigToExp[g0 ]] Exp[- τ ] // ExpandAll
```

$$2 - 2\,e^{-2\,\tau}$$

```
nd1 = n1 / d1
```

$$\frac{-e^{-zg\,\tau - zk\,\tau} + e^{-2\,\tau + zg\,\tau - zk\,\tau} + e^{-zg\,\tau + zk\,\tau} - e^{-2\,\tau + zg\,\tau + zk\,\tau}}{2 - 2\,e^{-2\,\tau}}$$

```
n2 = Map[Factor, Numerator[nd1], {1, 3}]
```

$$e^{(-2+zg-zk)\,\tau} + e^{-(zg-zk)\,\tau} - e^{(-2+zg+zk)\,\tau} - e^{-(zg+zk)\,\tau}$$

```
nd2 = n2 / Denominator[nd1]
```

$$\frac{e^{(-2+zg-zk)\,\tau} + e^{-(zg-zk)\,\tau} - e^{(-2+zg+zk)\,\tau} - e^{-(zg+zk)\,\tau}}{2 - 2\,e^{-2\,\tau}}$$

```
gt = nd2 - (1 / 2) (Exp[-τ (zg - zk)] - Exp[-τ (zg + zk)] - Exp[-τ (2 - zg - zk)] )
```

$$\frac{e^{(-2+zg-zk)\,\tau}+e^{-(zg-zk)\,\tau}-e^{(-2+zg+zk)\,\tau}-e^{-(zg+zk)\,\tau}}{2-2\,e^{-2\,\tau}}+\frac{1}{2}\left(e^{-(2-zg-zk)\,\tau}-e^{-(zg-zk)\,\tau}+e^{-(zg+zk)\,\tau}\right)$$

- **Fig2b: In the integral representation of the Green's function, Eq.(27)**

```
gm =
```

$$\frac{-e^{-2\,\tau-zg\,\tau-zk\,\tau}+e^{-2\,\tau+zg\,\tau-zk\,\tau}+e^{-2\,\tau-zg\,\tau+zk\,\tau}-e^{-4\,\tau+zg\,\tau+zk\,\tau}}{2\left(1-e^{-2\,\tau}\right)}$$

$$\frac{-e^{-2\,\tau-zg\,\tau-zk\,\tau}+e^{-2\,\tau+zg\,\tau-zk\,\tau}+e^{-2\,\tau-zg\,\tau+zk\,\tau}-e^{-4\,\tau+zg\,\tau+zk\,\tau}}{2\left(1-e^{-2\,\tau}\right)}$$

```
gm - gt // Together // Simplify
```

0

```
Map[Factor, gm, {3, 4}]
```

$$\frac{e^{(-2+zg-zk)\,\tau}+e^{-(2+zg-zk)\,\tau}-e^{(-4+zg+zk)\,\tau}-e^{-(2+zg+zk)\,\tau}}{2\left(1-e^{-2\,\tau}\right)}$$

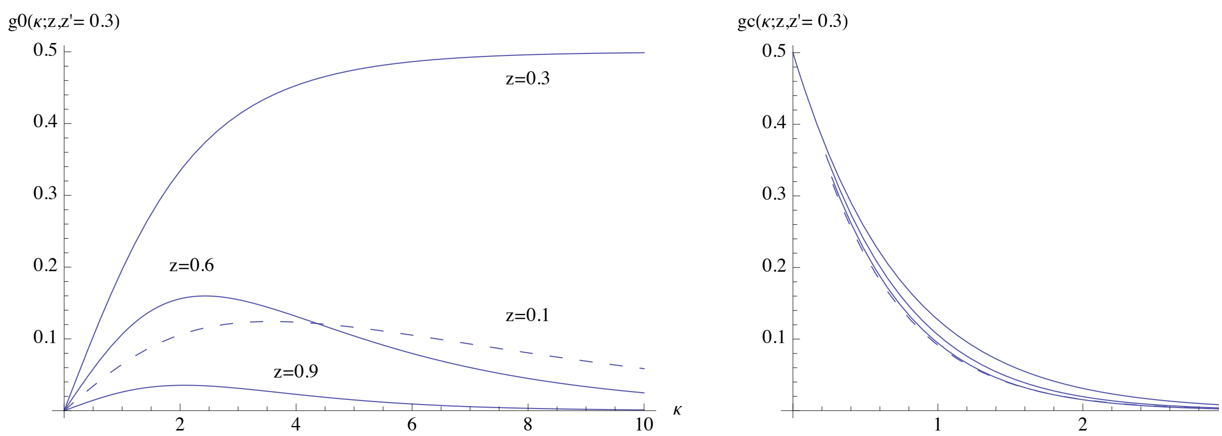
```
gm1 = gm /. zk → .3
```

$$\frac{-e^{-2.3\,\tau-zg\,\tau}+e^{-1.7\,\tau-zg\,\tau}-e^{-3.7\,\tau+zg\,\tau}+e^{-2.3\,\tau+zg\,\tau}}{2\left(1-e^{-2\,\tau}\right)}$$

```
q39 = Table[ Plot[Evaluate[gm1 /. zg → 0.1 k], {τ, 0, 4}, PlotLabel →  .1 k], {k, 3, 9, 3}];

Show[q39, PlotRange → All]

gm2 = gm /. zg → .3
```

$$\frac{-e^{-2.3\,\tau-zk\,\tau}+e^{-1.7\,\tau-zk\,\tau}-e^{-3.7\,\tau+zk\,\tau}+e^{-2.3\,\tau+zk\,\tau}}{2\left(1-e^{-2\,\tau}\right)}$$

```
q1 = Plot[Evaluate[gm2 /. zk → 0.1`], {τ, 0, 4}, PlotStyle → Dashing[{0.02`, 0.03`}]]

fig2b = Show[q39, q1, PlotRange → {0, 0.5`},
  PlotLabel → None, AxesLabel → {"κ", "gc(κ;z,z'= 0.3)"}]

fig2 = Show[GraphicsRow[{fig2a, fig2b}]]
```



## Evaluation of the Sommerfeld integral

```
it = Integrate[BesselJ[0, τ ka] Exp[- τ α],
  {τ, 0, Infinity}, Assumptions → α > 0 && ka > 0]
```

$$\frac{1}{\sqrt{ka^2 + \alpha^2}}$$

```
Clear[P]
```

```
i1 = it l / (4 π) /. {ka → ϰ P, α → zg - zk}
```

$$\frac{l}{4 \pi \sqrt{(zg - zk)^2 + P^2 \varkappa^2}}$$

```
i2 = - it l / (4 π) /. {ka → ϰ P, α → zg + zk}
```

$$-\frac{l}{4 \pi \sqrt{(zg + zk)^2 + P^2 \varkappa^2}}$$

```
i3 = - it l / (4 π) /. {ka → ϰ P, α → 2 h - zg - zk}
```

$$-\frac{l}{4 \pi \sqrt{(2 h - zg - zk)^2 + P^2 \varkappa^2}}$$