Elektrodynamics Exercises WS 2015-16

Problem set 7, 22.01.2016

Problem 1: Lorentz transformation of fields I

An infinitely long cylinder with cross section area S carries a homogenous current density $\mathbf{j}$ and a homogenous charge density ρ.

a) Choose a reference frame S' with velocity $\mathbf{v}$, such that the current density in this frame vanishes ($\mathbf{j}' = 0$). Under what conditions is this possible?
Determine the charge density ρ', the electric field $\mathbf{E}'(\mathbf{r}')$ (Hint: Gauss theorem) and the scalar potenital $\phi'(\mathbf{r}')$ in this frame S' (only outside of the cylinder).

b) Using an inverse Lorentz transformation (which is defined by inverting the velocity, $\mathbf{v} \to -\mathbf{v}$), calculate the potentials ϕ and $\mathbf{A}$ from ϕ'. Calculate also the fields $\mathbf{E}$ and $\mathbf{B}$.

c) Calculate the fields $\mathbf{E}$ and $\mathbf{B}$ also directly from their transformation laws (see lecture notes).

Problem 2: Lorentz transformation of fields II

A charged particle with charge q is moving at constant speed $\mathbf{v} = v\ \mathbf{e}_x$ in the laboratory frame S. At time $t = 0$ the particle is at the origin $\mathbf{r} = \mathbf{0}$.

a) Using Coulomb's law, calculate the scalar potential $\phi'(t', \mathbf{r}')$ in the rest frame S' of the particle.

b) Calculate the potentials $\mathbf{A}(t, \mathbf{r})$ and $\phi(t, \mathbf{r})$ in frame S using the Lorentz transformations. Be sure to write the potentials in the unprimed coordinates $(t, \mathbf{r})$).

c) Show that these potentials fulfill the Lorenz gauge.

d) Show furthermore that the potentials can be written in covariant formulations:

$$A_\nu = \frac{q}{c} u_\nu / D, \qquad D = \sqrt{c^{-2}(x^\mu u_\mu)^2 - x^\mu x_\mu}.$$

Here, u_μ is the vector of velocity of the particle, and x_μ the space-time coordinate.

Problem 3: Lorentz invariance

Based on the energy-momentum tensor, find for which value of a, $E^2 + aB^2$ is Lorentz scalar.