Elektrodynamics Exercises WS 2015-16

Problem set 1, 23.10.2015

Exercise 1: Dirichlet Problem

We consider a potential in the region $z \geq 0$, with Dirichlet boundary conditions at $z = 0$ (and at infinity).

a) Write down the Dirichlet Greens function $G(\mathbf{r}, \mathbf{r}')$.

b) Now consider the following: In the plane $z = 0$, let the potential Φ be finite with $\Phi = V$ inside a circle with radius a, and $\Phi = 0$ outside. Derive an integral expression for the potential at a given point P, using cylindrical coordinates (r, ϕ, z).
Note that there is no point charge!

c) Show that the potential along the axis perpendicular to the circle is given by

$$\Phi = V \left(1 - \frac{z}{\sqrt{a^2 + z^2}} \right) \tag{1}$$

d) Determine the surface charge density $\sigma = -\varepsilon_0 E_z(z = 0)$ for $r \gg a$ (only at leading order).

e) Show that for large distances ($r^2 + z^2 \gg a^2$) the potential can be expanded in powers of $(r^2 + z^2)^{-1}$. The leading terms should be given by

$$\Phi = \frac{Va^2}{2} \frac{z}{(r^2 + z^2)^{3/2}} \left[1 - \frac{3a^2}{4(r^2 + z^2)} + \cdots \right] . \tag{2}$$

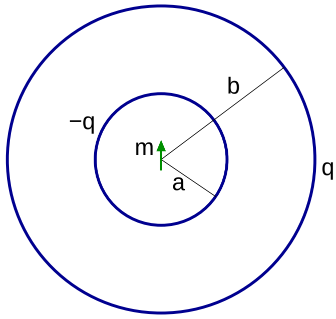
Show that the results of c) and e) are consistent in region of space where both limits are valid.

Exercise 2: Stress tensor

Calculate the Maxwell stress tensor in the plane between two equally charged point charges q.

Show that the force between the two charges can be written as the integral of the flux of the stress tensor through the plane between the charges.

Exercise 3: Angular momentum



Consider two concentric metallic spheres in vacuum, with radii a and b (c.f. figure). The inner sphere carries charge q, the outer one $-q$. In the center is a magnetic dipole with dipole moment $\mathbf{m}$.

a) Calculate the electric field $\mathbf{E}$, the magnetic field $\mathbf{B}$, and the Poynting vector.

b) Caculate the angular momentum $\mathbf{L}_F$ of the electromagnetic field.