Elektrodynamics Exercises WS 2015-16

Problem set 6, 08.01.2016

Problem 1: Transformation of velocities

Consider two coordinate systems S and S'. S' is moving with velocity β along the x-axis ($c = 1$). A particle is now moving within the system S with velocity $\mathbf{u}$, where the vector of this velocity has an angle θ with the x-axis.

a) Show that the velocities transform in the following way:

$$u' \cos\theta' = \frac{u\cos\theta - \beta}{1 - \beta u \cos\theta} \qquad u' \sin\theta' = \frac{u\sin\theta}{\gamma(1 - \beta u \cos\theta)}$$

b) Photons move at the speed of light, $u = 1$. Show that in that case we have also $u' = 1$. Calculate for this case the transformation of the angle θ.

c) Neutral π-mesons decay into 2 γ rays. Since mesons have no spin, the γ rays are emmited isotropically in the rest frame (S') of the mesons. Show that the angular distribution in the laboratory frame of the γ -rays arising from mesons of velocity $v = \beta c$ is:

$$P(\theta)d\theta = \frac{\sin\theta d\theta}{2\gamma^2(1 - \beta\cos\theta)^2}$$

Follow the steps:

c1) Consider (S) the lab. frame, and (S') the meson's rest frame. Because of isotropical emmision in the rest frame $P(\theta', \phi') = \frac{1}{4\pi}$. Calculate $P(\theta')d\theta'$, by integrating $P(\theta', \phi')$ over $\phi' = 0 \to 2\pi$.

c2) The probability is a conserved quantitiy, i.e. is Lorentz invariant: $P(\theta')d\theta' = P(\theta)d\theta$. Using the transformation of the angle obtained in (a), calculate $d\theta'/d\theta$ and, hence, determine $P(\theta)$.

d) Plot $P(\theta)$ as function of θ. Find the angle, for which the distribution has its maximum as a function of β.

Observe that with increasing velocity, β, the maximum is shifting towards smaller angles. Also, larger the β, the maximum peak is higher. This phenomena is called **beaming**. We can understand that the radiation emmited by a particle at relativistic speeds is not isotropic as compared to when the particle is in the rest frame (where the emmision of radiation is isotropic).

Problem 2: General Lorentz transformation

Determine the Lorentz transformation between two rest frames S and S' that move with respect to each other with velocity $\mathbf{v}$. Note that the velocity $\mathbf{v}$ is not parallel to a coordinate axis. For this purpose, split the vector $\mathbf{r}$ into two components, parallel $\mathbf{r}_{\parallel}$ and perpendicular $\mathbf{r}_{\perp}$ to the velocity. The parallel component is Lorentz transformed, the perpendicular component is unchanged. Write the solution in vector notation $\mathbf{r} = \ldots$ and $t = \ldots$.

Problem 3:Special relativity

A rod of length L_0 (frame S') is parallel to the surface of a photographic plate and moves, with velocity v, in a direction along its length. A light flash of negligible duration, illuminates the plate (frame S) at normal incidence while the rod is in front of it.

a) Since the length is determined by measuring both ends simultaneously, find the dimension of the shadow in the frame of the plate (S).

b) According to an observer in the frame (S'), which moves with the rod find the dimension of shadow.

c) How does the observer in (S') reconcile the observed dimension of the shadow with the known length of the rod?