Elektrodynamics Exercises WS 2015-16

Problem set 5, 18.12.2015

Problem 1: Rotating dipole

Consider the radiation field of a rotating electric dipole in far-field approximation. The dipole rotates in the xy-plane,

$$\mathbf{d}(t) = d_x \cos(\omega t)\mathbf{e}_x + d_y \sin(\omega t)\mathbf{e}_y .$$

a) Write down $\mathbf{d}(t)$ in complex notation.

b) What is the time-dependent magnetic field $\mathbf{B}(\mathbf{r}, t)$? Write it explicitly in complex and real notation, and use Cartesian coordinates.
Hint: Use electric dipole approximation.

c) Calculate the Poynting vector (in Cartesian coordinates).

d) Calculate the time average of the Poynting vector (in Cartesian coordinates).

e) Show that in the limiting cases in spherical coordinates you will obtain

e1) for circular rotating dipole ($d_x = d_y = d_0$):

$$\bar{\mathbf{S}}_{sym}(\mathbf{r}) = \frac{\mu_0 d_0^2 \omega^4}{16\pi^2 c} \frac{(1+\cos^2\theta)}{2r^2}\mathbf{N}, \quad \mathbf{N} = \mathbf{r}/r$$

e2) for linear dipole ($d_x = d_0$ and $d_y = 0$) (c.f. 16.24 in the lecture notes):

$$\bar{\mathbf{S}}_{lin} = \frac{\mu_0 d_0^2 \omega^4}{16\pi^2 c} \frac{\sin^2\alpha}{2r^2}\mathbf{N} .$$

Here α is angle between dipole (x-direction) and $\mathbf{N}$.

e) Calculate the total flux Φ of $\bar{\mathbf{S}}(\mathbf{r})$ for both circular rotated dipole as well as for the linear dipole and determine the ratio Φ_{sym}/Φ_{lin}

Problem 2: Usage of the multipole expansion

The potential of unknown charge distribution, of infinitesimal volume (at $\mathbf{r} = 0$) is measured on the (imaginary) surface of the sphere with radius R and center at $\mathbf{r} = 0$. According the measurement

$$\Phi(R, \theta, \phi) = a(\cos\theta)^2 . \qquad (1)$$

a) Calculate the potential for arbitrary r, θ and ϕ.
Hint: Φ fulfills Laplace-equation everywhere except of $\mathbf{r} = 0$ and vanishes for $\mathbf{r} \to \infty$. You can expand general solution of the Laplace-equation in spherical functions and use conditions mentioned above. (The presence of charge at $\mathbf{r} = 0$ is signaled by singularity at $\mathbf{r} = 0$). Then determine the coefficients so that Eq. (1) fulfilled. You just need terms with $l = 0, 1, 2$ and $m = 0$.

b) Now consider the same charge distribution surrounded by metallic sphere with radius ρ and center at $\mathbf{r} = 0$. The metallic sphere is grounded, therefore its potential is zero ($\Phi(\rho, \theta, \phi) = 0$). Please calculate potential inside the sphere.
Hint: Expand potential in spherical functions. In this case total potential contains terms which you obtain in (a), plus contribution from the charge from the metallic surface. This contribution fulfills Laplace-equation, but it must be regular at $\mathbf{r} = 0$.